EC2020-Fall 2011
Problem Set 3

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When you write up your answers, your goal should be to (1) be correct, and (2) convince your reader that your answer is correct. **Please STAPLE pages together so that we do not lose them.**

(This problem set updated: 17 August 2011).

Problems:

1. Solve the following optimization problem:

$$\begin{aligned} \max \quad & (x_1 x_2)^{1/2} \\ \text{s.t.} \quad & 2x_1 + x_2 \leq 1 \\ & x_1 + 2x_2 \leq 1 \end{aligned}$$

   In particular,

   (a) Write out the Kuhn-Tucker conditions. Let λ be the multiplier for the first constraint, and let ϕ be the multiplier for the second constraint.

   (b) Four cases are possible, neither constraint binds, one constraint binds, or both constraints bind. Determine whether or not it is possible to satisfy the conditions in each case, and if it is possible, find the solution.

   (c) Present an alternative graphical solution.

   (d) Show that $zD^2uz \leq 0$ for all $z \in R^2$.

   (e) Show that the D^2u satisfies the relevant determinant based test of Theorem M.D.2.

   (f) Show that u is quasi-concave by showing that $zD^2uz \leq 0$ for all $z \in R^2$ with $z\nabla u(x) = 0$.(Do not make reference to your work above here – that defeats the point of the exercise.)

   (g) Show that u is quasi-concave using the determinant based tests based on the bordered Hessian.

   (h) Show that u is concave by showing that the characteristic roots of D^2u are all non-positive

2. Prove that if $u(x)$ and $x(p,w)$ are continuous functions then $V(p,w) = u(x(p,w))$ must also be a continuous function.

3. Suppose that f is a discontinuous function from $(0,1)$ to $(0,1)$. Show that f is not concave.