University of Toronto
Economics 336Y – Public Economics

Problem Set #5

1. A monopolist has an opportunity to produce a new product. The fixed RD cost is K and the demand for the new product is estimated to be $X = A - p$. The MC is zero.
   (a) What price would be chosen by the monopolist if the project is undertaken?
   (b) What is the socially efficient price and quantity?
   (c) Under what conditions would the monopolist choose not to produce the good, while a benevolent government would do so? Is the converse also possible?

(a) $MR = A - 2p^* = 0$ implies $p^* = A/2$ and $X = A/2$. $CS = A^*/8$ and $TR = A^2/4$.
(b) $p = 0$ and $X = A$ yields $CS = A^2/2$.
(c) Monopolist produces if $A^2/4 > K$ and government if $A^2/2 > K$. The monopolist would never innovate if the government would not. This conclusion might be reversed in an imperfectly competitive industry.

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2. Building a new bridge would entail a fixed cost $K = 24$ and each car on the bridge would entail a marginal cost of $c = 1$. The demand for trips over the life of the bridge is $Q(P) = 9 - P$. (You may imagine that trips and fixed costs are measured in millions.)
   (a) Calculate the first-best optimal price of bridge crossings, and the associated number of crossings. Do you recommend that the bridge be built?
   (b) Suppose that the toll rate must be set so that toll revenues cover fixed plus marginal costs of the bridge. Calculate this second-best toll rate. Do you recommend that the bridge be built?
   (c) What if $K = 16$ instead?

(a) The first-best price is $P^* = 1$ implying $Q^* = 8$. Consumer surplus from crossing is

$$\frac{1}{2}(9-1)^2 = 32 > 24$$

implying the bridge should be built.
(b) For any fixed cost K, revenue from Q bridge crossings is $(9 - Q)Q$ and total costs are $K + Q$. The second-best number of crossing is the largest Q for which

$$(9-Q)Q - Q - K = -Q^2 + 8Q - K \geq 0$$

Applying quadratic formula, this inequality has a (real) solution if and only if

$$64 - 4K \geq 0$$

or $K \leq 16$. So there is no break-even bridge toll in part (b), even though the bridge increases welfare.
(c) The break-even toll when $K = 16$ is $\hat{Q} = 4$ implying $P(\hat{Q}) = 5$. (The loss in welfare, compared to the first-best, is 8.)

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3. The city of Toronto is considering whether to build a new hockey arena. The arena would have a capacity of 900 skaters per day, and the proposed admission fee is $6 per skater per day. The estimated average total cost of the arena is $4 per skater per day.

   Toroto has hired you to conduct a cost-benefit analysis of the project. A study of existing, similar arenas shows that the demand varies wit the price charged as follows:

price per day	skaters per day
$8	600
$10	300
$4	1,200
$6	900
$2	1,500

   (a) If the arena is built as planned, what would be the net benefit per day from the arena? What is total consumer (skater) surplus?
   (b) Given this information, is a 900-skater arena optimal? Explain.

The demand curve is linear with an intercept at $12 and a slope of −$1 per 150 skaters. Consumer surplus at $p = 6, D = 900$ is

$$CS = \frac{1}{2} \cdot 6 \cdot 900 = 2700$$

Profit is

$$\Pi = 2 \cdot 900 = 1800$$

So the net benefit is $4500 per day.

Assuming that the arena must cover its fixed as well as variable costs from revenues, then the optimal size is $1,200$, with an admission fee of $4.

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4. The capacity of a subway system is measured by the maximum number of passengers per direction per hour (ppdph) it can carry. Suppose that the capacity of the TTC subway system is fixed at K ppdph. The capital cost of the subway is cK, and operating costs are zero. The TTC has estimated that the inverse demand curve for total trips T on the subway at rush hour is given by the inverse demand function

$$w_r(T) = A_r - T$$

whereas at non-rush hour times it is given by

$$w_n(T) = A_n - T$$

If $A_r > K > A_n$, what price should be charged for trips on the TTC at rush hour? at non-rush hour times? Now suppose that the TTC has chosen capacity optimally. Will revenues be sufficient to cover the TTC's capital costs?

Since marginal cost is zero and there is excess capacity outside rush hour, it is optimal to set the fare then to $f_n = 0$. At rush hour, a fare of zero would result in excess demand, so fares should be used to ration trips among consumers (why use fares instead of queues, by the way?) If a "peak-load fare" of $f_r = w_r(K) = A - K$ is chosen, then supply equals demand at rush hour. For the last part of ther question, observe that the net benefit to society of subway capacity K is

$$B(K) = \frac{1}{2}K^2 + (A_r - K - c)K + \frac{1}{2}A_n^2$$

(Draw the graph and use the Harberger triangle formula to verify this). Therefore the optimal capacity sets $B'(K^*) = 0$ or $K^* = A_r - c$, and the optimal fare at rushhour is $f_r = c$. Revenues are $R = cK^*$ which exactly cover the capital costs of the system.
