

## Economics 336Y – Public Economics

### Problem Set on Clarke-Groves Taxes

1. The pivot tax is positive only if the report of taxpayer changes the total report from negative to positive (including zero) or vice versa. Furthermore the only reports in this case are  $r_i = -1000$  or  $r_i = 1000$ . So in this case the only time one resident can be pivotal is if  $R_i = -1000$  and the resident reports  $r_i = 1000$  (note that  $R_i = 0$  is impossible). So

$$T_i = \begin{cases} 1000 & \text{if } -r_i = R_i = -1000 \\ 0 & \text{otherwise} \end{cases}$$

2. Recall that the only time anyone can affect the outcome is if  $R = -1000$  and  $r = 1000$  (in which case the outcome will switch from not building the centre to building it). So Christina would never lie: if  $R = -1000$  and she reports  $r = 1000$  then she gets the centre (payoff -1000) *and* she pays a tax of 1000, which is worse than telling the truth and getting payoff zero. For Britney, if she reports the truth ( $r = 1000$ ) then she changes the outcome of the vote and so has to pay the pivot tax of \$1000; but she also gets the centre which has a net benefit to her of \$1000. So she is indifferent between telling the truth and underreporting her benefit ( $r = -1000$ ) which would result in the centre not being built. So both types get maximal utility by telling the truth.
3. We have shown that pivot taxes are only paid when there is a tie, with exactly 50 residents in favour of the centre and 50 residents against it. The probability of a tie of exactly 50 demanders out of 100 when demanders and non-demanders are equally likely is about 8 per cent (you can look that up). If there is a tie, then all demander must pay a pivot tax of \$1000. Therefore there the Clarke-Groves tax will result in a budget surplus of \$50,000 in about 8 per cent of cases like this.