Unemployment Insurance Eligibility, Moral Hazard and Equilibrium Unemployment

Min Zhang†

*Shanghai University of Finance and Economics*

June, 2010

Abstract

This paper shows that the Mortensen-Pissarides search and matching model can be successfully parameterized to generate observed large cyclical fluctuations in unemployment and modest responses of unemployment to changes in unemployment insurance (UI) benefits. The key features behind this success are the consideration of the eligibility for UI benefits and the heterogeneity of workers. With the linear utilities commonly assumed in the Mortensen-Pissarides model, a fully rated UI system designed to prevent moral hazard has no effect on unemployment. However, the UI system in the United States is neither fully rated nor able to prevent workers with low productivity from quitting their jobs or rejecting employment offers to collect benefits. As a result, an increase in UI generosity has a positive, but realistically small, effect on unemployment. This paper answers the Costain and Reiter (2008) criticism to the Hagedorn and Manovskii (2008) strategy of adopting a high value of non-market activities to generate realistic business cycles with the Mortensen-Pissarides model.

JEL classification: E24 E32 J64

Keywords: Search, Matching, Moral Hazard, UI Entitlement, Equilibrium Unemployment, Labor Markets

1 Introduction

Empirical studies regarding the impact of unemployment insurance (UI hereafter) program on workers’ incentive to work document that changes in UI benefits have significant

†This paper is part of my dissertation. It has been written under the supervision of Professor Miquel Faig, whom I thank for his generosity and encouragement. I also thank Shounyong Shi, Michael Reiter, Andres Erosa, Angelo Melino, Xiaodong Zhu, Diego Restuccia, Gueorgui Kambourov, Margarida Duarte and Michelle Alexopoulos for very useful comments. All errors are mine.
but modest effect on un(-employment) (see Solon, 1985; Moffitt and Nicholson, 1982; Meyer, 1990). However, a challenge has been posted in accounted for this observation in various models. For example, Ljungqvist and Sargent (2006) argue that, when the publicly-provided benefits ignored in Prescott (2002) is taken into account, with Prescott’s calibration of the parameters, the standard growth model generates larger movements in employment in Europe than it has experienced. More recently, a similar difficulty is found in the Mortensen-Pissarides search and matching model. Several authors, including Hornstein et al. (2005), Costain and Reiter (2008), and Zhang (2008), criticize that the calibration strategy argued by Hagedorn and Manovskii (2008) brings up some problems with the standard model although it fixes the volatility puzzle. Particularly, the high value of non-market activities required to generate labor market cycles observed in the United States induces dramatic responses of unemployment to labor policy changes. This paper addresses this issue by introducing some realistic institutional features of the UI system and worker heterogeneity into the Mortensen-Pissarides model.

The consideration of eligibility for UI benefits proves crucial to this success. In the standard model, workers automatically qualify for UI benefits while they are searching for a job. Thus, the UI benefits represent the opportunity cost of employment and, therefore, hurt employment. However, in reality, the UI entitlement must be earned by prior employment and the UI benefits do not last forever. When such features are taken into account, the UI benefits create a positive effect on the incentive to work, which imposes a downward pressure on unemployment.

The central insight for this positive effect lies in the workers’ desire to gain or retain UI entitlement. Job seekers who are not eligible for UI are eager to be hired in hope to earn UI entitlement through employment. This entitlement effect is stressed in Mortensen (1977). In addition, this contribution extends the entitlement effect to job seekers receiving UI benefits. Due to the positive possibility of losing UI entitlement and the option of retaining it by taking a job, a more generous UI system makes the UI recipients value the UI entitlement more and, thus, more willing to accept a job.

The UI system has distortion effects on moral hazard. When the government lacks perfect information on job offers and reasons for match dissolutions, workers eligible for UI might quit their jobs or reject job offers to collect UI. Worker heterogeneity is introduced into the model to capture the fact that the UI-seeking behavior largely occurs among low-skilled workers. The moral-hazard effect is reinforced by the presence of the UI

---

1 The entitlement effect is also studied by Burdett (1979), Hamermesh (1979), van Den Berg (1990), Albrecht and Vroman (2005), and Coles and Masters (2006).

2 Most papers look at the entitlement effect either on the side of the UI nonrecipients, such as that of Mortensen (1977), or on the side of the recipients, for example, that of Albrecht and Vroman (2005).

3 For instance, Green and Riddell (1997) find that many jobs terminate when workers approach the duration that permits a UI entitlement and the strategic terminations are most likely to happen among the low-skilled. Katz and Meyer (1990) report that a sharp increase in the escape rate from unemployment is observed among UI recipients when the benefits are likely to expire.
contribution fees. Intuitively, a large UI contribution fee required to finance the system reduces a worker’s desire to be employed.

Given the competing effects of the UI system on employment, this paper generalizes the validity of the irrelevance effect of the UI system established in Faig and Zhang (2008) with heterogeneity. When the UI system is fully funded, the rules of the UI provisions can prevent the moral hazard behavior in job retention and job acceptance, and workers are risk-neutral as commonly assumed in the Mortensen-Pissarides model, the presence and generosity of the UI system are irrelevant to the determination of vacancies, unemployment and output.

However, the UI system prevalent in the United States is neither fully rated nor able to prevent workers with low productivity from quitting their jobs or rejecting job offers. When the model is confronted with the data from the United States and the value of leisure is allowed to be as high as needed to reproduce the observed labor market cycles, the results show that moderate increases in the UI replacement rate lead to increases in the unemployment rate similar to those observed in the U.S. economy. For example, with UI benefits 1 percent (units of productivity) more generous than its current level, the predicted log unemployment rises by 3.3 percent, which squares well with the estimate of 2 in Costain and Reiter (2008). Intuitively, more generous UI benefits raise the disutility of working and triggers more moral hazard. Moreover, this effect is amplified by firms’ endogenous job-creation decision. The rise in job refusals lowers the firms’ expected profits from hiring the UI-eligibles, the majority of the unemployed, and leads to fewer vacancies for them, which slows down the overall transitions out of unemployment.\(^4\) However, the rise in unemployment is partially curbed by the entitlement effect in two ways: the ineligible workers’ desire to gain UI entitlement encourages the firm to create more jobs for UI nonrecipients. The eligible workers’ incentive to maintain UI entitlement curtails the degree of job rejections, which improves the firm’s profit and translates into more job opportunities for UI recipients.

The introductions of the entitlement qualification requirements and worker heterogeneity prove to be quantitatively important. When the moral hazard effect is removed from the model by setting the probability of collecting UI upon job quits and job rejections \(\pi\) to be zero, the predicted unemployment drops by one third to 3.85 percent. When both entitlement and moral hazard effects are shut down by setting UI benefits \(b\) to be zero. The decline in unemployment is much smaller than the one resulting from the reform with \(\pi = 0\). The difference is explained by the entitlement effect.

The paper most related to this one is Faig and Zhang (2008). However, that work considers homogeneous workers, which leads to counterfactual predictions about strategic quits.\(^5\) Also, that paper hinges on the positive correlation between the UI benefits and

\(^4\)This mechanism is consistent with the incentive of firms to delay rehiring workers who receive UI.

\(^5\)In Faig and Zhang (2008), because of the homogeneity in workers, if one worker decides to quit the
income taxes to generate a positive response of unemployment to the rise in UI benefits. Several recent papers propose alternative ways to reconcile the cyclical and policy-related variations in unemployment. For example, Costain and Reiter (2008) fix the problem with the help of match-embodied technological change. Hagedorn et al. (2008) reach a similar conclusion by exploring worker heterogeneity in skills.

The rest of the paper proceeds as follows: Section 2 lays out a stochastic version of the model. Section 3 studies the main properties and existence of equilibrium. The irrelevance effect of a UI program is then established with worker heterogeneity. Section 4 calibrates the model to data in the labor market in the United States. Finally, Section 5 concludes.

2 The Model

2.1 Model Environment

Consider a continuous time model economy with an infinite horizon, populated by a continuum of measure one of workers and a large measure of potential firms with free entry into the labor market. Both the workers and the firms are risk-neutral and discount future income at a common rate $r$. Firms with vacancies are identical in all respects, while workers searching for jobs differ in UI eligibility. Denote $e$ as the UI eligibility state: some of the unemployed workers are eligible for UI and receive benefits ($e = 1$), while others are not ($e = 0$). Workers can earn UI entitlement by working with firms for a while and lose it over a spell of unemployment, both of which follow a stochastic process with arrival rates $g$ and $d$, respectively.

