University of Toronto
Economics 336Y – Public Economics

Problem Set #5

1. (a) $MR = A - 2p^* = 0$ implies $p^* = A/2$ and $X = A/2$. $CS = A^*/8$ and $TR = A^2/4$.
   (b) $p = 0$ and $X = A$ yields $CS = A^2/2$.
   (c) Monopolist produces if $A^2/4 > K$ and government if $A^2/2 > K$. The monopolist would never innovate if the government would not. This conclusion might be reversed in an imperfectly competitive industry.

2. (a) The first-best price is $P^* = 1$ implying $Q^* = 8$. Consumer surplus from crossing is
$$\frac{1}{2}(9-1)^2 = 32 > 24$$
   implying the bridge should be built.
   (b) For any fixed cost K, revenue from Q bridge crossings is $(9-Q)Q$ and total costs are $K+Q$. The second-best number of crossing is the largest Q for which
$$(9-Q)Q - Q - K = -Q^2 + 8Q - K \geq 0$$
   Applying quadratic formula, this inequality has a (real) solution if and only if
$$64 - 4K \geq 0$$
   or $K \leq 16$. So there is no break-even bridge toll in part (b), even though the bridge increases welfare.
   (c) The break-even toll when $K = 16$ is $\hat{Q} = 4$ implying $P(\hat{Q}) = 5$. (The loss in welfare, compared to the first-best, is 8.)

3. The demand curve is linear with an intercept at \$12 and a slope of $-\$1$ per 150 skaters. Consumer surplus at $p = 6, D = 900$ is
$$CS = \frac{1}{2} \cdot 6 \cdot 900 = 2700$$
   Profit is
$$\Pi = 2 \cdot 900 = 1800$$
   So the net benefit is \$4500 per day.

4. Assuming that the arena must cover its fixed as well as variable costs from revenues, then the optimal size is $1,200$, with an admission fee of \$4.

5. Since marginal cost is zero and there is excess capacity outside rush hour, it is optimal to set the fare then to $f_n = 0$. At rush hour, a fare of zero would result in excess demand, so fares should be used to ration trips among consumers (why use fares instead of queues, by the way?) If a "peak-load fare" of $f_r = w_r(K) = A - K$ is chosen, then supply equals demand at rush hour. For the last part of ther question, observe that the net benefit to society of subway capacity K is
$$B(K) = \frac{1}{2}K^2 + (A_r - K - c)K + \frac{1}{2}A_n^2$$
   (Draw the graph and use the Harberger triangle formula to verify this). Therefore the optimal capacity sets $B'(K^*) = 0$ or $K^* = A_r - c$, and the optimal fare at rushhour is $f_r = c$. Revenues are $R = cK^*$ which exactly cover the capital costs of the system.

ps-pricing-a.tex
March 20, 2011