University of Toronto

Faculty of Arts and Science
December Examinations 2007
EC0314F - Turner
Duration - 2hours

Examination Aids: No notes or books are allowed, but you may use a calculator. Please turn your cell-phones off.

There is an automatic 10 point penalty for anyone working on their exam after the end of the exam is announced.

The exam is worth 100 points in all.

When you write up your answers, your goal should be to (1) be correct, and (2) convince your reader that your answer is correct. Answers which do not achieve these goals will not be awarded full credit. To accomplish the second objective, it is helpful if your work is legible and if all steps are presented, possibly with a line of explanation.

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1. The southeastern United States is currently experiencing a severe drought which is leading to conflict over water supplies. There are three parties with claims on the water for a particular watershed. Georgia, Florida, and dolphins. More specifically, Georgia is at the headwaters of the drainage (and so sees the surface water before the other parties), the dolphins are in between Florida and Georgia (and their claim to a certain amount of water is guaranteed by the Endangered Species Act), and Florida is at the bottom of the drainage, and so gets the water after it has flowed past the other users. The dolphins don't use up any of the water, they just swim in it.

   Let Georgia's demand for water be given by $D_g(p) = 10 - p$. Let Florida's demand for water be given by $D_f(p) = 12 - p$. The Dolphins are guaranteed 5 units of water by the Endangered Species Act.

   (a) (10) Suppose the supply of water is $W = 30$.

      i. Draw a graph showing the demand for water by all three parties and the open access equilibrium.
      ii. In the open access equilibrium, what is the cost of protecting the dolphins?
      iii. Draw a similar graph for a competitive market equilibrium in which the surface water is owned by many small agents.
      iv. In the market equilibrium, what is the cost of protecting the dolphins?
      v. What is the welfare loss associated with open access as opposed to a market equilibrium in this case?

   (b) (15) Now suppose that the supply of water is $W = 12$.

      i. Draw a graph showing the demand for water by all three parties and the open access equilibrium.
      ii. In the open access equilibrium, what is the cost of protecting the dolphins?
      iii. Draw a similar graph for a competitive market equilibrium in which the surface water is owned by many small agents.
      iv. In the market equilibrium, what is the cost of protecting the dolphins?
      v. What is the welfare loss associated with open access as opposed to a market equilibrium in this case?

   Continued next page.

2. Consider a small two period mine operated by a profit maximizing owner. Let interest rates, demand, extraction costs, and initial stock be given by:

$$\begin{aligned} r &\sim \text{interest rate} \\ p(q_t) &= 10 \\ c &= 2 + q_t^2 \\ S_0 &= 6 \end{aligned}$$

Suppose that firms in the industry have perfect foresight and maximize the discount present value of profit.

(a) (15) *Using the method of Lagrange*, solve for the level of extraction in periods 1 and 2.

(b) (5) At what rate does profit of the marginal unit of stock change over time?

(c) (5) Verify that the present value of the rent associated with each period's last unit of extraction is constant across periods.

3. Suppose that the growth equation for a stock of fish is

$$F(X) = r\left(X - X^3\right)$$

and the harvesting technology is

$$H = qXE.$$

Let c denote the cost of effort. To ease notation, suppose that the price of harvested fish is 1.

(a) (10) Solve for the steady state open access equilibrium level of stock, harvest, and effort.

(b) (10) Find the first order conditions that characterize the steady state private property equilibrium level of stock, harvest, and effort.

(c) (5) Explain why an open access fishery dissipates all rents in the fishery. (A good answer requires only a few sentences)

4. Consider a forest characterized by;

$$\begin{aligned} p &= \text{price per board foot of lumber} \\ c &= \text{cost of harvesting per board foot} \\ d &= \text{cost to replant the forest} \\ V(t) &= \text{growth equation for trees} \end{aligned}$$

Suppose that $V(t) = \frac{t+k}{t+k+1}$, for $k > 0$.

(a) (10) Find the expression for the present value of the forest conditional on rotation time I.