A firm-worker pair is required to form to carry out production. For this purpose, workers and firms with vacancies search in the labor market to look for a suitable partner. Following the standard directed-search theory, the search process is summarized as a two-stage game: all firms simultaneously post wage contracts that stipulate that worker’s wage is contingent on productivity of a formed match and the worker’s eligibility state over the spell of employment; after observing all wage contracts in the market, workers decide on which firm to apply to. Then, firms randomly pick workers from applicants and commit to the wage contracts posted. Any formed match produces a flow output $\hat{p}_p(e)$ until it

---

6The reasons for the choice of the directed search are as follows: First of all, it has become standard in the search theory to adopt the directed search approach to deal with heterogeneity issue. See Acemoglu and Shimer (1999), Shi (2002), Shimer and Wright (2004), and Moen and Rosen (2007). Secondly, the directed search approach is very attractive in that it explicitly models the tradeoff between the wage schedule and the match frequency associated with that wage. This mechanism brings some important benefits: It assures efficient allocations in equilibrium. In addition, it provides an interesting microfoundation for the wage determination. In contrast, the Nash-bargaining wage in the random search framework is determined by splitting the match surplus according to an exogenous bargaining power.
dissolves. The productivity $\hat{p}_p(\epsilon) = \overline{p}_p + \epsilon$ consists of two parts. One part $\overline{p}_p$ is common to all matches in the economy and the other part $\epsilon$ is assumed to be match-specific. The subscript $p$ in the common component of productivity denotes the state of the economy, which follows a Markov jump stochastic process with a constant arrival rate $\lambda$ and takes values in a finite support $P \in R^n_+$. Matches are formed as follows. Upon being paired up with firms, workers draw $\epsilon \in [\epsilon^0, \epsilon^1]$ from an exogenous cumulative distribution function $H(\epsilon)$ and decide whether to take the job or not. If the eligible workers turn down the offer, they are allowed to collect UI benefits with a probability $\pi$. Otherwise, an employment relationship is formed and $\epsilon$ stays constant during the spells of employment. Denote $\epsilon^+_p$ as the critical value of $\epsilon$. The ineligible (eligible) workers would take a job only when the realized match-specific productivity exceeds the critical value, $\epsilon > \epsilon^+_p (\epsilon^+_p)$. As long as the UI entitlement is valuable to workers, the value of $\epsilon^+_p$ is not smaller than $\epsilon^0$ simply because the option of rejecting an offer and continuing receiving benefits leads to a higher outside option for the eligible workers than for the ineligible ones. Job seekers of type $e$ are matched with firms as a result of a jump stochastic process with an arrival rate $f^e_p$. Thus, the effective job-finding rate for workers of type $e$ is $f^e_p (1 - H(\epsilon^+_p))$. Workers are assumed to retain their UI eligibility state upon taking a job.

Quitting jobs, or moral hazard quit, is allowed. Matches dissolve either due to exogenous separation shocks that come at an arrival rate $s$ or because of voluntary separations initiated by workers. Since the government cannot fully observe the reason for the match separations, eligible workers leaving a job voluntarily are able to collect full benefits with a probability $\pi$. For a given $\epsilon$, an employed worker who recently becomes eligible for UI is identical in all respects to the one who is eligible for UI at the time of forming a match, which implies that for a given productivity state $p$, moral hazard quits only happen among workers who have recently earned their entitlement and have the match-specific productivity $\epsilon$ falling in the interval $[\epsilon^0_p, \epsilon^1_p]$.

All unemployed workers receive a flow utility from leisure $l$ regardless of the UI eligibility state, while eligible workers also gain a flow utility from UI benefits $b$. UI benefits are provided by a government-run UI program that is financed with UI contribution fees $\tau_p$ paid by employed workers. The UI fees depend on both the aggregate state of the economy $p$ and the employed workers’ eligibility state $e$. The government can borrow or save at the interest rate $r$, so the UI program can run temporary deficits or surpluses for the time being. Later on, I will allow for permanent deficits or surpluses by introducing a public good and general taxation.

To facilitate the exposition, I assume here that active firms searching for workers post time-(or tenure)-independent wage contracts $\phi_p$ that specify wages contingent on the productivity state and matched workers’ individual states $(e, \epsilon)$: $\phi_p = \{w^0_p(e), w^1_p(e)\}$. More generally, the wage could be tenure-dependent in the contracts. However, as proved
in the Appendix, the optimal time-independent wage is optimal among a wider class of time-dependent contracts as well. Under contract $\phi_p$, eligible workers with $\epsilon \geq \epsilon_p^1$ accept the job and receive the wage $w_p^1(\epsilon)$ until the match breaks down. Analogously, ineligible workers with $\epsilon \geq \epsilon_p^0$ accept the job and receive the wage $w_p^0(\epsilon)$ until the match dissolves exogenously or until they gain the UI entitlement. Workers with a newly earned eligibility receive the $w_p^1(\epsilon)$ over the rest of the spell of employment if they choose not to quit; otherwise, they become unemployed and collect UI benefits $b$ with a probability $\pi$. An active firm posts a vacancy at a flow cost $c$. When a production process starts, the firm gains a flow profit $\bar{p}_p(\epsilon)$ net of the labor costs $w_p^e(\epsilon) + \tau_p^e$.

For tractability purpose, both $l$ and $b$ are assumed to be positive, and are assumed to satisfy $\bar{p}_p + \varepsilon - \tau_p^0 \geq l$ for all $p \in P$. Unlike Faig and Zhang (2008), the condition on leisure cannot guarantee the surplus from the match with ineligible workers is non-negative for all $p \in P$, which implies that the ineligible workers with low productivity will reject job offers. Hence, $\epsilon_p^0 \geq \varepsilon$ for all $p \in P$.

There are $m \geq 1$ submarkets. Suppose $\phi_{jp}$ is the wage contract posted in the $jth$ submarket for a given productivity state $p$. Workers choose from the set of posted wage contracts $\{\phi_{jp} : \text{for } j = 1, 2, ..., m, \text{ and } p \in P\}$. I refer to the set of firms posting $\phi_{jp}$ and the set of workers who direct their search to this wage contract as submarket $j$. In this particular submarket, for a given $p \in P$, denote $u_j^e$ and $v_j^e$ as the respective measure of searching workers of type $e$ and vacancies to be filled by type-$e$ unemployed workers, and $\theta_j^e$ as the vacancy-unemployment ratio (also called market tightness). Workers and firms are paired up together by a constant returns to scale matching function, which is Cobb-Douglas in the measure of type-$e$ unemployment $u_j^e$ and vacancies $v_j^e$. The symmetry across the workers in the submarket implies that the matching rate, $f(\theta_j^e)$, at which the workers are matched with jobs is equal to the number of matches divided by unemployment of type $e$. Likewise, the rate $q(\theta_j^e)$, at which the firms have the vacancies paired up with workers, is equal to the number of matches divided by the measure of vacancies.\footnote{I suppress for convenience the dependence on $p$ in the notations of the unemployment, vacancies, market tightness, and turnover rates.} The elasticity of the matching rate with respect to the market tightness, $1 - \eta$, satisfies $\eta \in (0, 1)$.\footnote{Since the matching function is Cobb-Douglas in $u$ and $v$, the value of $\eta$ is independent of $\theta$ and thus constant for any $p \in P$ and $e \in \{0, 1\}$.}

The rates $f(\theta_j^e)$ and $q(\theta_j^e)$ have the following relationship:

$$f(\theta_j^e) = \mu \cdot \left(\theta_j^e\right)^{1-\eta} = \theta_j^e \mu \cdot \left(\theta_j^e\right)^{-\eta} = \theta_j^e \mu \cdot f(\theta_j^e).$$

\section*{2.2 Bellman Equations}

In this part, I focus on a particular submarket $j$. In equilibrium, there exists a single market for each type of unemployed workers. To save on notation, I drop $j$ in the
subscript hereafter. Workers may be in one of four possible states depending on their employment state and UI eligibility state. Likewise, firms paired with workers may be in one of two possible states depending on their worker’s eligibility for UI. Contingent on the aggregate state $p$ and the realized match-specific productivity $\epsilon$, denote $W_p^e(\epsilon)$ and $U_p^e$ as the values of being an employed worker and an unemployed worker of type $e$, respectively. Similarly, denote $J_p^e(\epsilon)$ as the values of a firm hiring a worker of type $e$. Denote $E_p X_p \lambda$ as the expected values of $X (W(\epsilon), U, and J(\epsilon))$ conditional on $p$ when the economy experiences a change in the productivity state. The utility values are recursively defined by the following Bellman equations.

