

Outsourcing to Pool Risk

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Abstract

This paper shows how firms, through the use of relatively simple contracts, pool risk among themselves indirectly yet efficiently. The result suggests that the presence of opportunity to pool risk among firms does not necessarily imply that a potential merger of them would bring any efficiency gain. It also suggests risk-pooling as a novel explanation of outsourcing. (JEL: D23, L22, L23)

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1 Introduction

To what extent does risk-pooling explain the mode of organization? In particular, whether a decentralized organizational mode of production enables firms to pool risk indirectly.

Clarifying the distinction of risk-sharing and risk-pooling is useful. Risk-sharing is a strategy a group of participants employ to share risk from a *single* source among themselves under some agreed upon terms; whereas risk-pooling is a strategy a group of participants, each facing his own risk, employ to aggregate their risk from *different* sources to make the aggregate more certain. Participants benefit because each of them faces relatively lower risk by taking a share of the aggregate risk according to some pre-specified terms.

Arrow et al. (1972), and Mulligan (1983) study risk-pooling in the context of stochastic machine breakdown under a centralized mechanism. As the number of machines increases, the number of workable machines at any point in time is more predictable. A bigger firm with more machines, therefore, can be more confident in the number of repairmen it needs, and the amount of orders it can deliver to customers in any period of time. Eppen (1979) is the seminal paper in the management science literature that explicitly model how a centralized warehouse serving various retailers would gain from pooling risk.

Implicit in their arguments is that they take such statistical economies of scale as a potential gain that only large firms can enjoy. This paper, however, asks why only big firms can realize such a gain. For instance, if a group of small firms, each needing a similar kind of repairing services, contract with a repairing company, why would not the gain from risk-pooling be realized too? Empirically, indeed, video game centers in Hong Kong are doing exactly that. Small centers, facing the risk of video game machine breakdown, do not usually own a team of technicians. Their businesses rely on independent technicians who not only serve them, but also other companies.

Herriott (1998) also points out that it is reasonable to model a risk pool as in a cooperative framework using explicit normative group decision mechanism *not* because the market mechanism cannot pool risk efficiently, but because the market mechanism gives the same equilibrium outcome as if the participants are in a cooperative framework. In other words, decentralized tradings among participants within a risk pool gives the same outcome as if participants are involved in a cooperative game-theoretic framework. The author also characterizes sufficient conditions under which

cooperative mechanism and noncooperative market mechanism give different equilibrium outcomes.

To put the argument in a context, consider the following example. Each pizza is made up of a dough, a homogenous mixture of different ingredients.¹ Since fridge size is limited, dough-kneading is usually done before rush hours. Pizza demand fluctuates; there is a risk of not kneading enough dough for the evening.

Suppose in an evening a pizza store runs out of dough. The manager can re-allocate his staffs to knead dough again. It is a costly practice because staffs must be busy at the moment with their own work. They all wish they had prepared more dough in the afternoon.

Alternatively, the manager can call another pizza store to borrow dough. If the store he calls does not have any, the manager calls another pizza store.

There is yet another organizational mode, a likely more superior one. Suppose a boss runs a few pizza stores. The aggregate demand of all stores is more certain, a result of risk-pooling. He would coordinate dough kneading across stores, and deliver the dough to each store according to their early evening sales. The benefit is threefold: [1] it reduces the need to re-allocate busy labour in rush hour to knead dough, [2] it reduces the likelihood of wasting dough for any store, and [3] the aggregate level of safety number of dough for all stores is lower than if each store is kneading their own dough for the night.

Such explicit coordination requires information sharing. To determine the optimal number of dough, the store managers have to inform the boss their realized and forecast sales. The boss, by definition, has a formal authority to command such details from his store managers.² The presence of formal authority therefore facilitates information sharing.

If stores are owned independently, however, coordination seems to be compromised because without a formal authority, no one has the right to acquire information from others.

The fundamental distinction between a centralized and a decentralized mode of organization therefore rests on the information structure. Under the former, a boss has a formal authority to acquire information for risk-pooling; under the latter, no one has authority to acquire information

¹Pizza stores do not usually mix the ingredients by themselves. They buy it from manufacturers. They come and are stored in a pre-kneaded mode, which is much smaller, to save storage space.

²Masten (1988) points out that an employee has a legal obligation to disclose all cost information to his employer. By the same token, failure to disclose relevant demand information, resulting in unnecessary cost increase, can be considered as an action not acceptable based on the spirit of employment agreement. Whether the employer can verify the failure to disclose is another issue which involves specifying the monitoring cost and likely involves moral hazard problem that this paper would abstract from.

from any other parties.

