University of Toronto
Economics 336Y – Public Economics

Problem Set on Externalities

1. A steel firm produces smoke as a byproduct of production, and smokes production costs of a neighbouring laundry. The cost functions of the two firms are

$$C_s(y_s) = \frac{1}{2}y_s^2$$
$$C_l(y_l, y_s) = \frac{1}{2}y_l^2 + \frac{1}{3}y_s y_l$$

where y_s is steel output and y_l is laundry output.

Assume that both firms sell their output in a competitive market for a price of one. Calculate the level of steel output $\hat{y}_s$ that maximizes the profit of the steel firm; the level of steel output y_s^* that maximizes the joint profits of steel and laundry firms; and the Pigouvian tax on steel output t^* that would cause a private steel firm to choose the efficient output y_s^* as its profit maximizing output.

2. Acid rain pollution is produced from two sources, a power plant and a smelter. Studies show that the marginal cost of reducing acid pollution at the power plant is given by the function

$$MC_1(A_1) = 10 + A_1$$

where A_1 is the units of acid rain reduction (or "abatement") at the power plant; and the marginal cost at the smelter is

$$MC_2(A_2) = 20 + A_2.$$

Suppose that the initial level of pollution was 120 units, but the government has limited pollution to 70 units by selling pollution permits to the two polluters – so that the total amount of abatement is 50 units.

Calculate the total demand for permits to pollute at any permit price p per unit of acid rain. If the permit market is competitive, what is the equilibrium price of a permit? How many permits are bought by the power plant, and how many by the smelter? Draw a graph illustrating your answer.

3. Consider two people, Pollux and Victor. Pollux chooses the level of a damaging activity P, and Victor chooses a level of investment I. The greater the investment I, the greater the damage caused by P. Victor's profits $\pi_v(P, I) = I(2 - I - P)$. Pollux's profits: $\pi_p(P) = P(A - P/2)$ where $2 \geq A \geq 1$.
   (a) What is efficient investment and pollution (I^*, P^*)?
   (b) Suppose that Victor has property rights, and Pollux makes an offer of compensation for pollution which Victor wither accepts or rejects. If the offer is rejected, then no pollution is allowed. How much compensation $C(I, P)$ must Pollux offer Victor to accept a level of pollution P if the investment is at level I?
   (c) Suppose I is chosen before compensation is offered: What is equilibrium investment and pollution $(\hat{I}, \hat{P})$?

4. A steel company is considering building a new plant which could cause external damage to a neighbouring laundry. Let the benefit of the plant to the steel company (in additional profits) be B and the cost of the plant to the laundry (in lost profits) be C. The difficulty is that the only the companies, and not the government, observe B and C. Instead, the government believes that

$$B = \begin{cases} 1 & \text{with 50\% probability} \\ 3 & \text{with 50\% probability} \end{cases}$$

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and

$$C = \begin{cases} 0 & \text{with 50\% probability} \\ 2 & \text{with 50\% probability} \end{cases}$$

(a) If the government was able to learn the values of B and C, what decision rule should it use to determine whether the steel plant should be built or not, if it seeks to maximize aggregate profit of the two companies? What is the probability that the plant gets built, in terms of the above beliefs?

(b) Now suppose the government can observe that $C = 2$, but it is uncertain about B in the way described above. Can the government impose a tax on the steel plant, if it is built, to achieve the efficient outcome of part (a)? If so, what is the tax rate?

(c) Now suppose that the government is uncertain about both B and C, in the way described above. Does there exist a tax (or subsidy) policy that achieves the efficient outcome, in spite of government's uncertainty? Describe it in general terms and justify your answer by appealing to results proved in the lectures. For bonus points, write down the exact tax/subsidy system and prove that it achieves the efficient outcome.