

EC2020-Fall 2011  
Problem Set 5

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When you write up your answers, your goal should be to (1) be correct, and (2) convince your reader that your answer is correct. **Please STAPLE pages together so that we do not lose them.**

*(This problem set updated: 17 August 2011).*

**Problems:**

MWG Chapter 4: B.1, B.2

MWG Chapter 5: B.1-3, 5.C.1, 5.D.2

MT.4.1 Consider a set of  $I$  consumers indexed by  $i$ , each of whom consume two goods  $x$  and  $y$  and have income  $w_i$ . Suppose each consumer has a utility function  $u_i(x,y) = x^\alpha y^{1-\alpha}$  for some  $\alpha \in (0,1)$ .

- Verify that  $u_i$  represents homothetic preferences.
- Suppose that the income distribution for the economy is given by  $(w_1, \dots, w_I)$ . Find aggregate demand. Does this demand vary with the wealth distribution?
- Suppose that we give income  $W = \sum_I w_i$  to consumer 1. Find consumer 1's demand functions.
- Suppose the planner a welfare function given by  $W = \sum u_i$ . Does  $W$  depend on the distribution of wealth in the economy?
- Using the planner's welfare function, calculate the compensating variation of a change in the price of  $x$ . Calculate the sum over all  $i$  of the individual compensating variations for the same price change.

MT.5.1 Prove each of the seven parts of proposition 5.C.1. Provide three different proofs for Hotelling's lemma. For part (vi) you can skip the proof that  $\pi$  is differentiable.

MT.5.2 Consider a consumer with preferences over two goods,  $x$  and  $y$  and  $u(x,y) = y + x^{\frac{1}{2}}$ . Let  $p_x$  and  $p_y$  denote the prices of the two goods. Ignore corner solutions in this problem.

- Evaluate  $\frac{\partial}{\partial w} x(p,w)$  and  $\frac{\partial}{\partial w} y(p,w)$
- Evaluate CV and EV for a change in  $p_x$
- Evaluate CV and EV for a change in  $p_y$

MT.5.3 Prove that the lagrange multiplier in the for the firm's cost minimization problem is equal to the marginal cost of output. That is,  $\lambda = \frac{\partial c}{\partial q}$ .