EC314-Fall 2010
Problem Set 4

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When you write up your answers, your goal should be to (1) be correct, and (2) convince your reader that your answer is correct. Answers which do not achieve these goals will not be awarded full credit. To accomplish the second objective, it is helpful if your work is legible and if all steps are presented, possibly with a line of explanation. **Please STAPLE pages together so that we do not lose them.** *(This problem set updated: 18 August 2010)*.

Problems

1. In class and in the text we talked about a monocentric city in a plane. In this problem we look at the easier case of a monocentric city on a line. Assume that: (1) The city consists of farmers who bring their pigs to market at $x = 0$, where there is a port. (2) The price of pigs at the port is p and that the cost of transporting a pig a unit distance is c, so that the revenue net of transportation for a pig produced at x is $p - c|x|$. (3) Pigs are produced only from land and each unit of land produces a pigs. The rent for land is $r(x)$ per unit. In all this means that the profits per unit of land for a farmer at x are given by

$$\pi(x) = (p - c|x|)\,a - r(x).$$

   (a) Explain (briefly) why, in equilibrium, every unit of land in the city must yield the same profit.
   (b) Suppose that all farmers in the city make exactly zero profits.
      i. State a farmer's profits before he pays land rent as a function of position. Plot these profits as a function of position.
      ii. Solve for $r(x)$. Plot the rent gradient as a function of position.
      iii. How long is the city.

2. In class and in the text it was claimed that the Leontief production technology has some very particular properties. This exercise verifies some of those claims. Let $f(n,l) = \min\left\{\frac{n}{a}, \frac{l}{b}\right\}$ where a,b are constants and n,l are labor and land. Let w, and r denote the prices labor and land.

   (a) Draw two graphs which shows isoquants for f and iso-cost lines. Use different costs to draw the isocost lines in each graph.
   (b) Find the cost function $c(q,w,r)$ for this technology. That is, the minimum that can be spent on inputs to produce output q. You should find that cost minimization requires that the inputs be used in the same proportions regardless of prices or q.
   (c) Let $q(w,p)$ denote the profit maximizing supply of output from this technology. Recall that profit maximization implies cost minimization and argue that $\frac{q}{l} = k$, where k is a constant. What is k?

3. Now consider the monocentric semi-circular city that we discussed in class and in the book. As in problem 1, suppose that (1) the city consists of farmers who bring their pigs to market at $x = 0$, where there is a port. (2) The price of pigs at the port is p and that the cost of

transporting a pig a unit distance is c, so that the revenue net of transportation for a pig produced at x_i is $p - cx$. Here x refers to radial distance from the port. (3) Pigs are produced only from land and each unit of land produces a pigs. The rent for land is $r(x)$ per unit. In all this means that the profits per unit of land for a farmer at x are given by

$$\pi(x_i) = (p - cx)\, l(x)\, a - r(x)\, l(x).$$

where $l(x)$ denotes the amount of land at distance x_i that the farmer rents. Suppose that all farmers in the city make exactly zero profits.

(a) State a farmer's profits per unit of land, before he pays land rent, as a function of position.

(b) Solve for $r(x)$. Plot the rent gradient as a function of position.

(c) How big is the city.

4. Suppose now that there are two activities in the city of problem 3, and that they have bid rent functions $r_i(x)$ and $r_j(x)$ with $r_i(x) \neq r_j(x)$. Draw a graph illustrating the rent gradient for the city, the edge of the city, and the boundary where one activity gives way to the other.

5. (Harder) Toronto's city council recently approved a 'land transfer tax', a tax of about 1% on the sale price of a property each time it changes hands. To understand some of the implications of this tax, consider a simple linear city. Assume that:

- The city consists of economics professors who come from Labrador to work downtown at $x = 0$, where there is a university, and who live at some location x which is x units of distance from the university.
- A professor's wage at the university is w.
- Each professor lives on exactly one unit of land, and rents this land from an absentee landlord for $r(x)$. Land rent outside the city is exactly zero.
- A professor has the opportunity to move to Labrador and secure wage 0 with no commuting.
- A professor's utility at location x is given by $u(x) = w - cx - r(x)$, and is zero in Labrador.

(a) Find and illustrate the equilibrium size of the city and the equilibrium rent gradient.

(b) Suppose that the Mayor of the city decides to collect a property tax of t% on residential land and burn the money. To make things more awkward, however, this tax is not collected every year, but only in years when professors move. Professors move exactly once every 10 years, in years 9,19, 29... and their landlords must pay tax $tr(x)$ in these years. Let δ denote the market discount factor. Use i to index time, and suppose that time starts at zero.

  i. Let V denote the discount present value of this land transfer tax. In doing this calculation you may find it helpful to let $\alpha = \delta^{10}$.

  ii. Find the annual property tax rate T which generates the same discount present value of revenue as the land transfer tax t.

(c) Now suppose that the Mayor collects tax $Tr(x)$ on every residential land transaction, with the rest of the description of the city the same. Let $R(x)$ denote the after tax rent gradient. Find the equilibrium size of the city, the equilibrium rent gradient and the equilibrium tax gradient. How does the city with taxes differ from the city without?

(d) Finally, to make things really awkward, suppose that the Mayor decides to tax more expensive property more highly. In particular, suppose that properties are subject to rate T each year, but that locations with rent above $r*$ are subject to a rate surcharge s. Thus, the tax annual property tax is $Tr(x) + s\max\{0,(r - r^*)\}$. Find the equilibrium size of the city, the equilibrium rent gradient and the equilibrium tax gradient. How does the city with taxes differ from the city without? (To keep things simple, this problem is about annual tax rates. You don't need to duplicate part (b) for this non-linear tax).