Options: General

[1] Define the following terms associated with options:
   a. Option – An option is a contract which gives the holder the right to buy or sell an underlying asset at a fixed price on or before a given date.
   b. Exercise – Exercise is the act of buying or selling the underlying asset through the option contract.
   c. Strike price – The strike price/exercise price is the fixed price at which the holder can buy or sell the underlying asset.
   d. Expiration date – The maturity date of the option contract. It is the last date on which an American option can be exercised. It is the only date on which a European option can be exercised.
   e. Call option – A call is an option contract which gives the holder the right to buy an underlying asset at a fixed price on or before the contract’s expiration date.
   f. Put option – A put is an option contract which gives the holder the right to sell an underlying asset at a fixed price on or before the contract’s expiration date.

[2] What is the difference between European options and American options?

American Options can be exercised on any date before and including the exercise date. A European option can be exercised only on the expiration date.

[3] Alicia holds American put options on Google (Option code: GOQ). The exercise price of the put is $400 and Google stock is selling for $395.03 per share. If the put sells for $4.30, what is the best strategy for Alicia? (Hints: How can Alicia make the most profit possible?)

There is an arbitrage opportunity for her to exploit. She exploits such opportunity by the following strategy: [i] Buy one put contract (i.e., paying 100x$4.30 =$430), and [ii] buy 100 shares of Google that costs her $39,503, and [iii] immediately exercise the put option contract to sell 100 shares of Google at $400. She will get back $40,000. Her net profit from such strategy if she does it once is: $40,000 - $39,503 - $430 = $67.

Of course, to fully exploit this opportunity, she’ll have to execute such series of transactions either many times or inflate the amount involved as much as she can in one such series of transactions. In short, there is an arbitrage opportunity and she should rationally exploit it.
A call option on Intuitive Surgical stock (Option code: AXQ) currently trades for $6. The expiration date is December 17 of this year. The exercise price of the option is $95.

a. If this is an American option, on what dates can the option be exercised?

Any date before and including the expiration date.

b. If this is a European option, on what dates can the option be exercised?

Only on Dec 17, 2005

c. Suppose the current price of Intuitive Surgical stock is $80. Is this option worthless?

Today is already Nov 14, and there is only 1 month left for the stock price to climb from $80 to above $95 for a call option to be “in-the-money”. The chance is slim. But as long as there is some positive probability, the option is not worthless.

Janet sold 10 IBM put contracts (Option code: IBM) (Note: not options, but contracts) and bought 5 IBM call contracts with the proceeds of the sales. Both options have the same exercise price of $85 and the same expiration date. Draw the payoff diagram of her zero-investment portfolio. (Zero in the sense that her net cash outflow is zero)

Her payoff would look like the following (try to figure out why I use these numbers):

![Payoff Diagram]

The strike price of a call option on Sony Corporation common stock (Option code: SNE) is $40.

a. What is the payoff at expiration on this call if, on the expiration date, Sony stock sells for $35?
b. What is the payoff at expiration of this call if, on the expiration date, Sony stock sells for $45?

$5/option, $500/option contract.

c. Draw the payoff diagram for this option.


a. What is the payoff at expiration of this put if, on the expiration date, Sony stock sells for $45?

$0

b. What is the payoff at expiration of this call if, on the expiration date, Sony stock sells for $25?

$15/option, $1500/option contract.

c. Draw the payoff diagram for this option.
[8] You hold a European call option contract on Coca-Cola (Option code: KO) stock. The exercise price of the call is $50. The option will expire in moments. Assume there are no transactions costs or taxes associated with this contract.
   a. What is your profit on this contract if the stock is selling for $51?

   $1 \times 100 = $100

   b. If Coca-Cola stock is selling for $49, what will you do?

   Do not exercise. The option is thus expired without exercise.

[9] Merrill Lynch’s common stock currently sells for $66. Both puts and calls on Merrill Lynch are being traded. These options all expire six months from today, and they have a strike price of $70. Six months from today, Merrill Lynch common stock will sell for $77 with a probability of 0.5. It will sell for $55 with a probability of 0.5. You own a put contract (again, it is a contract, not just one option) on Merrill Lynch. Now, you are becoming nervous about the risk to which you are exposed.
   a. What other transactions should you make to eliminate this risk?

   To completely eliminate the risk of the put, sell a call contract (with the same expiration date and strike price) and buy 100 shares of Merrill Lynch right now. Such strategy ensures you the same net payoff whether Merrill Lynch’s stock price rises to $77 or falls to $55.

   b. What is the expected payoff at expiration of the strategy you developed in (a)?
<table>
<thead>
<tr>
<th>Stock price at expiration = $77</th>
<th>Stock price at expiration = $55</th>
</tr>
</thead>
<tbody>
<tr>
<td>Long 100 stock shares</td>
<td>$7,700</td>
</tr>
<tr>
<td>Long 1 put contract</td>
<td>$0</td>
</tr>
<tr>
<td>Short 1 call contract</td>
<td>-$700</td>
</tr>
<tr>
<td>Net payoff at expiration</td>
<td>$7,000</td>
</tr>
</tbody>
</table>

[10] Suppose you observe the following market prices:

- American Call (Strike = $30) $3
- Stock $35

a. What should you do?