The literature has also stressed the importance of information structure on the study of organizational mode. Riordan and Sappington (1987) and Riordan (1990) point out that the crucial consequence of vertical integration is that the downstream firm has better information about upstream costs. Aoki (1986) compares the efficiency of two information structures from distinctive modes of organizations among technologically interrelated shops whose costs are uncertain. The important implication from this branch of literature is that since information structure has changed, whether one organization mode is more superior than the other depends crucially on how contracts perform, and whether contracts can bridge the informational difference.

This paper elaborates on this implication by arguing that whether the presence of risk-pooling opportunity necessarily entails the emerge of a centralized production mode, i.e., bringing transactions within one firm, hinges on whether independent firms under decentralized mode can write contracts that substitute the role of a formal authority to share information indirectly yet efficiently.

A model formalizes this insight. The model involves firms with ex post production flexibility as defined in Appelbaum and Lim (1985), and Aivazian and Berkowitz (1992, 1998), i.e., the ability to produce *after* demand is realized. Specifically, all firms require a common input for their own outputs, for which they can all produce equally well. The fact that their demand is not perfectly correlated suggests that they may pool risk by jointly produce the common input. If firms are in fact “divisions” of a single firm, they would do exactly that. However, if firms are independent, then there is no formal authority to make sure necessary information for risk-pooling would in fact be shared among parties.

Indeed, if independent firms can write relatively simple contracts, they can indirectly coordinate and realize the benefit from risk-pooling just as good as if a formal authority is present. This paper gives two such contracts that achieve this purpose. The result has an important implication to anti-trust issue. It questions whether firms applying for a merger on the basis of risk-pooling is a valid argument. Such an argument, the result suggests, can only hold if firms are seriously constrained by its contractual environment and thereby cannot even write simple contracts among themselves.

Aside from the study of organization mode under risk-pooling, this paper also suggests a novel way of understanding outsourcing. In addition to the economies of scale, and productivity differences, the opportunity to take advantage of risk-pooling can also lead firms to outsource.

2 Model

There are three risk-neutral firms labeled $i = a, b, c$. A common input, called y , is used in all of their production. Specifically, each firm transforms a unit of input y to a unit of their own output. Without loss of generality, the transformation cost is normalized to zero for all firms. Denote p_i as the price of output x_i .³

There are two periods. The discount factor is 1. Firms' output demands are unknown in period 0, while each firm observes its own realized demand privately at the beginning of period 1.

To focus on the strategic interaction in the input market, I assume that their output demands $X = (x_a, x_b, x_c)$ follows a joint distribution with cdf F independent of firms' input decisions. Normalize $x_i \in [0, 1]$ for all i . Denote F_i as the corresponding cdf of x_i derived from F . Assume it has no mass point for all i .

At the end of period 1, each firm transforms inputs to outputs and makes sales. It can neither sell more than its realized demand, nor the inventory level of input y accumulated so far. In each period $t = 0, 1$, each firm i decides the amount of input to acquire, either from in-house production (denoted as y_i^t) or by buying from another firm (denoted as q_i^t), or a mix of both. Regardless of the way, all acquired y will be delivered at the end of period 1 to be ready to transform to output.

I focus on the way input production is organized. Assume that all firms are able to produce y , and they all do so equally well, i.e., they have the exactly same cost structure in producing y .

Assume also that the production technology does not exhibit economies of scale: there is no fixed cost, and the marginal cost is constant and depends on the time the order is placed. For instance, if firm i decides to produce y_i^0 units of input in period 0, then the marginal cost is 1.⁴ If firm i decides to produce y_i^1 units in period 1, the marginal cost is $(1 + \theta)$, where $\theta > 0$.⁵ The total production cost would then be $y_i^0 + (1 + \theta)y_i^1$. Finally, assume $p_i > (1 + \theta)$.⁶ This implies that in period 1, if the realized demand is larger than the input inventory level, the firm will acquire more y ex post to satisfy all the realized demand.

The main trade-off here is between producing early with lower production cost but less infor-

³To simplify, I assume price is fixed, or it is set before firms made their input decisions. Abstracting from price setting allows us to focus on the strategic interactions in the input market.

⁴Normalize the marginal cost of y in period 0 to 1, and the price is relative to this numeraire.

⁵This assumption is based on Proposition 7 in Alchian (1959).

⁶This implies that $\theta < \frac{p-1}{1}$

mation and producing later with more information but higher production cost.