**Workers’ Problem**

An unemployed worker ineligible for UI receives a flow utility from leisure plus the expected gains or losses from being matched with a firm and a change in productivity, which happen with arrival rates $f_0^p$ (or $f(\theta_p)$) and $\lambda$, respectively. The ineligible workers with $\epsilon \geq \epsilon_0^p$ accept job offers. Otherwise, they remain unemployed.

$$r U_p^0 = l + f(\theta_p) \left[ \int_{\epsilon_0^p}^{\theta} W_p^0(\epsilon) dH(\epsilon) + H(\epsilon_0^p) U_p^0 - U_p^0 \right] + \lambda (E_p U_p^0 - U_p^0).$$

(2)

An unemployed worker receiving UI earns a flow utility from both leisure and UI benefits. The expected gains or losses come from being matched with a firm, losing UI entitlements, and experiencing a productivity change. The associated arrival rates are $f_1^p$, $d$, and $\lambda$, respectively. Upon being paired with a firm, the worker with $\epsilon \geq \epsilon_1^p$ accepts the job. Otherwise, the worker rejects the offer and continues collecting UI benefits with probability $\pi$.

$$r U_p^1 = \ell + b + f(\theta_p) \left[ \int_{\epsilon_1^p}^{\theta} W_p^1(\epsilon) \partial H(\epsilon) + H(\epsilon_1^p) \left( \pi U_p^1 + (1 - \pi) U_p^0 \right) - U_p^1 \right] + d \left( U_p^0 - U_p^1 \right) + \lambda (E_p U_p^1 - U_p^1).$$

(3)

An employed worker ineligible for UI receives a wage $w_p^0(\epsilon)$ plus the expected gains or losses from exogenously losing the job, becoming eligible for UI and experiencing a change in productivity, which occur with the respective arrival rates $s$, $g$ and $\lambda$.

$$r W_p^0(\epsilon) = w_p^0(\epsilon) + s \left[ U_p^0 - W_p^0(\epsilon) \right] + g \left[ W_p^1(\epsilon) - W_p^0(\epsilon) \right] + \lambda \left[ E_p W_p^0(\epsilon) - W_p^0(\epsilon) \right], \forall \epsilon. (4)$$

An employed worker with UI eligibility chooses whether to quit the job or not. If the worker quits, he or she becomes unemployed and collects full benefits after quitting the job with probability $\pi$. Otherwise, the worker receives a wage $w_p^1(\epsilon)$ plus the expected gains or losses from exogenously losing the job at an arrival rate $s$ and a productivity
change at an arrival rate $\lambda$.

$$r W^1_p(\epsilon) = \max \left\{ r (\pi U^1_p + (1 - \pi) U^0_p), \ w^1_p(\epsilon) + s [U^1_p - W^1_p(\epsilon)] + \lambda [E^1_p W^1_p(\epsilon) - W^1_p(\epsilon)] \right\}, \ \forall \ \epsilon. \tag{5}$$

### Firms’ Problem

A firm hiring an ineligible worker obtains the flow profits $(\hat{p}_p(\epsilon) - w^0_p(\epsilon) - \tau^0_p)$ plus the expected gains or losses from the exogenous match dissolution, the worker’s gaining UI eligibility and a productivity change. The associated arrival rates for these events are $s$, $g$ and $\lambda$.

$$r J^0_p(\epsilon) = \hat{p}_p(\epsilon) - w^0_p(\epsilon) - \tau^0_p - s J^0_p(\epsilon) + g [J^1_p(\epsilon) - J^0_p(\epsilon)] + \lambda [E^0_p J^0_p(\epsilon) - J^0_p(\epsilon)], \ \forall \ \epsilon. \tag{6}$$

A firm with an eligible worker either gains nothing if the worker quits the job, or receives the flow profits $(\hat{p}_p(\epsilon) - w^1_p(\epsilon) - \tau^1_p)$ plus the expected gains or losses from an exogenous match separation and a productivity change that occur at arrival rates $s$ and $\lambda$, respectively.

$$r J^1_p(\epsilon) = \max \left\{ 0, \ \hat{p}_p(\epsilon) - w^1_p(\epsilon) - \tau^1_p - s J^1_p(\epsilon) + \lambda [E^1_p J^1_p(\epsilon) - J^1_p(\epsilon)] \right\}, \ \forall \ \epsilon. \tag{7}$$

A firm posts vacancies in the submarket with workers of type $\epsilon$ until the flow cost of posting a vacancy equals the expected gains from filling it, which occurs at an arrival rate $q^e_p \left(1 - H(e_p)\right)$. Since the free entry condition drives the value to be zero, the value of a firm with a vacancy is defined by

$$c = q^e_p \int_{c^e_p}^\tau J^e_p(\epsilon) dH(\epsilon), \ \text{for } e = 0, 1. \tag{8}$$

### 2.3 Competitive Search Equilibrium

In equilibrium, if a worker of type $\epsilon$ enters the $j$th submarket, this submarket must yield the worker the highest $U^e_p$. Let $U^e_p$ denote the equilibrium utility of being a type-$\epsilon$ unemployed worker conditional on $p$, then it must satisfy: $U^0_{p,j} = U^0_p$ and $U^1_{p,j} = U^1_p$, $\forall j = 1, 2, ..., m$.

For expositional purposes, conditional on $p$ and $\epsilon$, denote $R^e_p(\epsilon)$ as the worker’s *ex post* gains from a match for a given $\epsilon$ and $R^e_p$ as the worker’s *ex ante* gains from a match. Analogously, conditional on $p$ and $\epsilon$, denote $S^e_p(\epsilon)$ and $V^e_p$ as the firm-worker pair’s *ex post* match gains for a given $\epsilon$ and *ex ante* match gains, respectively. Note that due to strategic quits, $R^e_p(\epsilon) = 0$ and $S^e_p(\epsilon) = 0$ for $\epsilon \in [\xi, e^*_p]$. Hence, $R^e_p \equiv \int_{c^e_p}^\tau R^e_p(\epsilon) dH(\epsilon) = \int_{c^e_p}^\tau R^e_p(\epsilon) dH(\epsilon)$ and $V^e_p \equiv \int_{c^e_p}^\tau S^e_p(\epsilon) dH(\epsilon) = \int_{c^e_p}^\tau S^e_p(\epsilon) dH(\epsilon)$. Therefore, in equilibrium,
\[ R_p^e = \int_{Z}^{\infty} R_p^0(e) dH(e) = \int_{Z}^{\infty} [W_p^0(e) - U_p^0] dH(e), \text{ for } e = 0. \]  

\[ R_p^1 = \int_{Z}^{\infty} R_p^1(e) dH(e) = \int_{Z}^{\infty} [W_p^1(e) - \pi U_p^1 - (1 - \pi) U_p^0] dH(e), \text{ for } e = 1. \]  

\[ V_p^e = \int_{Z}^{\infty} S_p^e(e) dH(e) = \int_{Z}^{\infty} [R_p(e) + J_p^e(e)] dH(e), \text{ for } e = 0, 1. \]  

Substituting (9) and (10) into (2) and (3) gives:

\[ rU_p^0 = l + f_p^0 R_p^0 + \lambda (E_p U_p^0 - U_p^0). \]  

\[ rU_p^1 = l + \frac{rb}{r + d + f_p^1 (1 - \pi)} + \frac{r [f_p^1 R_p^1 + \lambda (E_p U_p^1 - U_p^1)]}{r + d + f_p^1 (1 - \pi)} + \frac{[d + f_p^1 (1 - \pi)] [f_p^0 R_p^0 + \lambda (E_p U_p^0 - U_p^0)]}{r + d + f_p^1 (1 - \pi)}. \]  

The critical value of \( \epsilon_p^e \) is determined by:

\[ S_p^e(\epsilon_p^e) = 0, \text{ if } \epsilon_p^e \in [\xi, \tau], \]  

\[ \epsilon_p^e = \xi, \text{ if } S_p^e(\xi) > 0, \]  

\[ \epsilon_p^e = \tau, \text{ if } S_p^e(\tau) < 0. \]  

