University of Toronto

Faculty of Arts and Science
December Examinations 2009
ECO314H1F - Turner
Duration - 2hours

Examination Aids: No notes or books are allowed, but you may use a calculator. Please turn your cell-phones off.

There is an automatic 5 point penalty for anyone working on their exam after the end of the exam is announced.

The exam is worth 100 points in all.

When you write up your answers, your goal should be to (1) be correct, and (2) convince your reader that your answer is correct. Answers which do not achieve these goals will not be awarded full credit. To accomplish the second objective, it is helpful if your work is legible and if all steps are presented, possibly with a line of explanation.

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1. Let T denote a stock of South American forest. Suppose that each unit of forest sequesters s tones of atmospheric carbon each year and that the social value of each ton of sequestered carbon is v (estimates of v typically range between 20$ and 100$). Let δ denote the social discount factor.

   (a) (15) Suppose that this stock of forest is being cut down at rate ρ, so that $T_{t+1} = (1-\rho)T_t$. Calculate the discounted present value of carbon sequestered by this forest.

   (b) (10) The United Nations Program to Reduce Deforestation and forest Degradation (REDD) is intended to reduce the rate of deforestation. Suppose it succeeds in reducing the rate by 50%, that is, to $\rho/2$. What is the most we should be willing to pay to fund the program?

2. Consider the problem of choosing optimal cutting times for a parcel of forest land. To describe the problem , let D denote the cost of planting a unit of land with trees, c be the cost per unit volume of lumber of harvesting trees, and p the market price per unit volume of lumber, r the interest rate. Finally, let $V(t) = \alpha t$ be volume of lumber per unit of land as a function of the time t since the trees were planted.

   (a) (10) Let $(t_0, t_1, t_2, ...)$ be a list of the times when the parcel of land is harvested and replanted. Let W denote the discount present value of this plan. Write an expression for W.

   (b) (5) Argue, briefly but persuasively, that for any harvest plan that maximizes W, $t_{k+1} - t_k$ is the same for all k.

   (c) (5) Write down the condition that any optimal rotation period I^* must satisfy.

   (d) (5) Verify that the optimal rotation period is finite.

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3. Consider a mining firm facing interest rate r, elastic demand p, unit extraction costs, $c(q) = q^2$, initial stock level S_0, and stock level in period t, S_t. Suppose that the firm has perfect foresight and that, while the mine may in principle be exploited indefinitely, in each period the firm maximizes the discount present value of profits *over a two-period horizon*. That is, in period t the industry acts to maximize

$$\Pi_t(q_t, q_{t+1}) = (pq_t - q_t^2) + \frac{1}{1+r}(pq_{t+1} - q_{t+1}^2)$$

   (a) (15) Let $(q_t^{t*}, q_{t+1}^{t*}, q_{t+2}^{t*}, ...)$ denote the industry's optimal exploitation plan in period t. Find the industry's optimal exploitation plan in period t.

   (b) (10) Say that the industry's behavior is *dynamically consistent* if at time $t+1$ it is optimal for the industry to follow an optimal plan made in the preceding period. That is, if we let q_t^{s*} be the optimal plan for period t made at time s, then the firm's behavior is dynamically consistent if

$$(q_{t+1}^{t*}, q_{t+2}^{t*}, q_{t+3}^{t*}, ...) = (q_{t+1}^{t+1*}, q_{t+2}^{t+1*}, q_{t+3}^{t+1*}, ...),$$

   Is the firm's behavior dynamically consistent? Explain.

4. 10 units of water are to be allocated between urban and rural dwellers. Demand for the two groups is,

$$\begin{aligned} P_u(w_u) &= 10 - w_u \\ P_r(w_r) &= 5 - w_r/3. \end{aligned}$$

   (a) (10) Find the allocation of water that maximizes social surplus graphically.

   (b) (5) Suppose that rural dwellers own water rights and sell them in a market. Find the equilibrium allocation and price graphically.

   (c) Suppose the rural dwellers are upstream from the urban dwellers and have the right to take as much water as they want, but cannot sell excess water.

      i. (5) Draw a graph to illustrate the resulting allocation of water.

      ii. (5) Find the magnitude of the lost social surplus (relative to the private property regime) and illustrate this quantity on your graph.

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Solutions

1.