2.1 Individual Optimization vs. Joint Optimization

Assuming their output demands are not perfectly positively correlated, then there is a positive gain from risk-pooling: their aggregate production cost of y would be lower if they jointly produce y and share whatever information necessary for optimizing the production, in contrast to producing y alone.

Individual Optimization. To show this point, we first derive the optimal strategies for the case in which firms produce y in-house.

By backward induction, first consider firm i 's strategy in period 1. Given a realized output level x_i , a level of period 0 and period 1 input production, denoted by y_i^0 and y_i^1 respectively, firm i 's profit is

$$\pi(x_i, y_i^0, y_i^1) = p_i \min\{x_i, y_i^0 + y_i^1\} - y_i^0 - (1 + \theta)y_i^1 \quad (1)$$

By the assumption that $p_i > (1 + \theta)$, it would choose

$$y_i^{1*} = \begin{cases} 0 & \text{if } x_i \leq y_i^0 \\ x_i - y_i^0 & \text{if } x_i > y_i^0 \end{cases}. \quad (2)$$

This says firm i would optimally satisfy all of its demand, even if the production level in period 0 falls short of the realized demand. Its sales therefore is always equal to x_i .

In period 0, it chooses y_i^0 to maximize the expected profit $E\pi(y_i^0)$, where

$$E\pi(y_i^0) = \int_0^1 p_i x_i dF_i(x_i) - \int_{y_i^0}^1 (1 + \theta)(x_i - y_i^0) dF_i(x_i) - y_i^0. \quad (3)$$

The first order condition is

$$F_i(y_i^{0*}) = \frac{\theta}{1 + \theta}. \quad (4)$$

The intuition is to produce y at a level that equates the marginal cost and the expected marginal benefit. Given y_i^0 , the marginal cost is 1; the benefit is positive if the realized demand is larger than this level in which the firm saves $(1 + \theta)$ by avoiding the need to produce an extra unit of y in period 1. This happens with probability $1 - F_i(y_i^0)$. The expected marginal benefit is the product

of the two. Rearranging will give us (4).

By the no mass point assumption, the solution is unique. Firm i 's optimal strategy is thus a pair of (y_i^{0*}, y_i^{1*}) as specified in (2) and (4).

Define $\pi_i^* \equiv E\pi(y_i^{0*})$ as the expected profit under individual optimization.

Joint Optimization. Under joint production, “firms” are “divisions” of a single firm. They share information necessary to optimize the aggregate production of y across periods, and if needed, they make any transfer of y costlessly among themselves. The structure of the problem is the same as if we were looking at the individual firm’s optimization. The only difference is that we replace the individual firm’s demand with the aggregate demand.

Denote the sum of the realized demands of “divisions” as s , where $s = \sum_{i=a,b,c} x_i$ and $s \in [0, 3]$. Denote G_s as its cdf and assume that it has no mass point.

The optimal level of aggregated y production in period 0, y^{0**} , is the solution to

$$G_s(y^{0**}) = \frac{\theta}{1 + \theta}. \quad (5)$$

The optimal level of aggregated y production in period 1 is

$$y^{1**} = \begin{cases} 0 & \text{if } s \leq y^0 \\ s - y^0 & \text{if } s > y^0 \end{cases}. \quad (6)$$

By the assumption that the transfer of y among “divisions” is costless, it does not matter how y^{0**} and y^{1**} are distributed among the n firms.

Denote π^{**} as the expected profit under joint optimization such that

$$\pi^{**} \equiv \sum_{i=a,b,c} \int_0^1 p_i x_i dF_i(x_i) - \int_{y^{0**}}^3 (1 + \theta)(s - y^{0**}) dG_s - y^{0**}. \quad (7)$$

Gain from risk-pooling. The gain from risk-pooling Δ is

$$\Delta = \pi^{**} - \sum_{i=a,b,c} \pi_i^*. \quad (8)$$

Note that $\Delta \geq 0$ because under joint optimization, firms can always mimic the strategies of

those under individual optimization. This result is not surprising and has been documented in the literature (for example, see Spulber (1985), and Lim (1981)). It is also the reason why it is optimal to hold common inventories when retailers face demands which are not perfectly correlated.⁷

2.2 Organization of Production

To realize such a gain, coordination of production and sharing of information are crucial. First of all, the sharing of sales information is crucial because period 1 production of y should be the level specified in (6). That means any shortage of y in period 1 has to be met by any excess of y before production occurs. Secondly, the aggregated production of y among firms in period 0 should be the level specified in (5). No firm should have any incentive to produce beyond this level.