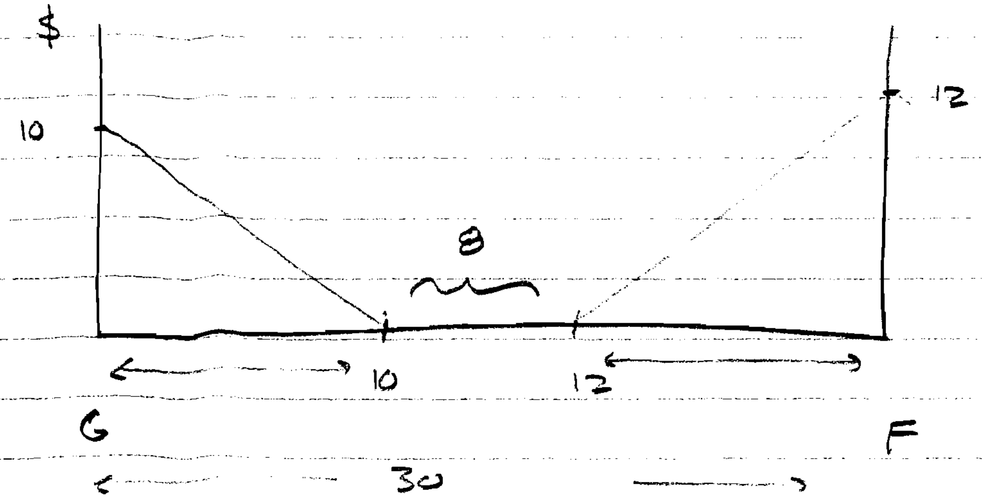
(b) (10) Find the first order condition that determines the rotation time that maximizes the discount present value of the forest.

(c) (5) How does the growth function of trees change as k changes?

2.

(a)

(i)



(ii) THERE ARE 8 UNITS OF SURPLUS WATER, 3 MORE THAN THE DOLPHINS NEED.
THE COST OF PROVIDING THE DOLPHINS WITH 5 UNITS OF WATER IS ZERO.

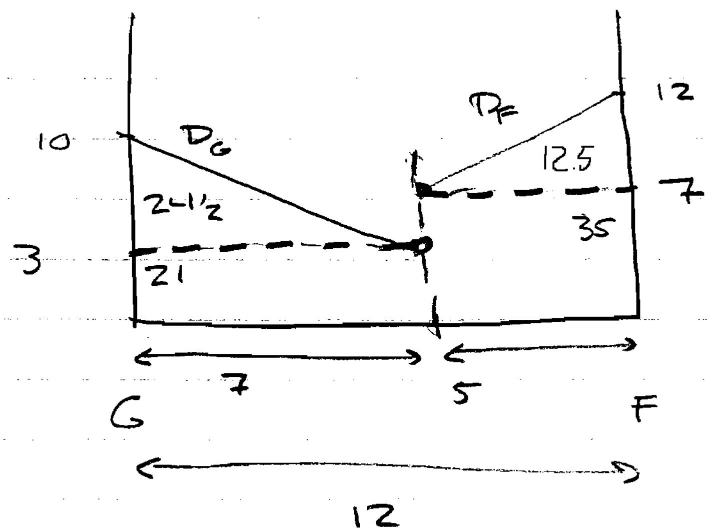
(iii) THE MARKET PRICE OF WATER IS ZERO. MARKET EQUIL AND OPEN ACCESS COINCIDE.

(iv) THERE IS NO COST OF PROTECTING DOLPHINS SINCE THERE IS SURPLUS WATER.

(v) THE TWO OUTCOMES ARE THE SAME. THERE IS NO WELFARE LOSS.

[b]

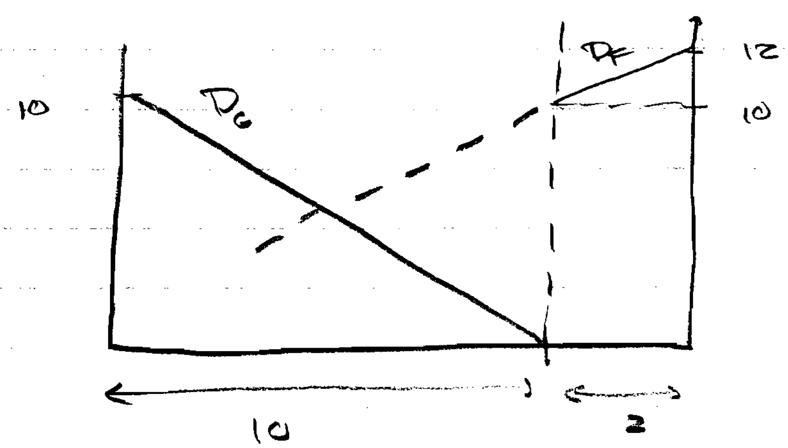
(i)