(a) IN EACH YEAR THE VALUE OF CARBON SEQUESTERED IS

$$(1-\rho)^t T_0 s v$$

THE DISCOUNT P.V. IS

$$W_\rho = \sum_{t=0}^{\infty} \delta^t (1-\rho)^t s v T_0$$

$$= s v T_0 \sum_{t=0}^{\infty} \delta^t (1-\rho)^t$$

LET $\gamma = \delta(1-\rho)$

THEN $\sum_{t=0}^{\infty} \delta^t (1-\rho)^t = \sum_{t=0}^{\infty} \gamma^t = \frac{1}{1-\gamma}$

$$= \frac{1}{1-\delta(1-\rho)}$$

THUS WE HAVE $W_\rho = \dfrac{s v T_0}{1-\delta(1-\rho)}$

(b) WRITING $\rho/_2$ IN PLACE OF ρ IN (a)

WE HAVE $W_{\rho/2} = \dfrac{s v T_0}{1-\delta(1-\rho/2)}$

WE SHOULD BE WILLING TO PAY

$W_{\rho/2} - W_\rho$ FOR THE PROGRAM.

2.

$V(t) = \alpha t$ ~ GROWTH EQN

C ~ COST OF HARVEST

P ~ PRICE

D ~ REPLANTING COST

(a) $$W = \left[(p-c)V(t_0)e^{-rt_0} - D\right] +$$

$$\left[(p-c)V(t_1)e^{-r(t_1-t_0)} - D\right]e^{-rt_0} +$$

$$\left[(p-c)V(t_2)e^{-r(t_2-t_1)} - D\right]e^{-rt_1} + \cdots$$

(b) SINCE c, p, D, V ARE CONSTANT ACROSS PERIODS, THE INTERVAL THAT MAXIMIZES THE PROFIT OF ANY CYCLE MUST MAXIMIZE PROFIT OF ALL CYCLES. THUS

$t_{k+1} - t_k$ CONSTANT ALL $k \geq 1$.

(c) FIRST WRITE $W(I)$

$$W(I) = \left[(p-c)V(I)e^{-rI} - D\right] +$$

$$e^{-rI}\left[(p-c)V(I)e^{-rI} - D\right] +$$

$$e^{-r2I}\left[(p-c)V(I)e^{-rI} - D\right] +$$

$$\vdots$$

$$+ e^{-rkI}\left[(p-c)V(I)e^{-rI} - D\right] + \cdots$$

$$\Rightarrow \quad W(I) = \left[(p-c)V(I)e^{-rI} - D\right] + e^{-rI}\Bigg[\left[(p-c)V(I)e^{-rI} - D\right] + \left[(p-c)V(I)e^{-rI} - D\right]e^{-rI} + \left[\quad '' \quad\right]e^{-r2I} + \cdots\Bigg]$$

$$\Rightarrow \quad W(I) = \left[(p-c)V(I)e^{-rI} - D\right] + e^{-rI}W(I)$$

$$\Rightarrow \quad W(I) = \frac{1}{1-e^{-rI}}\left[(p-c)V(I)e^{-rI} - D\right]$$

(c)

ANY OPTIMAL I^* MUST SATISFY $W'(I^*) = 0$, SO FIND $W'(I)$

$$W'(I) = 0$$

$$\Rightarrow (p-c)\left[-re^{-rI}V(I) + e^{-rI}V'(I)\right]\frac{1}{1-e^{-rI}} + \left[(p-c)e^{-rI}V(I) - D\right]\left(\frac{1}{1-e^{-rI}}\right)^2(-re^{-rI}) = 0$$

$$\Rightarrow \quad (p-c)\left[-rV(I) + V'(I)\right] + \frac{-r}{1-e^{-rI}}\left[(p-c)e^{-rI}V(I) - D\right] = 0$$

(SKIPPING SOME ALGEBRA)

$$\Rightarrow \quad -rV(I) + V'(I) - e^{-rI}V'(I) + \frac{rD}{p-c} = 0$$

SINCE $V(I) = \alpha I$, $V'(I) = \alpha$

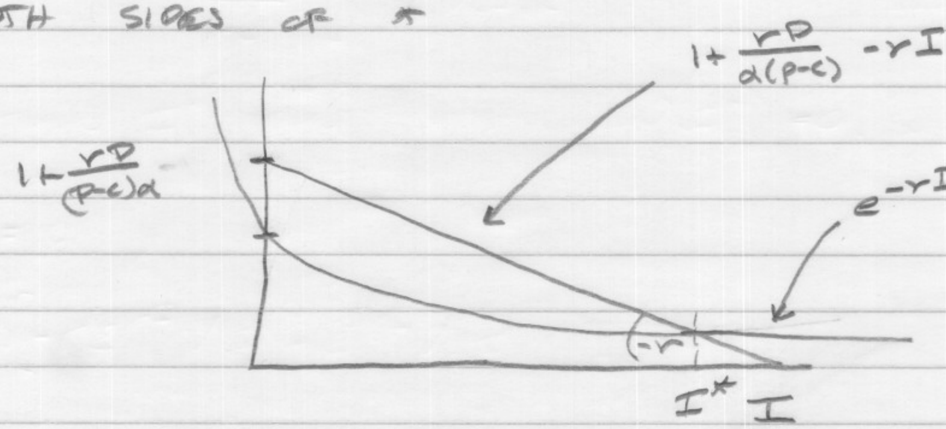
$$\Rightarrow \quad -r\alpha I + \alpha - e^{-rI}\alpha + \frac{rD}{p-c} = 0$$