The liaison in production and information becomes so critical that most would naturally expect a centralized production mode, i.e., all three firms are “divisions” of the same parent company. This is reasonable, as getting updated sales information, sales forecast, and production information from one’s own subsidiaries are much easier through a formal authority than from independent firms.

Such an argument appears to be sound. It has, however, an implicit premise: that a decentralized market mechanism is not equally good in pooling risk among firms.

If one firm is able to offer contracts that elicit first-best coordination of production, and indirectly share information, then the emerge of formal authority, or a centralized production mode, is *not* necessary. The next section investigates the simplest set of contracts that enables firms to do so.

3 Outsourcing

I refer to firm i as outsourcing if it has a positive possibility to procure y from another firm. This definition includes cases in which firms procure inputs exclusively from other firms, as well as cases in which firms procure inputs from other firms, and at the same time maintain its own in-house

⁷When would $\Delta = 0$? One sufficient condition, obviously, is when firms’ demands are perfectly positively correlated. In generally, Δ increases as the firms’ demands are increasingly negatively correlated. That means the more negatively correlated the firms’ demand, the bigger the gain from risk-pooling.

production.⁸

The timing of the game is as follows. Introduce three more stages before period 0. First, the nature chooses the joint stochastic distribution F . Second, one firm is randomly chosen to offer contracts to the other two firms. Third, other firms decide to accept or reject.

The game then proceeds to period 0 in which firms decide the amount of y to produce in-house, and for firms who have accepted the contract, the amount to buy from the other firm. At the beginning of period 1, all demands are realized and privately observed by firms. In period 1, again, firms decide the amount of y to produce in-house, and for firms who have accepted the contract, the amount to buy from the other firm. At the end of period 1, all delivery is made. All payments agreed upon in contracts are made. Inputs are transformed instantaneously to outputs and sales revenue is realized.

The information structure is such that (i) prices for each firm's output and F are public information; they are therefore able to derive their own, as well as others', distribution F_i , (ii) contracts, contract agreements, and order quantities are private information of the contracting parties, (iii) the levels of in-house production and realized demand are private information of the firm.

3.1 The Feasible set of contracts

A contract specifies the payments to be made conditional on specified events that occur while it is in force. To be enforceable, courts must be able to verify whether those events have occurred. We assume throughout that courts can verify whether or not trade has occurred, with whom, and the specified payment and quantity, but not that of the demand realization of individual firm, or any in-house production.

Without loss of generality, assume firm c is randomly chosen. Recall that q_i^t denotes the order quantity i place in period t , for $i = a, b$, and $t = 0, 1$. A set of contracts is then $\{T_a, T_b\}$, where $T_i = T_i(q_i^0, q_i^1)$ for $i = a, b$. The notion T_i is the amount of money firm i pays to firm c at the end of period 1, contingent on firm i 's order quantities. Since the realized demand is a private information, no contract can specify payment contingent on it.

A set of contracts is "efficient" in terms of risk-pooling if it elicits the levels of aggregate

⁸The more important but subtle point is that outsourcing contracts usually do not stipulate clauses that strictly prohibits in-house production.

production of y in both period 0 and period 1 equal to those specified in (5) and (6) respectively, i.e.,

$$\sum_{i=a,b,c} y_i^t = y^{t**}, \quad (9)$$

for $t = 0, 1$.

The solution concept is weak perfect Bayesian equilibrium.

3.2 Unit price contracts

The simpler is the contract, the easier for firms to draft and the court to enforce. We focus on simple linear contracts. The simplest contract is one that only specifies a unit price, i.e.,

$$T_i(q_i^0, q_i^1) = w_i(q_i^0 + q_i^1) \quad (10)$$

for $q_i^0, q_i^1 \geq 0$.

Specifically, firm c , being randomly chosen to offer contracts, offers to sell y at a unit price of w_i to firm i . The price w_i should fall between 1 and $(1 + \theta)$. If $w_i < 1$, firm c will definitely make a loss. If $w_i > (1 + \theta)$, firm a and b would not outsource to firm c .