$$SS = 35 + 12.5 + 21 + 24\tfrac{1}{2} = 93$$

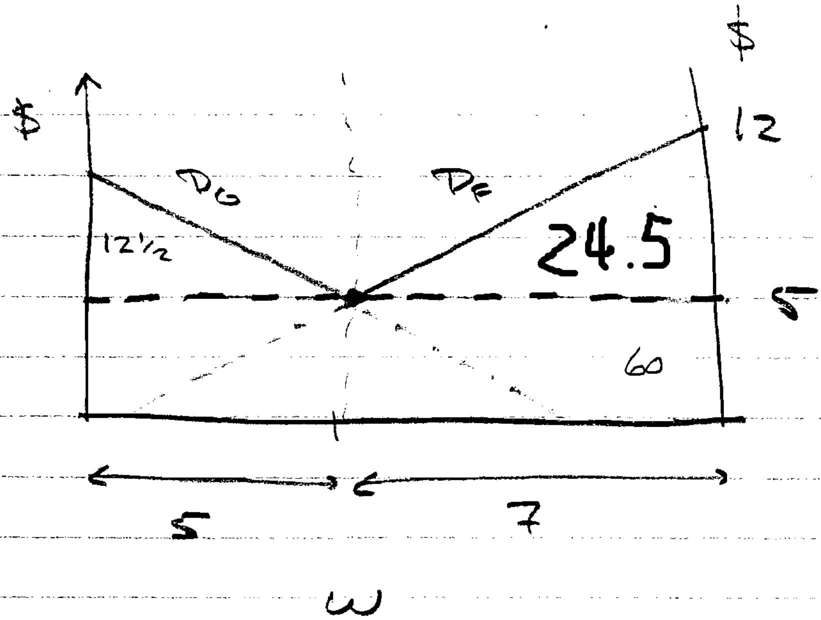
(ii) IN OPEN ACCESS, GEORGIA USES WATER UNTIL ONLY THE 5 UNITS IT MUST LEAVE FOR THE DOLPHINS REMAIN. FLORIDA GETS THE REST.

IF WE REMOVED THE REQUIREMENT THAT GEORGIA LEAVE 5 UNITS, OPEN ACCESS WOULD LOOK LIKE THIS:



IT IS EASY TO SEE THAT CONSUMER'S SURPLUS IS LARGER WHEN THE DOLPHINS ARE ENTITLED TO 5 UNITS THAN WHEN THEY ARE NOT ⟹ PROTECTING THE DOLPHINS DOESN'T COST ANYTHING AT ALL. ⟶

[b] (iii)



$$\begin{array}{r} S.S. = \\ 60 \\ + 24\tfrac{1}{2} \\ + 12\tfrac{1}{2} \\ \hline 97 \end{array}$$

IN A MARKET EQUIL, FLORIDA GETS 7, GEORGIA GETS 5, AND THE PRICE IS 5.

(iv) AGAIN THERE IS NO COST TO PROTECTING DOLPHINS SINCE THE ALLOCATION THAT MAXIMIZES SURPLUS LEAVES 5 UNITS FOR THE DOLPHINS.

(v) WE NEED TO CALCULATE THE CHANGE IN SURPLUS BETWEEN PARTS (i) AND (iii). THIS DIFFERENCE IS 97 - 93 = 4 \$, OR ABOUT 4%.