Let \( \Phi \) denote the set of wage contracts \( \phi_p \) in all submarkets for any \( p \in P \), and \( \Phi^I \) the set of feasible wage contracts that satisfy the participation constraints of the worker and the firm. From a worker’s perspective, a worker of type \( e \) enters the submarket that offers the highest expected utilities \( U_p^e \). Equations (12) and (13) imply that the attractiveness of a submarket (or a wage contract) can be summarized by the expected gains \( R_p^e \). From a firm’s perspective, taking \( U_p^e \) as given, a firm chooses wage contract \( \phi_p \) to maximize \( V_p^e \). Hence, contingent on \( p \), the firm’s maximizing problem can be expressed as

\[ \max_{R_p^e(U_p^e)} \left\{ \max_{\phi_p \in \Phi^I(R_p^e)} \left[ -c + q_p^e(f_p^0(R_p^e)) \int_{\epsilon_p^e}^{\tau} J_p^e(\epsilon) dH(\epsilon) \right] \right\}, \text{ for } e = 0, 1, \text{ and } p \in P. \]  

The resulting value of a firm with a vacancy under the free entry condition can be written as

\[ c\beta = (1 - \beta) q_p^e(R_p^e(U_p^e)) R_p^e(U_p^e), \text{ for } e = 0, 1. \]  

**Definition 1:** The competitive search equilibrium is a vector of \((U_p^e, f_p^e, R_p^e)\), \(p \in P, e \in \{0, 1\}\), and a wage contract \( \phi_p^* \) which solve the maximization problem

---

\(^9\)When \( S_p^e(\xi) > 0 \), a worker of type \( e \) with match-specific productivity \( \xi \) receives positive gains from forming a match, so \( \epsilon_p^e = \xi \). Similarly, when \( S_p^e(\tau) < 0 \), a worker of type \( e \) and match-specific productivity \( \tau \) suffers losses from forming a match, so \( \epsilon_p^e = \tau \).
(15) and satisfy (16).

**Proposition 1 (Validity of the Hosios Rule)** In the competitive search equilibrium in the submarkets with workers of type $e$, the Hosios condition holds.

The validity of the Hosios rule in each submarket implies that the optimal wage contract in the competitive search equilibrium is equivalent to the Nash bargaining wage in an economy with undirected search. In that economy, unemployed workers are separated into two labor markets according to their eligibility state; firms and workers search randomly in each labor market; and the wage is determined bilaterally by a generalized Nash bargaining rule upon forming a match. The Hosios rule suggests that a worker’s bargaining power in wage negotiation equals his contribution to contacting a firm, which is characterized by the parameter $\eta$ in the matching function. Suppose the worker’s bargaining power is $\beta$, then $\beta = \eta \in (0, 1)$. Therefore, in the competitive search equilibrium, conditional on productivity state $p$ and the employed worker’s individual state $(e, e)$, the worker and firm share the match surplus according to the following rule:

$$R_p^e(e) = \beta S_p^e(e), \text{ and } J_p^e(e) = (1 - \beta) S_p^e(e), \forall e = 0, 1.$$

**Definition 2:** For a given productivity $p$, the competitive search equilibrium is a set of functions $(w_p^e(e), \theta_p^e, U_p^e(e), W_p^e(e), J_p^e(e), S_p^e(e))_{e=0,1}$ and $\epsilon_p$, which satisfy the Bellman equations (2)-(7), the free entry condition (8), the definitions of match surplus (9)-(11), the equation (14) determining the critical value $e_p$ and the surplus sharing rule (17) with $\beta$ satisfying the Hosios’ rule. Define $\hat{U}_p \equiv U_1^p - U_0^p$. This system of equations can be rewritten by the following functional equations:

$$c\theta_p^e = f(\theta_p^e) (1 - \beta) V_p^e, \text{ for } e = 0 \text{ and } 1.$$  

$$\hat{U}_p = \max \left\{ \frac{b + f(\theta_p^1) \beta V_p^1 - f(\theta_p^0) \beta V_p^0 + \lambda(E_p \bar{U}_{p'} - \hat{U}_p)}{r + d + (1 - \pi) f(\theta_p^1)}, 0 \right\}. $$  

$$\hat{B}_p(e) = \max \left\{ -S_p^0(e) + \pi \hat{U}_p, \frac{\tau_p^0 - \tau_p^1 + s \hat{U}_p + \lambda(E_p \bar{B}_{p'}(e) - \hat{B}_p(e))}{r + s + g} \right\}, \forall e.$$
\[ S_p^0(\epsilon) = \max \left\{ \hat{p}_p(\epsilon) - \ell - f(\theta^0_p) \beta V_p^0 + g\hat{B}_p(\epsilon) - \tau_p^0 + \lambda(E_{p'p^0}^0(\epsilon) - S_p^0(\epsilon)) \right\}, \forall \epsilon. \]  \hspace{1cm} (21)

\[ S_p^1(\epsilon) = S_p^0(\epsilon) + \hat{B}_p(\epsilon) - \pi\hat{U}_p, \forall \epsilon. \]  \hspace{1cm} (22)

\[ S_p^e(\epsilon^e_p) = 0. \]  \hspace{1cm} (23)

\[ \hat{B}_p = \int_{\zeta}^{\bar{\epsilon}} \hat{B}_p(\epsilon) \partial H(\epsilon). \]  \hspace{1cm} (24)

\[ V_p^e = \int_{\zeta}^{\bar{\epsilon}} S_p^e(\epsilon) \partial H(\epsilon), \text{ for } e = 0 \text{ and } 1. \]  \hspace{1cm} (25)

Equations (22), (24) and (25) imply:

\[ \hat{B}_p = V_p^1 - V_p^0 + \pi\hat{U}_p. \]  \hspace{1cm} (26)

Moreover, \( \hat{B}_p(\epsilon) = B_p^1(\epsilon) - B_p^0(\epsilon) \), where

\[ B_p^1(\epsilon) = \frac{s\hat{U}_p - \tau_p^1 + \lambda\left(E_{p'p^1}\epsilon - B_p^1(\epsilon)\right)}{r + s}. \]

\[ B_p^0(\epsilon) = \frac{g\left(B_p^1(\epsilon) - B_p^0(\epsilon)\right) - \tau_p^0 + \lambda\left(E_{p'p^0}\epsilon - B_p^0(\epsilon)\right)}{r + s}. \]  \hspace{1cm} (27)

(see Appendix for the derivation of 19-21).

**Proposition 2 (Property and Existence of Equilibrium)** If unemployed workers can give up UI eligibility voluntarily, then an equilibrium exists where \( V_p^0 > 0 \) for all \( p \in P \). Furthermore, the unemployed will not voluntarily give up eligibility (\( \hat{U}_p > 0 \)) if one of the following three conditions hold: (i) contribution fees are such that \( \hat{B}_p = \pi\hat{U}_p \) for all \( p \in P \), (ii) \( \pi \leq s/(r+g+s+\lambda) \) and \( \tau_p^0 \geq \tau_p^1 \) for all \( p \in P \), and (iii) \( \lambda \approx 0, \pi > s/(r+g+s+\lambda) \) and \( \tau_p^0 \geq \tau_p^1 \) for all \( p \in P \). Finally, the employed who are eligible for UI benefits will receive positive expected gains from matches (\( V_p^1 > 0 \)) for all \( p \in P \) under conditions (i) and (ii).

Proposition 2 states two conditions under which eligible workers have no incentive to quit current jobs or to reject job offers. One is \( \hat{B}_p = \pi\hat{U} \). In this case, the value of keeping current jobs is strictly positive, the same as what workers gain before their gaining UI.
eligibility \( (V^1_p = V^0_p > 0) \). The irrelevance of UI would be established in this case in the following subsection. The other one is the probability \( \pi \) is sufficiently small and the UI fees paid by ineligible workers are sufficiently high. In this case, eligible workers have small chance to obtain UI benefits after quitting jobs or turning down offers. In addition, they expect to receive lower UI fees if they keep working or accept offers. Both of these make moral hazard unemployment less desirable.

2.4 Irrelevance of the UI System

In this part, I study a case where the UI fee \( \tau^e \) is an endogenous variable such that it adjusts to fully finance the UI system. In a theoretical case, I show that the UI generosity would have no effect on the labor market outcomes if the rules of UI provisions can eliminate moral hazard from becoming or remaining unemployed \( (S^e_p (\epsilon) > 0 \text{ for } \epsilon = 0, 1) \), and the UI system is fully funded.\(^{10}\)

**Definition 3:** A fully funded UI system is one in which the expected present discounted value of net benefits from the UI system for a worker who is newly hired but not yet entitled to UI is zero.