Buy the call, exercise it immediately and immediately sell the stock you get from exercising the option to realize a pure profit. Again, this is an arbitrage opportunity. Exploit it.

b. What is your profit or loss?

$200 per each series of such transaction. ($200 instead of $2 because you cannot buy one option, but an option contract with 100 options inside it)

c. What do opportunities such as this imply about the lower bound on the price of American calls?

That means a call option value cannot fall below the difference between stock price and strike price.

d. What is the upper bound on the price of American calls? Explain.

The upper bound is the stock price (the price of the underlying asset), because no one would be willing to pay more than the price of the stock for the right to receive the stock.

[11] General Electric has both call and put options traded on the CBOE (Option code: GE). Both options have the same exercise price of $40 and the same expiration date. The options will expire in one year. The call is currently selling for $2 per share and the put is selling for $1 per share. The interest rate is 10 percent. What should the stock price of General Electric in order to prevent arbitrage opportunities?
Use the put-call parity. Plug in the numbers to the equation to find out the current stock price. It should be easy for you to do. The important message is, if the put-call parity does not hold, arbitrage opportunity emerges.

List the factors that determine the value of an American call option. State how a change in each factor alters the option's value.

Strike price – the value of an American call must be at least the difference between the stock price and the exercise price. For a given stock price, a higher exercise price will reduce the value of the call.

Expiration date – the time to expiration of a call affects the price of the option. Compare two calls which are identical except for the time to expiration. The longer term option has all the rights and benefits of the shorter term option, plus more. It has all of those benefits and rights for a longer period of time. Thus, as the time to expiration increases, the value of the call increases.

Stock price – the value of an American call must be at least the difference between the stock price and the exercise price. For a given strike price, a higher stock price will increase the value of the call.

Volatility of stock price – the higher is the volatility of the price of the underlying asset, the higher is the probability that the call will be in-the-money at the expiration date. Thus, higher volatility enhances the option's value.

Interest rate – If you buy a call, you do not have to pay the strike price until the expiration date. The delay in the payment has value. As interest rate rises, the delayed payment has more value. To convince yourself, consider what else you can do with the strike price until the expiration date (your opportunity cost). You can put that money in an account and earn interest on the amount until the expiration date. If the interest rate increases, you will earn more interest.

List the factors that determine the value of an American put option. State how a change in each factor alters the option's value.

Strike price – the value of an American put must be at least the difference between the exercise price and the stock price. For a given stock price, a higher exercise price will increase the value of the call.

Expiration date – the time to expiration of a put affects the price of the option. Compare two puts which are identical except for the time to expiration. The longer term option has all the rights and benefits of
the shorter term option, plus more. It has all of those benefits and rights for a longer period of time. Thus, as the time to expiration increases, the value of the put increases.

Stock price – the value of an American put must be at least the difference between the exercise price and the stock price. For a given strike price, a higher stock price will reduce the value of the put.

Volatility of stock price – the higher is the volatility of the price of the underlying asset, the higher is the probability that the put will be in-the-money at the expiration date. Thus, higher volatility enhances the option's value.

Interest rate – If you buy a put, you have the right to sell the stock for a fixed price in the future. The present value of the delayed receipt decreases as the interest rate rises. Thus, if the interest rate rises, the value of the put will fall.

[14] a. If the risk of a stock increases (that means the stock price is more volatile), what is likely to happen to the price of call options on the stock? Why?

The price of call options will rise. As the volatility of the stock price rises, the value of a call increases. Call holder gain only if the stock price is greater than the exercise price. Call holders do not lose if the stock price is less than the exercise price. The volatility increases the probability that the call will be in-the-money.

b. If the risk of a stock increases, what is likely to happen to the price of put options on the stock? Why?

Same idea as above.

The Binomial Option Pricing
[15] You bought a call contract (Again and again, this is a contract, not 1 single option) three weeks ago. The expiration date of the calls is five weeks from today. On that date, the price of the underlying stock will be either $120 or $95. The two states are equally likely to occur. Currently, the stock sells for $96; its strike price is $112. You are able to purchase 32 shares of stock. You are able to borrow money at 10 percent per annum. What is the value of your call contract?