(a) $$\max_{q_1 q_2} \left[10q_1 - (2+q_1^2)\right] + \frac{1}{1+r}\left[10q_2 - 2q_2^2\right]$$

S.T. $$q_1 + q_2 = 6$$

$$\mathcal{L} = \left[10q_1 - (2+q_1^2)\right] + \frac{1}{1+r}\left[10q_2 - 2q_2^2\right] + \lambda(6 - q_1 - q_2)$$

$$\frac{\partial \mathcal{L}}{\partial q_1} = 0 \Rightarrow 10 - 2q_1 - \lambda = 0 \quad (1)$$

$$\frac{\partial \mathcal{L}}{\partial q_2} = 0 \Rightarrow \frac{1}{1+r}\left[10 - 2q_2\right] - \lambda = 0 \quad (2)$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = 0 \Rightarrow q_1 + q_2 = 6 \quad (3)$$

(1) + (2) $\Rightarrow$ $$10 - 2q_1 = \frac{1}{1+r}\left[10 - 2q_2\right] \quad (4)$$

(4) + (3) $\Rightarrow$ $$10 - 2(6 - q_2) = \frac{1}{1+r}\left[10 - 2q_2\right]$$

$$10 - 12 + 2q_2 = \frac{10}{1+r} - \frac{2}{1+r}q_2$$

$$-2 + 2\left[q_2 + \frac{1}{1+r}q_2\right] = \frac{10}{1+r}$$

$$q_2\left[\frac{2+r}{1+r}\right] = \left[\frac{10}{1+r} + 2\right]\frac{1}{2}$$

$$q_2 = \left[\frac{5}{1+r} + 1\right]\left[\frac{1+r}{2+r}\right]$$

$$\boxed{q_2^* = \left[\frac{6+r}{2+r}\right]} \quad (5)$$

$\rightarrow$

(5) → (3) $\Rightarrow$ $q_1^* + \frac{6+r}{2+r} = 6$

$$\Rightarrow \boxed{q_1^* = 6 - \frac{6+r}{2+r}}$$

FROM (4) WE SEE THAT

$$10 - 2q_1 = \frac{1}{1+r}(10 - 2q_2)$$

BUT THIS IS JUST

$$\frac{\partial \pi_1}{\partial q_1} = \frac{1}{1+r} \frac{\partial \pi_2}{\partial q_2}$$

SO PROFIT FROM MARGINAL UNIT INCREASES AT RATE $1+r$.

AGAIN, BY INSPECTION OF (4) WE SEE THAT P.V. OF PROFIT FROM MARGINAL UNIT OF STOCK IS CONSTANT.

A FISHERY HAS GROWTH EQUATION

$$F(x) = k(x - x^3)$$

AND HARVESTING TECHNOLOGY

$$H = q\,x\,E \qquad (0)$$

PRICE OF FISH IS 1, COST OF EFFORT IS c.

(a) IN S.S. WE HAVE

$$H = F$$

$$\Rightarrow \quad q\,x\,E = k\,x(1 - x^2)$$

$$\Rightarrow \quad qE = k(1 - x^2) \qquad (1)$$

IN OPEN ACCESS WE HAVE

$$TR = TC$$

$$\Rightarrow \quad H = cE$$

$$\Rightarrow \quad q\,x\,E = cE$$

$$\Rightarrow \quad \boxed{x_{OA} = c/q} \qquad (2)$$

$$(2) \to (1) \quad \Rightarrow \quad qE = k - k\left(\frac{c}{q}\right)^2$$

$$\Rightarrow \quad E_{OA} = \frac{1}{q}\left(k - k\left(c/q\right)^2\right)$$

$$\Rightarrow \quad \boxed{E_{OA} = \frac{k}{q}\left(1 - \left(\frac{c}{q}\right)^2\right)} \qquad (3)$$

$$(2) + (3) \to (0) \quad \Rightarrow \quad \boxed{H_{OA} = q\left(\frac{c}{q}\right)\left(\frac{k}{q}\left(1 - \left(\frac{c}{q}\right)^2\right)\right)}$$

(b) IN PRIVATE PROPERTY WE HAVE

$$\text{MAX } \pi$$
$$\text{S.T. } H = F$$

$$\Rightarrow \quad \text{MAX } qXE - cE$$
$$\text{S.T. } qXE = kx(1-x^2)$$

$$\mathcal{L} = (qXE - cE) + \lambda\left(k(x-x^3) - qXE\right)$$

$$\frac{\partial \mathcal{L}}{\partial E} = 0 \Rightarrow qX - c - \lambda qX = 0 \qquad (1)$$

$$\frac{\partial \mathcal{L}}{\partial x} = 0 \Rightarrow qE + \lambda\left(k(1-3x^2) - qE\right) = 0 \qquad (2)$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = 0 \Rightarrow qE = k(1-x^2) \qquad (3)$$

(c) IN AN OPEN ACCESS FISHERY EACH UNIT OF EFFORT, INCLUDING THE MARGINAL UNIT, IS PAID ITS AVERAGE REVENUE PRODUCT.