**Proposition 3 (Irrelevance of UI System)** If the UI system is fully funded, contribution fees can be designed to render the UI system neutral in the sense that the level of UI benefits, the duration of these benefits and the time it takes to become eligible for UI are all irrelevant for the determination of output, vacancies, and unemployment. In particular, if the UI contribution fees are such that \( \tau^0_p = g\bar{B}_p \) and \( \bar{B}_p = \pi\bar{U}_p \), then the UI system is fully funded and neutral.

In the deterministic version of the model, it is interesting to remark that with \( \pi > \frac{\bar{B}}{r+s} \), the irrelevance result requires that the UI system gives a subsidy to the UI-eligibles, and collects the UI fees only from UI-ineligibles.\(^{11}\) This scheme of contribution system is optimal. Intuitively, large value of \( \pi \) implies low cost of job quits and job rejections. Therefore, it is desirable for eligible workers to reject offers or to quit their current jobs, which discourages job creation activity by firms. To restore the optimum, the UI agency provides the eligible workers subsidies to induce them to engage in market activities, which would increase the profits received by the firm and raise vacancies in equilibrium. Finally, two types of workers pay the same UI fees when \( \pi = \frac{\bar{B}}{r+s+g} \).

However, the conditions under which the irrelevance result holds seem too strong to be satisfied in reality. It is less likely that the UI provisions can completely rule out moral

---

\(^{10}\)The irrelevance result of the UI system with homogeneous workers is established in Faig and Zhang (2008).

\(^{11}\)In the deterministic model, the contribution scheme that ensures an irrelevance of UI is \( \tau^0 = g\bar{B} \), and \( \tau^1 = (s - \pi (r + s))\bar{U} \).
hazard behavior, and the prevalent UI system in the United States may not fully funded by the UI fees in the way stated in Proposition 3. Hence, the realistic UI system does affect the key variables in the labor market, such as output, vacancies, and unemployment; and the final result depends on whether the entitlement effect dominates the two disincentive effects: moral hazard effect and financial costs effect.

3 A Computation of the Benchmark Equilibrium

This section calibrates a discrete time version of the model laid out in Section 2 with $\pi = 1$. The calibration targets aim to replicate the main rates and flows in the labor market and, in a stylized way, the key features of the taxation and UI systems in the United States. The model period in the simulations is set to be one week. The consecutive periods are aggregated to construct monthly or quarterly series to match the implications of the model with properties of empirical series observed at those frequencies.

3.1 Parameterization

The interest rate is set to target the annual rate of 5.2 percent. To calibrate the value of leisure $l$, I pick the value to fit the standard deviation of the aggregate vacancy-unemployment ratio conditional on productivity, 0.151 as reported in Shimer (2005). With respect to the flow turnover cost $c$, it is set to be one by following a similar strategy in Shimer (2005).

As to the technology and matching parameters, the elasticity parameter in the matching function $\eta$ is set to match the observed volatility of unemployment conditional on productivity, 0.0775 as reported by Shimer. Intuitively, for a given standard deviation of vacancy-unemployment ratio, a decline in $\eta$ increases the standard deviation of job finding rate, and then raises the volatility of unemployment. The value of $\mu$ is chosen by matching the average aggregate unemployment rate over the period 1951-2003: $U^{as} = 5.67\%$. The value of worker’s bargaining power, $\beta$, is pinned down by applying the Hosios rule, so $\beta = \eta$.

As for the exogenous job separation rate $s$, it is set to match the average short-term unemployment rate over the period 1951-2003, 2.44%, which is measured as the ratio of the unemployment less than 5 weeks to the total unemployment.

It remains to specify the parameters in productivity. The match-specific productivity $\epsilon$ is assumed to be drawn from a uniform distribution with the lower bound $\epsilon$ normalized to zero (so $\bar{p}_p$ is the lowest productivity in a match). As to the upper bound $\bar{p}$, since the

---

12 For more general cases where $\pi < 1$, see Zhang and Faig (2010).
13 The normalization adopted by Shimer (2005) of setting average $\theta$ equal to one yields identical results except for the calibrated value of $\mu$.
14 The choice of the distribution does not affect the main qualitative results in this paper.
spread of the match-specific productivity affects the degree of moral hazard behavior, and then the volatility of job separations, the value of $\tau$ is chosen to match the observed standard deviation of job separation rate conditional on productivity, which is 0.0393 as given by Shimer (2005). The aggregate productivity, for a given $p$, is the weighted average of the expected productivity of two types of matches: 

$$\hat{p} = \frac{E^0}{E^1 + E^0} \left( \bar{p} + \frac{\tau}{2} \right) + \frac{E^1}{E^1 + E^0} \left( \bar{p} + \frac{\tau+1}{2} \right),$$

where $E^e$ measures the number of employed workers of type $e$. The median of the weekly productivity $\hat{p}$ is normalized to one. Following Shimer (2005), the common part of productivity $\bar{p}$ is assumed to follow a stochastic process that satisfies: 

$$\bar{p} = l + \tau^0 + e^y(\bar{p}^* - l - \tau^0),$$

where $\bar{p}^*$ is the mean of $\bar{p}$ and is determined by targeting the normalized median of $\hat{p}$; $y$ is a zero mean random variable that follows an eleven-state symmetric Markov process in which transitions only occur between contiguous states. As detailed in Zhang (2008), the transition matrix governing this process is fully determined by two parameters: the step size of a transition $\nabla$, and the probability that a transition occurs $\lambda$. The parameters $\nabla$ and $\lambda$ are picked to fit the moments of the quarterly productivity, namely the standard deviations 0.020 and the autocorrelation 0.878.

For the parameters of the UI program, the calibrations aim to be consistent with the average time it takes for a worker to gain UI eligibility, the average duration of UI benefits and the average actual replacement rates of UI benefits. In the United States, UI eligibility takes around 20 weeks of work and the maximum duration of benefits is around 24 weeks. The actual replacement ratio $(b/w)$ is measured as the ratio of the average weekly UI benefits paid to the eligible unemployed workers over the average weekly insurable earnings paid to the employed workers, which is around 0.357 over the period of 1972-2003 as reported in Zhang (2008). So $b/w = 0.357$. Finally, the values of $\tau^e$ are assumed to be the same and be proportionate to wages in all the simulations for reasons of simplicity, so $\tau = \tau^0 = \tau^1$. The parameter $\tau$ is interpreted as a general tax including the UI contribution fees and is determined to target the general tax burden relative to GDP, which is $\tau = 30\%$. So, the government is using a large fraction of $\tau^e$ to finance a public good, which yields separable utility to the constituents of the economy.

---

15See Card and Riddell (1992) and Osberg and Phipps (1995) for the weeks needed to gain eligibility. The number of weeks eligibility lasts is an average over the period 1951-2003 reported by annual report and financial data from the U.S. Department of Labor Employment and Training Administration (column 27). It is available at http://workforcesecurity.doleta.gov/unemploy/hb394.asp.

16Notice that since leisure is not taxed, income taxes can be considered as part of the opportunity cost of employment. Defining $t = \tau l / (1 - \tau)$, the opportunity cost of employment can then be decomposed into three components: the value of leisure $l$, the value of UI benefits $b$, and a term that captures the effect of taxes $t$.

The values of \( \{r, c, \xi, g, d, \pi, \tau\} \) follow directly from the stated targets described above. The values of the remaining parameters \( \{l, \mu, \eta, \beta, s, \nabla, \lambda, \overline{p}^*, b\} \) are obtained with the following iterative procedure. First, an initial guess about the values of these parameters is formed. Using this guess the model is simulated for a long horizon (144,000 weeks), and the initial guess is then revised. This process continues until the predictions of the model match the targets. Of particular note is that in simulations, the short-term unemployment and total unemployment are calculated under the following assumption: once a contact between a worker and a firm is created, a job match is formed regardless whether the worker accepts the job offer or not. Under this assumption, the spell of unemployment is interrupted as long as a contact with a firm is made. This assumption captures the periods of tryouts and probations observed in the reality. Workers, particularly the low-skilled, try jobs for a short period and then quit (or are fired) if the match is not desirable.\(^{18}\) In the simulations, I limit the tryout period to one week. Table 1 reports the calibrated values of the parameters.

