University of Toronto
Economics 336Y – Public Economics

Problem Set on charitable contributions

1. Ahmed and Bob are the only two residents on a street in a new subdivision, who must decide how much to spend on street lighting, a pure public good. Suppose that they have identical preferences for wattage of street lighting G and composite private consumption x_i given by the utility function

$$u_i(G, x_i) = 50 \log G + x_i \qquad (i = A, B)$$

   Street lighting costs 1 unit of private consumption per Watt, and each has the same total income, equal to Y.

   (a) $\sum MRS_i = 2 \times (50/G^{eff}) = 1$ implies $G^{eff} = 100$.
   (b) $MRS_i = 50/G^{eq} = 1$ implies $G^{eq} = 50$.
   (c) The previous answer shows G^{eq} is independent of private incomes. In both cases, however, individual contributions are not uniquely determined: Ahmed might contribute or not.
   (d) We now have $G^{eff} = 500$ and $G^{eq} = 50$. The efficient provision level is proportional to number of citizens, while the equilibrium level is independent of the number of citizens. This is a rather extreme version of the free-rider problem. The model would change if: (i) there were a "warm glow effect" of private contributions; and (ii) if we considered different preferences that caused some (e.g. low-income) consumers to reduce their contributions to zero as the number of contributors became large.

2. B contributes 0 and A chooses G such that

$$\frac{1}{2} 20 G^{-1/2} = 1$$

   So $G = 100$.

3. (a)

$$MRS_i = \frac{x_i}{3C}$$

   Since A and B have identical preferences and the MRS is increasing in private consumption, we know that the unique Nash equilibrium involves both contributing the same amount and setting $MRS_i = 1$. Therefore

$$\frac{70 - \hat{C}/2}{3\hat{C}} = 1$$

   or $\hat{C} = 20$.

   We can check this argument is correct by solving for the best response functions $c_A^*(c_B)$ and $c_B^*(c_A)$: They solve

$$\frac{70 - c_A^*}{3(c_A^* + c_B)} = \frac{70 - c_B^*}{3(c_A + c_B^*)} = 1$$

   so

$$c_A^* = \frac{70}{4} - \frac{3}{4} c_B = \frac{1}{4}\frac{70}{4} + \frac{9}{16} c_A^* = 10$$

   and similarly $c_B^* = 10$ so $\hat{C} = 20$.

   The efficient level solves

$$\frac{x_A}{3C^*} + \frac{x_B}{3C^*} = 1 \implies 3C^* = x_A + x_B = 140 - C^* \implies C^* = 35$$

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(b) Define $\tilde{C} = \hat{C} + 15$ as total provision. Since each pays a tax of 7.5, the Nash equilibrium solves

$$\frac{70 - \hat{C}/2 - 7.5}{3(\hat{C} + 15)} = \frac{70 - \tilde{C}/2}{3\tilde{C}} = 1$$

Therefore $\tilde{C} = 20$: there is no change in total provision.

(c) Since there is no tax for the extra concerts, now the equilibrium solves

$$\frac{70 - \tilde{C}/2 + 7.5}{3\tilde{C}} = 1 \implies \tilde{C} = 22\frac{1}{7}$$

The anonymous benefactor finances some of the concerts which is equivalent to an increase in community income. So concerts rise somewhat, as concerts are a normal good.