Practice exercises #1

1. For the following economies, find an equation for the set of feasible, Pareto efficient allocations and graph it.

   (a) Society has 100 units of a private good available to share between two consumers with utility functions

   $$u_A(x_A) = \sqrt{10x_A}$$
   $$u_B(x_B) = \sqrt{20x_B}$$

   (b) There are two private goods which are in fixed supplies X and Y, and two consumers A and B with utility functions

   $$u_A(x_A, y_A) = x_A$$
   $$u_B(x_B, y_B) = \min\{x_B, y_B\}$$

2. A consumer supplies labour and buys gin and rum. Suppose that the initial tax on rum is $t_r > 0$, and a positive tax on gin is introduced. Label the change in excess burden in the two markets, and write down the formula for it. Show that the change in EB is negative if t_g is small.

3. A worker's labour supply function is given by

   $$H(w) = 100w$$

   where H is hours worked per year and W the after-tax hourly wage.

   (a) Calculate the worker's Marshallian consumer surplus (or if you prefer "producer surplus") as a function of the wage rate.

   (b) Now suppose that the worker can earn a pre-tax wage of $20 per hour, but must pay a proportional wage income tax of 25 per cent. Calculate the revenue raised by the tax and the Harberger triangle measure of its excess burden. Draw a graph illustrating the results.

   (c) Do you think the assumed labour supply function is realistic? Justify your answer.

4. Let compensated demand for x be a general linear function of price p:

   $$x = a - bp$$

   where a, b are fixed, and let the marginal cost be fixed at c. Calculate an approximate formula the excess burden of a specific tax t on x. (Hint: Draw the graph.) How does the excess burden change with a and b? How does a change in b affect the elasticity of demand for x?

5. A taxpayer has utility function $U(x, h) = \sqrt{x} - h$ where h is hours of labour supply and x is consumption. The taxpayer earns a wage of $4 per hour worked.

   (a) Suppose that the government imposes a proportional tax at rate t on labour income, so that the taxpayer's budget constraint is

   $$x = (1 - t)4h$$

   Solve for the optimal labour supply and consumption (h^*, x^*) as a function of t. What is the revenue raisedby the tax? What is the taxpayer's level of utility, as a function of t?

(b) Suppose that the government imposes a lump-sum tax T so that the taxpayer's budget constraint is

$$x = 4h - T$$

Solve for the optimal labour supply and consumption $(\hat{h}, \hat{x})$ as a function of R in this case. Calculate the value of T (as a function of t) that equivalent for taxpayer utility to the proportional tax of the last part.

(c) Based on your answers, compute the excess burden of any proportional tax rate t. Could you have obtained this answer by integrating above the labour supply curve? Explain why or why not.

6. The market for tires is perfectly competitive, and the demand for tires is given by the function

$$D(p) = \frac{8}{p}$$

while the market supply of tires is given by the function

$$S(p^s) = 2p^s$$

where $p^s = p - t$ is the producer price and t is the excise tax rate. Calculate the equilibrium price of tires when there is no tax, and when the tax is $t = 3$. What do you conclude about the economic incidence of this tax?

7. Firms in a small open economy combine domestic labour (which is supplied inelastically) and imported capital to produce output under constant returns to scale. Studies show that, if K units of capital are employed per worker, the wage received by domestic workers is

$$W(K) = 10K - \frac{1}{2}K^2 - rK$$

where r is the rental price paid per unit of imported capital. Find an expression for the optimal demand for capital $K^*(r)$. Calculate the equilibrium wage rate when there is no tax on imported capital, and the world rental price of capital is $r_w = 4$. Now suppose that foreign capitalists must pay a tax of $t = 2$ on each unit of capital that they rent to firms in this country, and that the revenues generated are divided equally among the workers. Calculate the after-tax wage rate $W(K^*) + tK^*$. Is this a good tax? Explain your answer.