\(^{18}\)When this assumption is relaxed, that is, the job rejections do not interrupt a recorded unemployment spell, the main quantitative results are unchanged. However, as shown in Zhang and Faig (2010), in the presence of training costs, this assumption helps improve the model’s explanatory power.
3.2 Benchmark Results

The upper section of Table 2 shows some results that the parameterization was chosen to match, which shows the benchmark parameterization are well behaved. Particularly, the predicted unemployment is 5.67 percent. The standard deviation of \( \theta \) conditional on productivity at weekly frequency is 0.151. The model implies that the weekly finding rate is 0.129, yielding a monthly rate of 0.516, close to the value of 0.452 calculated by Shimer (2005). Meanwhile, the model predicts a weekly job separation rate 0.0077, which is equivalent to a monthly rate 0.030, almost the same as the one measured by Shimer. It is interesting to point out that the predicted weekly (effective) finding rate for the UI-nonrecipients is much higher than that for the UI recipients.\(^\text{19}\) This sharp contrast reflects various effects of the UI system on firms’ optimal job-creation behavior. The presence of job rejections in the market with the UI-recipients reduces the firm’s profit and discourages job creation activities. The desire to earn UI entitlement by the UI-nonrecipients raises the firm’s profits and promote job openings. Consequently, the entry into the market with the UI-ineligibles is more attractive to the firms. Lastly, the number of quits accounts for only a small fraction of job separations in the model, which can be explained by the small degree of heterogeneity in productivity among workers (small \( \bar{\tau} \)).

<table>
<thead>
<tr>
<th>Table 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Simulation Results</td>
</tr>
<tr>
<td>Benchmark Implications</td>
</tr>
<tr>
<td>Average unemployment rate</td>
</tr>
<tr>
<td>Standard deviation of ( \theta ) conditional on ( \hat{p} ) (weekly)</td>
</tr>
<tr>
<td>Weekly average job-finding rate ( f ) (aggregate)</td>
</tr>
<tr>
<td>Weekly average finding (or matching) rate for ineligible unemployed ( f_p^0 (1 - H_p^0) )</td>
</tr>
<tr>
<td>Weekly average finding rate for eligible unemployed ( f_p^1 (1 - H_p^1) )</td>
</tr>
<tr>
<td>Weekly average job separations</td>
</tr>
<tr>
<td>Weekly average quits (fraction of job separations %)</td>
</tr>
</tbody>
</table>

Effects of Alternative UI Systems
1. \( \Delta b = 0.01 \)
| \( \Delta \) log avg. unemployment rate (%) | 3.30 |
| \( \nabla \) log avg. finding rate (%) | 2.70 |
| \( \Delta \) log avg. separation rate (%) | 0.70 |
2. Unemployment if \( \pi = 0 \) (%) | 3.85 |
3. Unemployment if \( b = 0 \) (%) | 4.00 |

\(^{19}\)In the simulations, all ineligible workers take job offers for all \( p \in P \) although they are allowed to turn down the offers. This implies \( H_p^0 = 0 \) for all \( p \in P \).
3.3 The Impacts of the Change in the UI Replacement Rate

In the previous section, I establish that the model accounts well for the data in the labor market in the United States. Now I am in a position to conduct comparative statics to study if the model is able to generate positive and moderate response of unemployment to the rise in benefits. Meanwhile, I examine the relative importance of entitlement effect and moral hazard effect in determining unemployment. To this end, I set off by increasing the benefit payment by 1 percent units of productivity, and proceed with another two UI reforms: 1) shutting down moral hazard behavior by setting the probability of collecting benefits \( \pi \) to be zero; 2) removing both entitlement effect and moral hazard effect by setting \( b \) to be zero. In all the alternative UI systems, except for the changes in the policy parameter values as mentioned above, the remaining model parameters keep unchanged. The lower part of Table 2 delivers the results from these UI policy changes.

**Predicted Response of Unemployment to UI Policy Changes**

It is widely recognized that the effect on unemployment of a rise in benefits is modest. For example, Costain and Reiter (2008), based on cross-country regressions, estimates that the semi-elasticity of unemployment with respect to the UI replacement rate is around 2.\(^{20}\) My results are in line with this conventional view. When the UI benefit rises by 0.01, unemployment in logs rises by 3.3 percent.\(^{21}\)

Several papers, including Hornstein et al (2005), Costain and Reiter (2008), and Zhang (2008), criticize that the calibration method for the value of leisure proposed by Hagedorn and Manovskii (2008) causes dramatic reactions of unemployment to the labor policy changes in the standard model, although it resolves the volatility puzzle. By using a calibration strategy in the spirit of their argument, this model not only nicely preserves the business cycle properties of the standard Mortensen-Pissarides model, but also fixes the overreaction problem. The main reason for this success is because the entitlement effect curbs the rise in unemployment induced by the moral hazard effect and the cost of financing the UI system.

More specifically, unemployment reacts through the following channels. More generous UI benefits raise the disutility of working, which reduces the eligible worker’s expected surplus from a match and triggers more moral hazard. One can see this channel from the increase in the log of separation rate. Moreover, this effect is magnified by the change in the firm’s job-creation incentive. The rise in moral hazard unemployment lowers the firm’s expected profits and leads to fewer vacancies for the UI-eligibles, which slows down their transitions out of unemployment (lower \( f^1 \)). However, the predicted reaction of unemployment remains realistically modest because these positive effects on unemployment

\(^{20}\)Like most cross-country regressions, the estimate in Costain and Reiter (2008) is subject to the endogeneity problem. See Hagedorn et al. (2008) for further discussion.

\(^{21}\)In the presence of training costs, the predicted responses of unemployment are even close to the realistic ones. See details in Zhang and Faig (2010).
are partially offset by the entitlement effect. The increase in benefits makes a job offer more attractive to the UI-ineligibles, and urges them to take a job at even lower wages. This promotes the job creation for the UI nonrecipients and speeds up their escape from unemployment (higher $f^0$). Table 2 shows that the overall finding rate declines in response to the rise in $b$. This is because the majority of the unemployed are receiving UI benefits, which implies that the decrease in $f^1$, caused by workers threat to quit jobs or to reject offers, is quantitatively more important than the increase in $f^0$ resulting from the entitlement effect. The last channel at work is that the larger UI benefits make it more costly for the UI-eligibles to lose entitlement, and, therefore, restrain the job rejections.

**Contributions of Moral Hazard Effect and Entitlement Effect**

The results in the last two lines show that both entitlement effect and moral hazard quits are quantitatively important. When the moral hazard effect is missing from the model ($\pi = 0$), unemployment drops by one third to 3.85 percent. In the last line, one sees that in the absence of both entitlement effect and moral hazard effect, although the predicted unemployment drops, the overall effect is sizably smaller than the one with $\pi = 0$. This difference reflects the role played by entitlement effect. Setting $b$ to be zero shuts down both effects from the model. The absence of entitlement effect increases unemployment.

## 4 Concluding Remarks

This paper investigates the effects of UI generosity on the labor market outcomes in the Mortensen and Pissarides search and matching model where the realistic UI eligibility rules are endogenized and worker heterogeneity is introduced. This work illustrates the variety of effects that the UI system may have on unemployment. The entitlement effect arises since the presence of the UI system creates the desire for the UI nonrecipients to gain UI entitlement and the incentive for the UI recipients to retain UI eligibility, which facilitates forming employment relationships and reduces unemployment. The UI system has two unintentional effects. A more expensive UI system hurts employment due to the burden of the UI contribution fees required to finance the program. Also, a more generous UI system aggravates the moral hazard problem since the improved outside option induces more workers engaged in the low-productivity matches to quit their jobs and more workers paired up with bad jobs to turn down offers as long as they are entitled to UI.

These offsetting effects of the UI system on unemployment imply that under some conditions the irrelevance of the UI system emphasized in Faig and Zhang (2008) holds with heterogeneous workers. Like Ricardian Equivalence, this irrelevance result hinges on specific conditions that do not necessarily hold in reality and therefore it is not meant to characterize the UI system as irrelevant in reality. However, it can be used as a
benchmark to pinpoint the economic effect of the UI system on the labor market. That is, if the system does have some effects on the labor market outcomes, it must be related to the way it is financed since it would distort the firm’s job creation behavior. Or, it might be due to the rules of the UI provisions since it would trigger moral hazard quits or rejections. Lastly, it might be because workers are not risk-neutral.

