Merger Simulation: Bertand with Linear Demand

Workshop 2

U.S. 2010 HMG, p. 20

“In differentiated product industries, some products can be very close substitutes and compete strongly with each other, while other products are more distant substitutes and compete less strongly. For example, one high-end product may compete much more directly with another high-end product than with any low-end product.”

“A merger between firms selling differentiated products may diminish competition by enabling the merged firm to profit by unilaterally raising the price of one or both products above the pre-merger level. Some of the sales lost due to the price rise will merely be diverted to the product of the merger partner and, depending on relative margins, capturing such sales loss through merger may make the price increase profitable even though it would not have been profitable prior to the merger.”

Differentiated Goods Mergers

• Focus is on closeness of substitutes
  – Mergers among firms that produce close substitutes cause greater harm
  – Market shares and concentration (e.g. HHI or CR4) not informative
  – Demand estimated and used as input
  – With merger simulation tools, explore how varying the substitution patterns and identity of merging firms affects post-merger equilibrium
Linear, Differentiated Goods Bertrand Model: Basis for Merger Simulation

- Linear system of demand equations:
  \[ q_1 = a_1 - b_1 p_1 + d_2 + e_3 \]
  \[ q_2 = a_2 + d_1 - f_2 + g_3 \]
  \[ q_3 = a_3 + e_1 + g_2 - h_3 \]
- Constant marginal costs (can differ): \( c_i \)
- 3 Bertrand firms: Firms 1 & 2 propose merging
- Possible cost efficiencies from merger:
  - Post-merger marginal costs = \( e_1 c_1 \) and \( e_2 c_2 \)

See Sections 23.2.1 and 23.8 of C-W

Firms’ Profit Functions

\[ \pi_1 = (p_1 - c_1)q_1 = (p_1 - c_1)(a_1 - b_1 p_1 + d_2 + e_3) \]
\[ \pi_2 = (p_2 - c_2)q_2 = (p_2 - c_2)(a_2 + d_1 - f_2 + g_3) \]
\[ \pi_3 = (p_3 - c_3)q_3 = (p_3 - c_3)(a_3 + e_1 + g_2 - h_3) \]

Firms’ Pre-Merger FOCs

\[ -2b_1 p_1 + d_2 + e_3 = -a_1 - b c_1 \]
\[ d_1 - 2f_2 + g_3 = -a_2 - f c_2 \]
\[ e_1 + g_2 - 2h_3 = -a_3 - h c_3 \]

What are the unknowns?
How to solve?
Merged Firm Maximizes Profits

\[ \pi_{1&2} = (p_1 - e_1c_1)q_1 + (p_2 - e_2c_2)q_2 \]
\[ \pi_{1&2} = (p_1 - e_1c_1)(a_1 - bp_1 + dp_2 + ep_3) + (p_2 - e_2c_2)(a_2 + dp_1 - fp_2 + gp_3) \]

\[ \frac{\partial \pi_{1&2}}{\partial p_1} = a_1 - bp_1 + dp_2 + ep_3 - b(p_1 - e_1c_1) + d(p_2 - e_2c_2) \quad \text{set} \quad 0 \]
\[ \frac{\partial \pi_{1&2}}{\partial p_2} = d(p_1 - e_1c_1) + a_2 + dp_1 - fp_2 + gp_3 - f(p_2 - e_2c_2) \quad \text{set} \quad 0 \]

The merged firm has two FOCs?
Takes into account the substitutability of goods 1 and 2?

Firms’ Post-Merger FOCs

\[ -2bp_1 + 2dp_2 + ep_3 = -a_1 - be_1c_1 + de_2c_2 \]
\[ 2dp_1 - 2fp_2 + gp_3 = -a_2 - fe_2c_2 + de_1c_1 \]
\[ ep_1 + gp_2 - 2hp_3 = -a_3 - hc_3 \]

What are the unknowns?
How to solve?

Bertrand Merger Simulation

Spreadsheets

• Interactive work with two different versions of the Bertrand Merger Simulation workbooks
  – Demand parameters and pre-merger prices are the inputs
  – Demand parameters and pre-merger marginal costs are the inputs