$$\Rightarrow \quad -rI + 1 - e^{-rI} + \frac{rD}{(p-c)\alpha} = 0$$

$$\Rightarrow \quad \left(1 + \frac{rD}{(p-c)\alpha}\right) - rI^* = e^{-rI^*} \qquad *$$

AN OPTIMAL ROTATION PERIOD MUST SATISFY $W'(I^*)=0$.

(d) TO SEE THAT I^* MUST BE FINITE, PLOT BOTH SIDES OF $*$



BY INSPECTION OF THIS GRAPH, $*$ HOLDS ONLY FOR UNIQUE FINITE I^*.

3.

(a)

AT TIME t FIRM SOLVES

$$\text{MAX } (Pq_t - q_t^2) + \frac{1}{1+r}(Pq_{t+1} - q_{t+1}^2)$$

$$\text{S.T.} \quad q_t + q_{t+1} = S_0$$

LAGRANGIAN IS:

$$\mathcal{L} = (Pq_t - q_t^2) + \frac{1}{1+r}(Pq_{t+1} - q_t^2) + \lambda(q_t + q_{t+1} - S_0)$$

F.O.C. ARE:

$$\frac{\partial \mathcal{L}}{\partial q_t} = 0 \implies P - 2q_t = -\lambda \quad (1)$$

$$\frac{\partial \mathcal{L}}{\partial q_{t+1}} = 0 \implies \frac{1}{1+r}(P - 2q_{t+1}) = -\lambda \quad (2)$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = 0 \implies q_{t+1} + q_t = S_0 \quad (3)$$

$$(1)+(2) \implies P - 2q_t = \frac{1}{1+r}(P - 2q_{t+1})$$

$$\implies (1+r)(P - 2q_t) = P - 2q_{t+1}$$

$$\implies q_{t+1} = \frac{1}{2}\left[P - (1+r)(P - 2q_t)\right]$$

$$\implies q_{t+1} = \frac{1}{2}\left[\cancel{P} - \cancel{P} + 2q_t - rP + 2rq_t\right]$$

$$\implies q_{t+1} = \frac{1}{2}P + (1+r)q_t \quad (4)$$

(4) + (3) $\Rightarrow$ $\frac{r}{2}p + (1+r)q_t + q_t = S_0$

$\Rightarrow$ $(2+r)q_t = \frac{2S_0}{rp}$

$\Rightarrow$ $q_t^* = \frac{2S_0}{(2+r)rp}$ (5)

(5) $\to$ (4) $\Rightarrow$ $q_{t+1}^* = \frac{rp}{2} + \frac{(1+r)\,2S_0}{(2+r)rp}$

(b) CONSIDER THE FIRMS DECISION AT $t=0$.
FROM PART (a) WE HAVE THAT

$$q_0^{0*} = \frac{2S_0}{(2+r)rp}$$

$$q_1^{0*} = \frac{rp}{2} + \frac{(1+r)2S_0}{(2+r)rp}$$

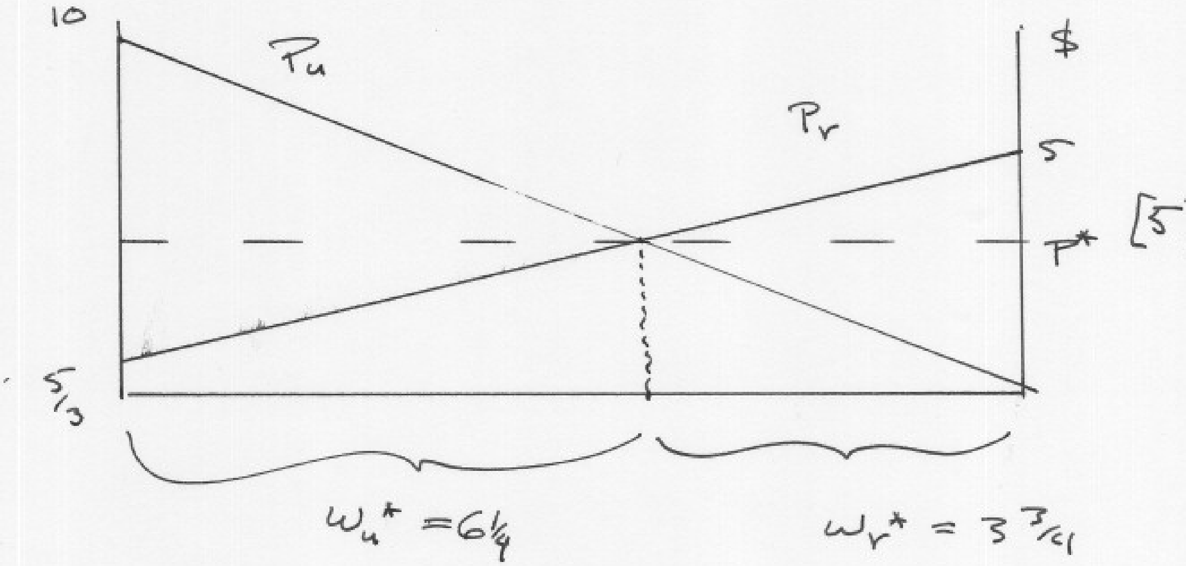
AND BY (3) $q_0^{0*} + q_1^{0*} = S_0$
SO IMPLICITLY, WE HAVE $q_2^{0*} = 0$.

WHEN WE GET TO $t=1$, THE FIRM SOLVES THE PROBLEM IN (a) AGAIN, BUT FOR $S_1 = S_0 - q_0^{0*}$

$\Rightarrow$ $q_1^{1*} = \frac{2S_1}{(2+r)rp}$, $q_2^{1*} = \frac{rp}{2} + \frac{(1+r)2S_1}{(2+r)rp}$

SINCE $q_2^{1*} > 0 = q_2^{0*}$ THE FIRM IS NOT DYNAMICALLY CONSISTENT.

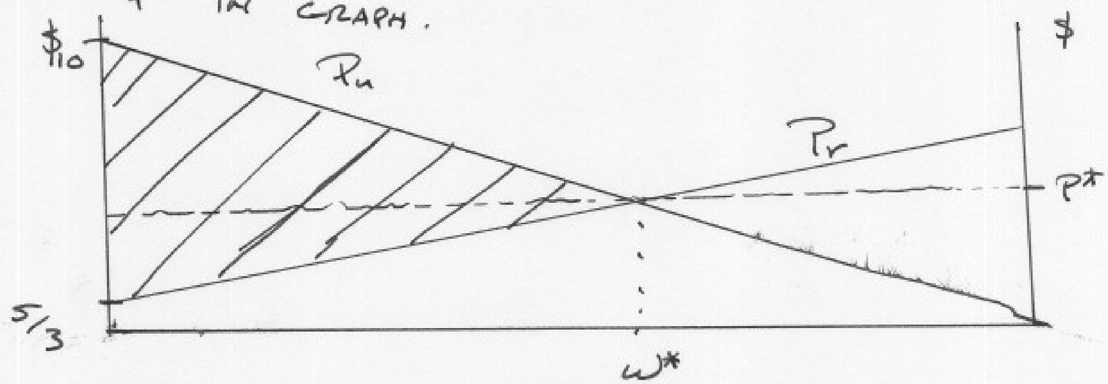
4.



[5

SOCIAL SURPLUS IS AREA BELOW UPPER ENVELOPE OF P_u AND P_r

MARKET ACHIEVES SOCIAL OPTIMUM ALLOCATION. PRICE IS P* IN GRAPH. [5



[10]

WITH OPEN ACCESS RURAL DWELLERS CONSUME ALL. LOSS OF SURPLUS IS HATCHED AREA.

TO FIND AREA OF TRIANGLE SOLVE FOR P^*, ω^*:

$$P_u(\omega_u^*) = P_r(10-\omega_u^*)$$

$$\Rightarrow \quad 10 - \omega_u = 5 - \frac{10-\omega_u}{3}$$

$$\Rightarrow \quad \boxed{\omega_u^* = \frac{25}{4} = 6\tfrac{1}{4}}$$

$$\Rightarrow \quad \boxed{P^* = 10 - 6\tfrac{1}{4} = 3\tfrac{3}{4}}$$

[3]

THE AREA OF THIS TRIANGLE IS: ~~[3]~~

$$\frac{1}{2}\left(\frac{25}{4}\right)\left(\frac{15}{4}-\frac{5}{3}\right)+\frac{1}{2}\left(\frac{25}{4}\right)\left(10-{}^{15}/_{4}\right)=\frac{625}{4}\approx 26\$$$