We know that binomial option pricing employs the strategy that buying m shares of stock per 1 short of call, the portfolio will be risk-free. In math, we see this formula, mS – C = K/(1+R)^T. A deeper look at
this equation will convince you that \( C = mS - K/(1+R)^T \), i.e., a call option will have a payoff the same as holding \( m \) shares of stock and borrow \( K \) amount of money. The exact value of \( m \) and \( K \) would be something we have to calculate. Take a look at the following table:

<table>
<thead>
<tr>
<th>Payoffs of buying a call contract</th>
<th>Stock price at expiration = $120</th>
<th>Stock price at expiration = $95</th>
</tr>
</thead>
<tbody>
<tr>
<td>(A contract covers 100 shares)</td>
<td>100 x ($120 - $112) = $800</td>
<td>$0 (not exercise)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Payoffs of Long 32 shares</td>
<td>32 x $120 = $3,840</td>
<td>32 x $95 = $3,040</td>
</tr>
<tr>
<td>Payoffs of borrowing $3,012.27</td>
<td>Repay $3040</td>
<td>Repay $3040</td>
</tr>
<tr>
<td>Net Payoff</td>
<td>$800</td>
<td>$0</td>
</tr>
</tbody>
</table>

The table indicates that buying 32 shares of stock and borrowing $3,012.27 would leave us the same payoff structure as we hold a call option contract. Thus, the cost of constructing this portfolio now should be the same as the value of the call option contract now. Thus, the value of the call option should be \( 32 x $96 - $3,012.27 = $59.73 \)

[16] Suppose Wells Fargo stock (It's a bank in US, Option code: WFC) is currently selling for $50 per share. It is expected that the stock price will be either $75 or $50 in six months. Treasury bills that will mature in six months yield 4.5%. What is the price of Wells Fargo European put option per share that has an exercise price of $60?

The first tricky thing is that the question asks put option, not call. We can do that by first calculate the call option price, and then apply put-call parity to calculate the corresponding put value. Let us do that now.

Set this equation:
\[
m^* S_{T}^{up} - (S_T - X) = m^* S_{T}^{low}
\]
\[
m^* \times $75 - ($75 - $60) = m^* \times $50
\]
\[
m^* = 0.6 \text{ (Portfolio with long 0.6 shares and short 1 call is risk-free)}
\]
The final payoff is 0.6 x $50 = $30 no matter which state (i.e., either $75 or $50) is realized.

Thus, solve this equation:
\[
m^* \times S_0 - C = (m^* \times $50) / (1+R)^T
\]
\[
0.6 \times $50 - C = $30/(1.045) \Rightarrow C = $1.29
\]

Using Put-call parity:
\[
S_0 - C + P = X/(1+R)^T
\]
\[
$50 - $1.29 + P = $60/(1.045)
\]
Assume only two states will exist one year from today when a call on Advanced Micro Devices expires (Option code: AMD). With equal probability, the price of AMD stock will be either $35 or $22 on that date. Today, AMD stock trades for $25. The strike price of the call is $28. The rate at which you can borrow is 7 percent. How much are you willing to pay for a contract of this call?

Set this equation:  
\[ m^* S_{T}^{\text{up}} - (S_T - X) = m^* S_{T}^{\text{low}} \]

\[ m^* x $35 - ($35 - $28) = m^* x $22 \]

\[ m^* = 0.5385 \text{ (Portfolio with long 0.5385 shares and short 1 call is risk-free)} \]

The final payoff is \( 0.5385 x \$22 = \$11.8462 \) no matter which state (i.e., either $75 or $50) is realized.

Thus, solve this equation:  
\[ m^* S_0 - C = (m^* x \$22) / (1+R)^T \]

\[ 0.5385 x \$25 - C = \$11.8462 / (1.07) \Rightarrow C = \$2.39 \]

The Black-Scholes Option Pricing Model

[18] Use the Black-Scholes model to price a call with the following characteristics:

- Stock price = $62
- Strike price = $70
- Time to expiration = 4 weeks
- Stock price variance = 0.35
- Risk-free interested rate = 0.05

\[ C = $1.50 \text{ (roughly)} \]

[19] Use the Black-Scholes model to price a call with the following characteristics:

- Stock price = $52
- Strike price = $48
- Time to expiration = 120 days
- Stock-price variance = 0.04
- Risk-free interest rate = 0.05

\[ C = $4.63 \text{ (roughly)} \]

[20] a. Use the Black-Scholes model to price a call with the following characteristics:

- Stock price = $70

C = (roughly)
Strike price = $90
Time to expiration = 6 months
Stock-price variance = 0.25
Risk-free interest rate = 0.06

C=$0.00131

b. What does put-call parity imply the price of the corresponding put will be?

P=$9.426(roughly)

[21] a. Use the Black-Scholes model to price a call with the following characteristics:

Stock price = $28
Strike price = $40
Time to expiration = 6 months
Stock price variance = 0.50
Risk-free interest rate = 0.06

C=$0.0024

b. What does put-call parity imply the price of the corresponding put will be?

P=$3.15443

[22] You have been asked by a client to determine the maximum price he should be willing to pay to purchase a Morgan Stanley call (Option code: MWD). The options have an exercise price of $55, and they expire in 120 days. The current price of Morgan Stanley stock is $52, the annual risk-free rate is 4 percent, and the estimated variance of the stock is 0.0576. No dividends are expected to be declared over the next six months. What is the maximum price your client should pay?

C=$0.13276