By backward induction, in period 1, since $w_i \leq (1 + \theta)$, firm $i = a, b$ would not produce in-house. In case it needs extra y , buying from firm c costs less. Therefore, $y_i^{1*} = 0$, and

$$q_i^{1*} = \begin{cases} 0 & \text{if } x_i \leq y_i^0 + q_i^0 \\ x_i - (y_i^0 + q_i^0) & \text{if } x_i > y_i^0 + q_i^0 \end{cases}. \quad (11)$$

In period 0, firm $i = a, b$ would not order from firm c ; it is a dominated strategy because firm i can always buy from firm c at the same price in period 1 *after* its demand is realized. Therefore $q_i^{0*} = 0$. It would produce some y in-house. It chooses y_i^0 to maximize the expected profit $E\pi(y_i^0|w_i)$, where

$$E\pi(y_i^0|w_i) = \int_0^1 p_i x_i dF_i(x_i) - \int_{y_i^0}^1 w_i (x_i - y_i^0) dF_i(x_i) - y_i^0. \quad (12)$$

The first order condition is

$$F_i(y_i^{0*}) = \frac{w_i - 1}{w_i}. \quad (13)$$

In period 1, firm c chooses y_c^1 to minimize production cost, i.e.,

$$y_c^{1*} = \begin{cases} 0 & \text{if } y_c^0 \geq x_c + \sum_{i=a,b,t=0,1} q_i^t \\ x_c + \sum_{i=a,b,t=0,1} q_i^t - y_c^0 & \text{if } y_c^0 < x_c + \sum_{i=a,b,t=0,1} q_i^t \end{cases}. \quad (14)$$

It says firm c only produces when the amount it needs in period 1 is larger than the amount it has produced in period 0.

Substituting $q_i^{0*} = 0$, the equation can be rewritten as

$$y_c^{1*} = \begin{cases} 0 & \text{if } y_c^0 \geq x_c + q_a^1 + q_b^1 \\ x_c + q_a^1 + q_b^1 - y_c^0 & \text{if } y_c^0 < x_c + q_a^1 + q_b^1 \end{cases}. \quad (15)$$

One can show that the aggregate production in period 1 is inefficient. Since the aggregate production in period 1 for all three firms, y_c^{1*} , is identical to y^{1**} in (6) if and only if $\sum_{i=a,b,c} y_i^0 = y^{0**}$ and $x_i \geq y_i^0$ for $i = a, b$. In other situations, $y_c^{1*} \neq y^{1**}$.

In words, even if the aggregate production in period 0 is efficient, the only situation in which the aggregate production in period 1 is efficient is when both firm a and b do not overstock. The probability of overstock, however, is positive for any positive levels of in-house production in period 0. Suppose firm a has overstocked while firm b has a shortage. Firm b would order from firm c , while firm a would not transfer its excess to firm b through firm c . The imperfect matching of excess and shortage causes the inefficiency in risk-pooling, for which I refer to as the atemporal bias.

The shortage-excess mis-match not only biases period 1 production, but also that of period 0, for which I refer to as the intertemporal bias. The following Proposition summarizes the findings.

Proposition 1. *Any unit price contract cannot be efficient in pooling risk.*

3.3 More complicated contracts

The result suggests that to avoid atemporal and intertemporal production biases, the set of contracts involved has to be richer than a simple unit price contract. What is the minimum a contract has to further include in order to elicit efficient production? First, it has to prevent in-house production in period 0. Second, it has to induce firms to transfer y among themselves in period 1.

To deal with the first concern, consider a two-part tariff that specifies a lump-sum amount t_i firm i pays if it accepts the contract, regardless of the order amount, and a unit price w_i for y , regardless of the timing firm i places its order, i.e.,

$$T_i(q_i^0, q_i^1) = t_i + w_i(q_i^0 + q_i^1) \quad (16)$$

for $q_i^0, q_i^1 \geq 0$.

To deal with the second concern, consider a buy-back contract that specifies two unit prices, one for buying from firm c (denoted as w_i), and one for selling to firm c (denoted as r_i), i.e.,

$$T_i(q_i^0, q_i^1) = w_i(\max(q_i^0, 0) + \max(q_i^1, 0)) - r_i(\min(q_i^0, 0) + \min(q_i^1, 0)). \quad (17)$$

Two-part tariff. Solving the game with two-part tariff contracts, I arrive at the following Proposition.

Proposition 2. *There exists a pair of efficient two-part tariff contracts the randomly-chosen firm is willing to offer.*

The proof of this Proposition builds on two lemmas and are shown in the appendix. I proves that it is incentive compatible for firms to produce at efficient levels, and that every firm is at least as well off having the contract as when they optimize individually.