Introducing the realistic institutional details of the UI system is crucial to improving the model’s empirical performance. With a large value of leisure, as argued by Hagedorn and Manovskii (2008), the model successfully reproduces different cyclical and UI policy-related variations in unemployment. This proves to be an insurmountable challenge in the standard model where unemployed workers receive UI unconditionally. This paper can meet this challenge mainly because the entitlement effect attenuates the rise in unemployment caused by the moral hazard and financial cost effects. However, this mechanism is absent from the standard model.

This work can be extended in several ways. For example, it can provide a framework to study to what extent the generosity of the UI system itself can explain the large disparity in the level and duration of unemployment between the United States and the European countries. It is well known that the European countries provide much longer UI benefits relative to the one in the United States. The model suggests that with everything else equal, the extension of the UI benefits from 24 weeks to 52 weeks raises unemployment from 5.55 percent to 6.86 percent. Also, in this paper the labor market transitions are limited to the changes over employment and unemployment. However, some empirical evidence shows that the UI generosity causes substantial flows into and out of the labor force (see Moothy 1989; Atkinson and Micklewright 1991; Andolfatto and Gomme 1996b). Since the driving forces underlying these flows could be entitlement and moral hazard effects as stressed in this paper, it is interesting to consider the state of being out of labor force, which is missing in this contribution, but likely important in enhancing our understanding in the behavioral effects of the UI system for labor market participants.

5 Appendix

5.1 Proof of Tenure-Independent Contract

Tenure-independent contract: The optimal dynamic contract repeats the static contract, provided that the firm can commit to not renegotiate the contract.

Proof: when wage contracts are assumed to be increasing with tenure, firms offer deferred compensation. However, the firm does not benefit from such a compensation. Because the worker’s opportunity cost of employment is time-invariant, the deferred compensation does not influence the worker’s participation constraint at the hiring margin (i.e., the incentive to take a job), but loosens the participation constraints in the firm’s optimal
contract decision. Consequently, there is no loss for the firm to restrict attention to tenure-independent contracts. An alternative proof is that given the linear preference, in the model what workers (firms) care about is the expected present discounted value of wages (profits) at the hiring margin. Hence, how wages evolve over the spell of employment does not matter for the equilibrium outcomes.\footnote{See Pissarides (2009) for a similar argument with discussion in greater detail. A formal mathematical proof is available upon request.}

5.2 Proof of Proposition 1

I first show proof in a deterministic version of the model \((\lambda = 0)\), and then relax this restriction later on. Mathematically, Step 2 can be formalized as: \(\max_{R^e} -c + q^e (f^e(R^e)) (V^e - R^e)\), s.t. \(U^e\).

FOCs with respect to \(R^e\) lead to

\[ q^e_{f^e} \cdot f^e_{R^e} \cdot (V^e - R^e) = q^e, \forall e = 0, 1. \tag{28} \]

Rearranging (28) yields

\[
\left( \frac{\partial q^e}{\partial f^e} \cdot \frac{f^e}{q^e} \right) \cdot \left( \frac{\partial f^e}{\partial R^e} \cdot \frac{R^e}{f^e} \right) (V^e - R^e) = R^e, \forall e = 0, 1. \tag{29} \]

The first term in brackets in (29) is the elasticity of the vacancy filling rate with respect to the job finding rate in a submarket with workers of type \(e\), denoted by \(\varepsilon_{q^e f^e}\). The second term in (29) is the elasticity of the job finding rate with respect to the worker’s expected gains from a match, denoted by \(\varepsilon_{f^e R^e}\). Recalling (12) and (13), it is easy to check that for a given \(U^e\), \(\varepsilon_{q^e f^e R^e} = -1, \forall e = 0, 1\). Besides, since \(f^e = \theta^e q^e\), \(\varepsilon_{q^e f^e} = \frac{\varepsilon_{q^e} q^e}{\varepsilon_{q^e f^e}}\) where \(\varepsilon_{q^e f^e}\) is the so-called elasticity of the finding rate with respect to the submarket tightness \(\theta^e\). Since \(\varepsilon_{q^e f^e} = -\eta \in (-1, 0)\), it is easy to check that \(\varepsilon_{f^e R^e} = 1 - \eta\). Hence, (29) can be expressed as:

\[
\frac{R^e}{V^e - R^e} = \frac{\eta}{1 - \eta}, \forall e = 0, 1. \tag{30} \]

Equation (30) suggests that in the submarket with workers of type \(e\), the Hosios condition holds. That is, the fraction of the total surplus from a match that goes to a worker is equal to the worker’s contribution to forming a match.

When \(\lambda = 0\) is relaxed, equations (12) and (13) show that \(\varepsilon_{q^e f^e R^e} = -1\) still holds for \(e = 0, 1\). Given that \(\varepsilon_{q^e f^e p} = -\eta\), it follows \(\varepsilon_{q^e f^e p^e} = 1 - \eta\). Hence, a stochastic version of (29) can be expressed as:

\[
\frac{R^e_{p^e}}{V^e_{p^e} - R^e_p} = \frac{\eta}{1 - \eta}, \forall p \in P \text{ and } e = 0, 1. \]

So the Hosios rule holds in a stochastic version of the model.
5.3 Derivation of Equations (19)-(21)

**Equation (19):** Subtracting (12) from (13) and combining with the surplus sharing rule (17) leads to (19).

**Equation (21):** Substituting equations (2), (4) and (6) into (9)-(11) with \( e = 0 \), and combining with the equation (22) and the sharing-rule (17) yields:

\[
S_p^0 (\epsilon) = \frac{\hat{p}_p (\epsilon) - l - \beta f^0_p V^0_p + g \hat{B}_p (\epsilon) - \tau^0_p + \lambda (E_p S^0_p (\epsilon) - S^0_p (\epsilon))}{r + s}, \forall \epsilon \geq e^0_p. \tag{31}
\]

**Equation (20):** For \( \epsilon \in [\epsilon, \epsilon^1_p] \), \( S_p^1 (\epsilon) = 0 \); otherwise, the *ex post* value of \( S_p^1 (\epsilon) \) can be derived by substituting equations (3), (5) and (7) into (9)-(11) with \( e = 1 \) and combining with the sharing-rule (17):

\[
S_p^1 (\epsilon) = \frac{\hat{p}_p (\epsilon) - \ell - \beta f^1_p V^1_p - \tau^1_p + \left[ d + (1 - \pi) \left( r + s + f \left( \theta (V^1_p) \right) \right) \right] \hat{U}_p}{-b - \lambda (1 - \pi) \left( E_p \hat{U}_p - \hat{U}_p \right) + \lambda \left( E_p S^1_p (\epsilon) - S^1_p (\epsilon) \right)}, \forall \epsilon \geq e^1_p. \tag{32}
\]

According to the definition of (22), combining the equations (19), (31) and (32) gives (20).

5.4 Proof of Proposition 2

This proof assumes a discrete number of aggregate productivities \( (n) \) and a discrete number of quality matches \( (m) \).

Define \( \theta (V) \) to be the real function that satisfies: \( c \theta = f (\theta) (1 - \beta) V \). The assumed properties of the matching function imply that \( \theta / f (\theta) \) is a strictly increasing function of \( \theta \) such that \( \lim_{\theta \to 0} [\theta / f (\theta)] = 0 \), so \( \theta (V) \) is well defined, continuous and increasing, and \( \theta (0) = 0 \). Using these definitions and \( S^e_p (\epsilon) \geq 0 \) for \( \epsilon \in [\epsilon^e_p, \bar{\epsilon}] \) and all \( p \in P \), the modified system of equations characterizing an equilibrium can be transformed into:

\[
\hat{U}_p = \max \left\{ \frac{b + f \left( \theta \left( V^1_p \right) \right) \beta V^1_p - f \left( \theta \left( V^0_p \right) \right) \beta V^0_p + \lambda E_p \hat{U}_p}{r + d + (1 - \pi) \left( \theta (V^1_p) \right) + \lambda}, 0 \right\}. \tag{33}
\]

\[
\hat{B}_p (\epsilon) = \max \left\{ -S^0_p (\epsilon) + \pi \hat{U}_p, \frac{\tau^0_p - \tau^1_p + s \hat{U}_p + \lambda E_p \hat{B}_p (\epsilon)}{r + s + g + \lambda} \right\}. \tag{34}
\]

\[
S^0_p (\epsilon) = \max \left\{ \frac{\hat{p}_p (\epsilon) - \ell - \tau^0_p - f \left( \theta \left( V^0_p \right) \right) \beta V^0_p + g \hat{B}_p (\epsilon) + \lambda E_p S^0_p (\epsilon)}{r + s + \lambda}, 0 \right\}. \tag{35}
\]
\[ S_p^1(\epsilon) = \max \left\{ \frac{\hat{p}_p(\epsilon) - \ell - \tau^1_p \beta V^0_p + \beta V^0_p + [s - \pi (r + s)] \hat{U}_p + \lambda \pi (E_p \hat{U}_p' - \hat{U}_p) + \lambda E_p S^1_p(\epsilon)}{r + s + \lambda}, 0 \right\}. \quad (36) \]

\[ V^*_p = \int_{\xi}^{\tau} S_p^1(\epsilon)dH(\epsilon), \text{ for } \epsilon \in \{0, 1\}. \quad (37) \]

Note that equations (35) to (37) imply that \( V^*_p \geq 0 \) for \( \epsilon \in \{0, 1\} \). Define \( \tau = \min \{ \tau^e_p \mid \epsilon \in \{0, 1\}, \ p \in P \} \), \( z = \max \{ \hat{p}_p + \tau - l - \tau \mid p \in P \} \). Given that \( \hat{p}_p + \epsilon - \tau^0_p \geq l \), for \( p \in P \), one has \( z > 0 \). Define \( \hat{V}^1 = \frac{z}{r + s} \) if \( \pi \geq s/(r + s) \); otherwise, \( \hat{V}^1 \) is the smallest positive root of the following continuous function:

\[ \Psi(V) = V - \frac{z}{r + s} - \left( \frac{s}{r + s + \pi} \right) \left( \max_{y \in [0, V^1]} \frac{b + \beta f(\theta(y)) y}{r + d + (1 - \pi) f(\theta(y))} \right) \quad (38) \]

The existence of this root is guaranteed because \( \Psi(0) < 0 \), and as \( V \to \infty \), the slope of \( \Psi(V) \) is strictly positive if \( s/(r + s) \geq \pi \), so \( \Psi(V) \) must be positive for \( V \) sufficiently large. Finally, define

\[ \hat{U} = \frac{b + \beta f(\theta(y)) y}{r + d + (1 - \pi) f(\theta(y))}, \quad \hat{B} = \min_{\mathcal{P} \in P} \left\{ \frac{\tau^0_p - \tau^1_p}{r + s} \right\}, \]

\[ \hat{B} = \hat{B} + \hat{U}, \text{ and } \hat{V}^0 = \frac{z + g \hat{B}}{r + s}. \]

For a set of functions \( \{ \hat{U}_p, \hat{B}_p(\epsilon), V^0_p, S^0_p(\epsilon), V^1_p, S^1_p(\epsilon) \} \), define the mapping \( F \) as follows. Let \( x \in R^{4n(1+m)} \) be the vector \( (\hat{U}_1, \ldots, \hat{U}_n, \hat{B}_1(\epsilon_1), \ldots, \hat{B}_n(\epsilon_1), \ldots, \hat{B}_n(\epsilon_m), V^0_1, \ldots, V^0_n, S^0_1(\epsilon_1), \ldots, S^0_1(\epsilon_m), \ldots, S^0_n(\epsilon_1), \ldots, S^0_n(\epsilon_m), V^1_1, \ldots, V^1_n, S^1_1(\epsilon_1), \ldots, S^1_1(\epsilon_m), \ldots, S^1_n(\epsilon_1), \ldots, S^1_n(\epsilon_m)) \), and \( F(x) \in R^{4n(1+m)} \) be the values of \( \{ \hat{U}_p, \hat{B}_p(\epsilon), V^0_p, S^0_p(\epsilon), V^1_p, S^1_p(\epsilon) \}_{p \in P, \epsilon \in [0, \tau]} \) on the right-hand-side of (33) to (37) when the left-hand-side of these equations is evaluated at \( x \). Define \( X \) as the subset of \( R^{3n(1+m)} \) that satisfies the following bounds: \( x_i \in [0, \hat{U}] \) for \( i = 1 \) to \( n \), \( x_i \in [\hat{B}, \hat{B}] \) for \( i = n + 1 \) to \( 2n (m + 1) \), \( x_i \in [0, \hat{V}^0] \) for \( i = 2n (m + 1) + 1 \) to \( 3n (m + 1) \), \( x_i \in [0, \hat{V}^1] \) for \( i = 3n (m + 1) + 1 \) to \( 4n (m + 1) \). The set \( X \) is non-empty, closed, bounded, and convex. The function \( F \) is continuous and maps \( X \) onto itself. Consequently, as a result of Brower’s fixed point theorem, \( F \) has a fixed point in \( X \). This proves existence.

The proof that \( V^0_p > 0 \) for all \( p \in P \) is by contradiction. Suppose that there were \( p \in P \) such that the solution to (33) to (37) satisfied \( V^0_p = 0 \). Definition (37) would then imply that \( S^0_p(\epsilon) = 0 \) for \( \epsilon \in [0, \tau] \). Equation (35) would then imply \( \hat{B}_p(\epsilon) < 0 \) for
$\epsilon \in [\epsilon^0, \bar{\epsilon}]$ (since $\bar{p}_p + \epsilon - \ell - \tau^0_p > 0$ for all $p \in P$). This would contradict (34) because if $V^0_p = 0$, the equation (34) implies that $\hat{B}_p (\epsilon) \geq \pi \hat{U}_p \geq 0$ for all $\epsilon$. As a result, it must have $V^0_p > 0$.

If $\hat{B}_p = \pi \hat{U}_p$ for all $p \in P$, the equation (26) implies that $V^1_p = V^0_p > 0$ for all $p \in P$. Furthermore, (33) implies that $\hat{U}_p > 0$ since $b > 0$.

Equations (34) and (22) give:

$$(r + s + g + \lambda) \left( S^1_p (\epsilon) - S^0_p (\epsilon) \right) \geq [s - \pi (r + s + g + \lambda)] \hat{U}_p + \left( \tau^0_p - \tau^1_p + \lambda E_p \hat{B}_p (\epsilon) \right).$$

Taking integral over $[\xi, \overline{\epsilon}]$ yields:

$$(r + s + g + \lambda) \left( V^1_p - V^0_p \right) \geq [s - \pi (r + s + g + \lambda)] \hat{U}_p + \left( \tau^0_p - \tau^1_p + \lambda E_p \hat{B}_p (\epsilon) \right). \quad (39)$$

If $s \geq \pi (r + s + g + \lambda)$ and $\tau^0_p \geq \tau^1_p$, then (39), together with $\hat{B}_p \geq \hat{B}$, implies $V^1_p \geq V^0_p$ for all $p \in P$, which together with (33) implies $\hat{U}_p > 0$ for all $p \in P$.

If $s < \pi (r + s + g + \lambda)$, then $V^1_p$ may be smaller than $V^0_p$. In the absence of shocks ($\lambda = 0$), if $V^0_p \leq V^1_p$, then (33) implies $\hat{U}_p > 0$. Otherwise, if $V^0_p > V^1_p$, then (39), together with $\lambda = 0$, $\tau^0_p \geq \tau^1_p$ and $s < \pi (r + s + g + \lambda)$, implies $\hat{U}_p > 0$. Finally, continuity ensures similar results for $\lambda \approx 0$.

5.5 Proof of Proposition 3

Proof: If $\hat{B}_p = \pi \hat{U}_p$, then equation (26) implies that $V^0_p = V^1_p$. This result, together with (18), (21), (25) and $\tau^0_p = g \hat{B}_p$, implies that both markets have identical vacancy-unemployment ratios determined by $V^e_p = \left[ \int_\xi^\overline{\epsilon} \hat{B}_p (\epsilon) \partial H (\epsilon) - \ell + \lambda (E_p V^e_p - V^e_p) \right] / [r + s + \beta f (\theta^e_p)]$ and (18). Since these are the equations that determine the vacancy-unemployment ratio in a model without UI, the UI system has no effect on $\theta^e_p$ for $e \in \{0, 1\}$ and for all $p \in P$. Hence, it has no effect on output, vacancies, and unemployment. With $V^0_p = V^1_p$, $\hat{U}_p$ is the present discounted value of expected UI benefits to be received by an eligible worker. So, the UI system is fully funded if $B^0_p (\epsilon) = 0$ in (27). Such equality is ensured if $\tau^0_p = g \hat{B}_p (\epsilon)$. Taking integral over $[\xi, \overline{\epsilon}]$ on both side of this condition, it follows that $\tau^0_p = g \hat{B}_p$.

References


