EC313-Fall 2011
Problem Set 6

(Updated 3 November 2011)

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When you write up your answers, your goals should be to (1) be correct, and (2) convince your reader that your answer is correct. It is always helpful if your work is legible and if all steps are presented, possibly with a line of explanation.

In the case of empirical exercises, your goal should be to provide enough information to allow a reader to replicate your answer. This requires a description of data and data sources as well as a description of your analysis of the data.

Answers which do not achieve these goals will not be awarded full credit.

Problems

1. Consider a fishery with N fishers. Let

$$\begin{aligned} K &= \text{stock of fish} \\ x_i &= \text{effort by fisher } i. \\ \left(\sum_{j=1}^{N} x_j\right)^{\alpha} K^{1-\alpha} &= \text{harvest, } \alpha < 1. \\ w &= \text{price of effort.} \end{aligned}$$

   This exercise asks you to characterize market and rent maximizing exploitation behavior.

   (a) Find the first order conditions for x_i^1 under open access. To do this assume symmetry, so that $X = \sum_{j=1}^{N} x_j$ and $x_j = X/N$. Make this substitution in the first order conditions and take the limit as $N \to \infty$.

   (b) Let $N = 1$ and find the first order conditions.

   (c) Explain the differences between the two sets of FOC's. Draw a graph like those that Gordon draws to illustrate both sets of FOC's.

2. Let Y_0 and E_0 denote world income and emissions of CO_2. Suppose that $Y_0 = 1$ and that the world consists of two countries, A and B that each account for half of emissions and income.

   Suppose that the relationship between income and mitigation is given by

$$\Lambda_i = \frac{2}{3}\mu_i^3$$

   where $\mu_i E_i$ is the reduction in emissions in country i and $\Lambda_i Y_i$ is the cost of this reduction.

   We would like to accomplish a reduction of αE_0 in world emissions, for $0 < \alpha < 1$.

   (a) Calculate the cost of this reduction if each country reduces its emissions by the same amount.

(b) Calculate the cost of this reduction if the entire reduction is accomplished by country A.

(c) Calculate the 'participation multiplier', that is, the number by which we must multiply the cost in part 1 to get the cost in part 2.

(d) The Kyoto protocol required emissions reductions from only a fraction of the world. Briefly explain why this problem suggests that this is (or is not) a good idea.

3. Current world income is about 63 trillion dollars. Consider the following three possible growth paths.

(a) World income grows at 1.5% forever. This path is the mitigation path – we magically solve the problem of warming at time 0 and live happily ever after. The discounted present value of world income on this path is

$$W_1 = \sum_{t=0}^{\infty} \left(\frac{1}{1+r}\right)^t (1.015)^t Y_0$$

(b) World income grows at 1.5% a year for $t = 1,..,49$ and at 0.5% per year thereafter. This is a stylized description of the path suggested by Dell et al.'s analysis in which warming stops growth in half the world. The discounted present value of world income on this path is

$$W_2 = \sum_{t=0}^{49} \left(\frac{1}{1+r}\right)^t (1.015)^t Y_0 + \left(\frac{1}{1+r}\right)^{50} \sum_{t=0}^{\infty} \left(\frac{1}{1+r}\right)^t (1.005)^t [(1.015)^{50} Y_0]$$

(c) World income grows at 1.5% a year forever, but in 100 years is subject to a 5% decrease. This corresponds (approximately) to the case Nordhaus analyzes: after it warms up, productivity drops, but growth continues largely unharmed. The discounted present value of world income on this path is

$$W_3 = \sum_{t=0}^{99} \left(\frac{1}{1+r}\right)^t (1.015)^t Y_0 + \left(\frac{1}{1+r}\right)^{100} \sum_{t=0}^{\infty} \left(\frac{1}{1+r}\right)^t (1.015)^t [0.95(1.015)^{100} Y_0]$$

The value of magical solution to the mitigation problem in W_1 is the difference between W_1 and W_2 or W_3. (Hint: recall that $\delta^t \gamma^t = (\delta\gamma)^t$)

(a) Evaluate $W_1 - W_2$ for $r = 2\%$ and $r = 5.5\%$.

(b) Evaluate $W_1 - W_3$ for $r = 2\%$ and $r = 5.5\%$.

(c) What is Nordhaus' estimate of the value of the carbon free energy? To which of the calculations above does this value most closely correspond?

(d) What do the calculations you performed here suggest about the role of the discount rate and the effect of climate on growth in evaluating mitigation policy?