One feasible pair of two-part tariff contracts involves a price $w_a = w_b = 1$. Firm c offers a price low enough such that firm a and b would have no incentive to produce in-house. They would also have no incentive to place order in period 0 because placing the order in period 1 is a strictly dominating strategy; for placing the order after the demand is realized ensures no overstocking. Profit-maximization requires them to order exactly the level of realized demand; ordering fewer entails the loss of sales, while ordering more increases cost but not sales. There is no atemporal

bias because ordering after the demand is realized implies no shortage or excess of inventory, and therefore negates the need to match shortage with excess among firms.

Expecting such a strategy profile, firm c produces input y for all firms. While no firm has an incentive to order a level other than the level of realized demand, firm c 's production decisions in both periods are essentially the same as those under joint optimization. There is no intertemporal bias.

The transfer t_i will be set accordingly to extract enough expected profit from firm i such that firm i is just indifferent between accepting or rejecting the contract offer. The equilibrium outcome is such that firm c gets all the gain from risk-pooling.

Risk-pooling, though without a centralized mechanism to explicitly coordinate, is efficient. The production of the inputs, however, is centralized. The next question is whether risk-pooling can be achieved without both explicit coordination, and centralized production.

Buy-back contract. This question can be addressed by considering the buy-back contract. Solving the game with buy-back contracts, I come up with another Proposition.

Proposition 3. *There exists a pair of efficient buy-back contracts the randomly-chosen firm is willing to offer.*

The proof of this Proposition, again, builds on two lemmas essentially the same as those in the two-part tariff case. The mechanism of risk pooling, however, is different. Specifically, one feasible set of buy-back contracts involves a price $w_a = w_b = (1 + \theta)$ and $r_a = r_b = 0$, which means firm c offers a price low enough such that firm a and b would have no incentive to produce in-house in period 1, while a buy-back price high enough to induce firms to sell any excess of y to firm c . All firms engage in in-house production in period 0, and they trade among themselves through firm c in period 1.

The fact that firms would sell to firm c when there is an excess means there is a channel through which the shortage of one firm can be met by the excess of another firm. It therefore avoids the atemporal bias, leading to an efficient level of aggregate production in period 1. Without atemporal bias, firm c would produce just enough such that the aggregate production in period 0 is optimal, holding a belief that firm a and b has produced in-house in period 0 as well. Firm c has no incentive to produce at any other level: with no atemporal bias in period 1, any other production level in

period 0 entails higher expected production cost. Therefore, intertemporal bias is absent too.

The equilibrium outcome is that all firms produce in-house in period 0, while only firm c produces in period 1. Firm a and b trade with firm c bilaterally in period 1 whenever their realized demands do not match their inventory levels. Production is therefore carried across all firms, and partial outsource is observed. Firms remain independent, yet they indirectly coordinate and pool risk efficiently. Firm c acquires all the gain from risk-pooling, while the other firms are as well-off as if they were under individual optimization.

The novelty of the mechanism is that there is no incentive for any firm to overstock in period 0. Because if firms are allowed to sell its overstock to others, a way for them to transfer their risk away, there seems to be an incentive for the firms to over-produce. However, even if firms over-produce, firm c would take that into account (in its belief), and produce at a lower level in period 0 in order to restore the optimal aggregate production in period 0.

4 Conclusion

This paper shows that despite the presence of private information, firms can fully realize gains from risk-pooling by indirectly coordinating through outsourcing contracts. The identified sets of contracts that can deliver this result is relatively simple, and therefore does not demand a particularly strong contractual environment for contract enforcement.

The result suggests that the mere presence of risk-pooling opportunity does not necessarily imply a centralized mode of organization. It is analogous to a well-known result in the finance literature. Under the assumption of perfect capital market, pooling risk by merging firms into a conglomerate cannot add value because individual investors would have been able to duplicate such a risk-pooling through personal portfolio diversification. However, the existence of various market imperfections may revert the results. For example, when it is costly to acquire information, to monitor a large number of risky assets, and when assets are indivisible, then the ability of individual investors to diversify personal portfolio is compromised, resulting in a positive gain from pooling risk when firms merge with one another.

Analogously, information sharing being essential to pool risk, one has to examine whether arms-length relationships are restricted to contracts that cannot indirectly share information. Only if it

is so can one conclude that merging is essential to realize the potential gains from pooling risk. This paper therefore calls for an examination of the contractual environment to determine the extent to which risk-pooling explains the mode of organization.

The paper also suggests risk-pooling as a reason for firms to outsource. Interestingly, the supplying firm does not enjoy more superior production technology. There is also no economies of scale. The need to pool risk is enough to drive equally productive (in terms of input production) firms to outsource to others.

The paper does not assume any other transaction cost other than information asymmetry. Without any other transaction cost, one can only state that a decentralized production mode can perform equally well. To further compare different organizational modes, additional transaction cost is essential. I discuss one such possible extension.

The model can be extended by introducing moral hazard. For instance, if each manager has to invest some effort to increase the chance of “observing” demand realization before it is too late to adjust output, then under a centralized mode, when a boss relies on his store managers to manage his stores, the manager may have no incentive to exert effort when effort is not observable. Such a moral hazard problem is more important the lower is the ex post adjustment cost, i.e., when θ is closer to zero. Further assume that there is a small fixed fee of enforcing outsourcing contract, then a decentralized mode is more preferable if moral hazard is more important, as when θ is small. On the other hand, a centralized mode of production is more preferable the less important is moral hazard, as when θ is large.

A Appendix - Proofs

A.1 PROOF OF PROPOSITION 2

Two lemmas establish the proof.

Lemma 4. *Incentive-compatibility(IC): any two-part tariff contract with $w_i \leq 1$ for $i = a, b$ would elicit first-best actions.*

Proof. We do backward induction. In period 1, firm c 's state variables are its own production level in period 0 (y_c^0), its realized demand x_c , and firm i 's orders in both periods if any (q_i^0, q_i^1 for

$i = a, b$). It chooses its production level in period 1 to maximize profit.

Obviously, if it has enough input from period 0's production to cover the sum of its demand and the total order, it would not have to produce anymore. On the other hand, if input from period 0's production is not enough, then firm c would produce just enough to cover the shortage. Therefore, we have the following optimal strategy:

$$y_c^{1*} = \begin{cases} 0 & \text{if } y_c^0 \geq x_c + \sum_{i=a,b} \sum_{t=0,1} q_i^t \\ x_c + \sum_{i=a,b} \sum_{t=0,1} q_i^t - y_c^0 & \text{if } y_c^0 < x_c + \sum_{i=a,b} \sum_{t=0,1} q_i^t \end{cases}. \quad (18)$$

Consider the optimal strategies of firm $i = a, b$ in period 1: choosing the levels of in-house production y_i^1 and the order amount q_i^1 . Firm i needs more y only if its realized demand x_i exceeds the level of y it acquires before, which includes both in-house production and ordering from firm c , y_i^0 and q_i^0 respectively. If $w_i \leq 1$, then firm i does so through ordering from firm c . To summarize, for $i = a, b$, if $w_i \leq 1$, then

$$q_i^{1*} = \begin{cases} 0 & \text{if } x_i \leq y_i^0 + q_i^0 \\ x_i - y_i^0 - q_i^0 & \text{if } x_i > y_i^0 + q_i^0 \end{cases}, \quad (19)$$

$$y_i^{1*} = 0. \quad (20)$$

In period 0, firm c 's state variables are q_a^0 and q_b^0 , which should be zero as argued below. It has a belief that both firm a and b did not engage in any in-house production. This belief and the expectation of firms' strategies in period 1 as specified in (19) and (20) lead firm c to expect $q_i^1 = x_i$. Therefore, firm c 's optimal action is the solution to this problem:

$$\begin{aligned} & \max_{y_c^0 \geq 0} \int_0^1 p_c x_c dF_c - y_c^0 - \int_{y_c^0}^3 (1 + \theta) (s - y_c^{0*}) dG_S \\ & + w_a \int_0^1 x_a^1 dF_a + w_b \int_0^1 x_b^1 dF_b + t_a + t_b, \end{aligned} \quad (21)$$

where again $s = x_a + x_b + x_c$ and G_S denotes its cdf. The first term is the expected revenue from its own output sales; the second is the production cost in period 0; the third is the expected production cost in period 1; the next two terms are the expected revenue from selling input y to

both firms, while the remaining two terms are the transfers.

In period 0, any actions for firm i to order any positive amount is strictly dominated by the action of ordering nothing. This is because ordering, regardless of the amount, associates with a positive chance of overstocking. Whereas ordering after demand is realized costs exactly the same and does not risk overstocking. Similarly, production is not optimal because ordering costs less. Therefore, $q_i^{0*} = y_i^{0*} = 0$ for $i = a, b$, which is consistent with firm c 's belief in period 0.

The solution to (21) is exactly equal to that of (5). Since $y_i^{0*} = 0$, therefore the aggregate period 0 production of y is efficient.

The aggregate period 1 production of y is equal to y_c^{1*} because $y_i^{1*} = 0$. Substituting $q_i^1 = x_i$ into (18), we know that y_c^{1*} is equal to (6). The aggregate period 1 production of y is therefore efficient too. \square

Lemma 5. *Participation constraint (PC): given $w_i \leq 1$, there is a non-empty set (t_a, t_b) such that the expected profit of each of the three firms is at least as high as that under individual optimization.*

Proof. It suffices to show that when $w_i = 1$, there is a pair of (t_a, t_b) that satisfies PC. First, the expected profit under a two-part tariff contract is $\pi_i^*(t_i, w_i)$ and $\pi_c^*(t_a, w_a, t_b, w_b)$ for firm $i = a, b$ and firm c respectively, where

$$\begin{aligned} \pi_c^*(t_a, w_a, t_b, w_b) &= \int_0^1 p_c x_c dF_c - y_c^{0*} - \int_{y_c^{0*}}^3 (1 + \theta) (s - y_c^{0*}) dG_S \\ &\quad + \sum_{i=a,b} \left(w_i \int_0^1 x_i^1 dF_i + t_i \right), \end{aligned} \quad (22)$$

$$\pi_i^*(t_i, w_i) = \int_0^1 (p_i - w_i) x_i dF_i - t_i. \quad (23)$$

The sum of the expected profit is equal to π^{**} . It implies that the mode of organization realizes all the surplus from risk-pooling.

For PC to hold, we need $\pi_c^*(t_a, w_a, t_b, w_b) \geq \pi_c^*$ and $\pi_i^*(t_i, w_i) \geq \pi_i^*$, where, for $i = a, b, c$,

$$\pi_i^* = \int_0^1 p_i x_i dF_i - y_i^{0*} - \int_{y_i^{0*}}^1 (1 + \theta) (x_i - y_i^{0*}) dF_i. \quad (24)$$

Suppose $w_i = 1$, and we want to set $\pi_i^*(t_i, w_i) = \pi_i^*(\widehat{t}_i, 1) = \pi_i^*$, then the solution is

$$\widehat{t}_i = y_i^{0*} + \int_{y_i^{0*}}^1 (1 + \theta) (x_i - y_i^{0*}) dF_i - \int_0^1 x_i dF_i. \quad (25)$$

Substitute $(\widehat{t}_a, \widehat{t}_b)$ back to $\pi_c^*(t_a, w_a, t_b, w_b)$, we check whether the constraint $\pi_c^*(t_a, w_a, t_b, w_b) \geq \pi_c^*$ still holds. Since the mode of organization realizes all the surplus from risk-pooling,

$$\begin{aligned} \pi_c^*(\widehat{t}_a, 1, \widehat{t}_b, 1) + \sum_{i=a,b} \pi_i^*(\widehat{t}_i, c) &= \pi^{**}, \\ \pi_c^*(\widehat{t}_a, c, \widehat{t}_b, c) &= \pi^{**} - \sum_{i=a,b} \pi_i^* \geq \pi_c^*. \end{aligned}$$

From line 1 to line 2, we substitute $\pi_i^*(\widehat{t}_i, c) = \pi_i^*$. The inequality holds because joint optimization yields a joint expected profit at least as high as that in individual optimization. Therefore, if $w_i = 1$, the pair $(\widehat{t}_a, \widehat{t}_b)$ satisfies PC. \square

A.2 PROOF OF PROPOSITION 3

Two lemmas establish the proof.

Lemma 6. *Incentive-compatibility(IC): any buy-back contract with $1 < w_i \leq (1 + \theta)$ and $r_i \geq 0$ would elicit first-best actions.*

Proof. We do backward induction. In period 1, firm c 's state variables are its own production level in period 0 (y_c^0), its realized demand x_c , and firm i 's orders in both periods if any (q_i^0, q_i^1 for $i = a, b$). It chooses its production level in period 1 to maximize profit. It only produces extra if its production of y in period 0 falls short of the demands; in which case it produces an amount exactly equal to that of the shortage. Its optimal strategy is therefore

$$y_c^{1*} = \begin{cases} 0 & \text{if } y_c^0 \geq x_c + \sum_{i=a,b,t=0,1} q_i^t \\ x_c + \sum_{i=a,b,t=0,1} q_i^t - y_c^0 & \text{if } y_c^0 < x_c + \sum_{i=a,b,t=0,1} q_i^t \end{cases}. \quad (26)$$

Consider the optimal strategies of firm $i = a, b$ in period 1: choosing the levels of in-house production y_i^1 and the order amount q_i^1 . