EC314-Fall 2010
Problem Set 2

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When you write up your answers, your goal should be to (1) be correct, and (2) convince your reader that your answer is correct. Answers which do not achieve these goals will not be awarded full credit. To accomplish the second objective, it is helpful if your work is legible and if all steps are presented, possibly with a line of explanation. **Please STAPLE pages together so that we do not lose them.** *(This problem set updated: 6 October 2010)*.

Problems

1. Prove that $\sum_{t=1}^{\infty}\left(\frac{1}{1+r}\right)^t = \frac{1}{r}$.

2. One problem with discounting is that is counts future benefits very little after we get more than a few years ahead. To see this, conduct the following three exercises:

   (a) Write the expression for the present value of a one hundred dollar payoff 100 years from now as a function of the interest rate.

   (b) Plot this present value as a function of r.

   (c) Policy makers actually use discounting when they are trying to make decisions. Commonly used discount rates range between 3% and 10%. What is the present value of our 100$ payoff 100 years from now at these two interest rates.

3. The preceding problem illustrates the problems with discounting. Here is the problem with not discounting. The problem with discounting is that it seems to give "too little" weight to the future. Suppose instead that we give all future years exactly the same weight as the current year. Thus, given streams of payoffs $(c_t)_{t=0}^{\infty}$ we will rank them according to their sum. There are two problems with this method.

   (a) Let $c_t = \begin{cases} -1000 & if \quad t=0 \\ 0.00001 & if \quad t \geq 1 \end{cases}$, and let $c'_t = 0$ for all t.
   If I do not discount future benefits, which policy should I choose?

   (b) Consider two policies
   $$c_t = \begin{cases} 1 & if \quad t \text{ odd and greater than } 1 \\ 2 & if \quad t \text{ even} \\ 1.1 & if \quad t = 1 \end{cases}$$
   and
   $$c'_t = \begin{cases} 2 & if \quad t \text{ odd} \\ 1 & if \quad t \text{ even} \end{cases}.$$

   If you use the undiscounted sum of payoffs can you rank these two policies?

4. Consider a mine owner who lives for two periods and has revenue from his mine for two periods. Let (x_1,x_2) denote the revenue from the mine in each period and let (c_1,c_2) denote the mine owner's consumption of a composite good in each period. Suppose that in each period one unit of mine output may be exchange for one unit of composite good, and suppose that the composite good may be borrowed or deposited at interest rate r.

   (a) Write down the mine owner's budget constraint.

   (b) Imagine that the mine owner is given a choice between two mines, one with revenue stream (x_1,x_2) and one with revenue stream (x_1',x_2'). Suppose that the mine owner's preferences for consumption are monotonically increasing (i.e. increasing in c_1 and c_2). Use your result in part 4a to argue that the mine owner should choose the mine with the largest discount present value.

   (c) Does this result still hold if the mine is to be owned by two mine owners whose utility functions are given by, $u_1 = c_1$, and $u_2 = c_2$? That is, mine owner one only cares about first period consumption and mine owner two only cares about second period consumption. Note that it doesn't really matter how ownership is arranged between one and two for this question.

5. Consumer 1 has preferences over goods x (sushi) and y (whale watching trips) and preferences given by $u_1 = x_1^{\alpha} y_1^{(1-\alpha)}$, for some $\alpha \in (0,1)$. Consumer 2 has preferences of x and y given by $(u_1)^3$. Thus, consumer 2's utility function is a monotonically increasing transformation of consumer 1's utility function (and therefore represents the same underlying preferences). Suppose that the price of x is 1, the price of y is p and both consumers have income w.

   (a) Find the inverse demand for good y for both consumers. To do this find the demand for y and invert it by solving for $y_i(p)$ for $i = 1,2$.

   (b) Write the expression for consumer's surplus as a function of $\overline{y}$ for each consumer, $CS_1(y)$ and $CS_2(y)$. This expression will involve integration. Do not evaluate the integrals, it will make things harder later.

   (c) The planner wants to divide one unit y between the two consumers to maximize the sum of consumer's surplus for the two consumers. Let $\overline{y}$ be the amount given to consumer 1. How should he choose $\overline{y}$. This requires you to solve a maximization problem. This problem is easy if you recall the fundamental theorem of calculus. This theorem states that if $f = DF/dx$ then,

$$\frac{d}{d\overline{y}}\left[\int_0^{\overline{y}} f(x)dx\right] = \frac{d}{d\overline{y}}\left[F(\overline{y}) - F(0)\right]$$
$$= f(\overline{y}).$$

   (d) Now suppose that $\alpha = 1/2$, that both consumers have one unit of x and the planner wants to divide one unit y between them to maximize $u_1 + u_2$. Show that the value of $\overline{y}$ that you found in the last problem does not satisfy the first order condition for this maximization problem.

   (e) This problem illustrates why maximizing social surplus (here consumer surplus) need not lead to maximization of intuitive welfare measures. Explain briefly why this occurs.