THIS MEANS THAT EFFORT IS DRAWN INTO THE FISHERY UNTIL

$$ARP(E) = MC(E)$$

MULTIPLY BY E WE HAVE $TR = TC$, or ALL FISHERY RENTS DISSIPATED.

(a) $$W = \left[e^{-rI}(p-c)V(I) - d\right] + e^{-rI}\left[e^{-rI}(p-c)V(I) - d\right]$$
$$+ e^{-r2I}\left[\qquad \right]$$
$$\vdots$$
$$+ e^{-rkI}\left[\qquad \right]$$
$$\vdots$$

$$\Rightarrow \quad W = \left[e^{-rI}(p-c)V(I) - d\right] + e^{-rI} W$$

$$\Rightarrow \quad W = \frac{1}{1-e^{-rI}}\left[e^{-rI}(p-c)V(I) - d\right]$$

(b) THE FIRST ORDER CONDITION IS

$$\frac{dW}{dI} = 0$$

$$\frac{dW}{dI} = \frac{d}{dI}\left[\frac{1}{1-e^{-rI}}\left[e^{-rI}(p-c)V(I) - d\right]\right] = 0$$

$$\Rightarrow \quad = \frac{d}{dI}\left[\frac{1}{1-e^{-rI}}\right]\cdot\left[e^{-rI}(p-c)V(I) - d\right] +$$
$$\frac{1}{1-e^{-rI}}\frac{d}{dI}\left[e^{-rI}(p-c)V(I) - d\right] = 0$$

$\rightarrow$

$$\Rightarrow \left(\frac{1}{1-e^{-rI}}\right)^2 re^{-rI}\left[e^{-rI}(p-c)V(I)-d\right] +$$

$$\left(\frac{1}{1-e^{-rI}}\right)\left[-re^{-rI}(p-c)V(I) + e^{-rI}V'(I)(p-c)\right] = 0$$

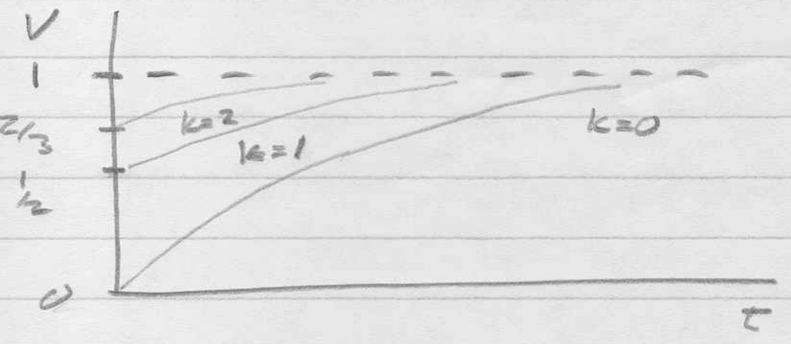
$$\Rightarrow \frac{1}{1-e^{-rI}} r\left[e^{-rI}V(I) - \frac{d}{p-c}\right] +$$

$$\left[-rV(I) + V'(I)\right] = 0$$

But $V(I) = \dfrac{I+k}{I+k+1}$

$$\Rightarrow \frac{dV}{dI} = \frac{(I+k)-(I+k+1)}{(I+k+1)^2} = \frac{-1}{(I+k+1)^2}$$

$$\Rightarrow \boxed{\frac{1}{1-e^{-rI}} r\left[e^{-rI}\left[\frac{I+k}{I+k+1}\right] - \frac{d}{(p-c)}\right] + \left[-r\left[\frac{I+k}{I+k+1}\right] + \frac{-1}{(I+k+1)^2}\right] = 0}$$

c) 

IT GETS FLATTER AND INITIAL SIZE IS LARGER.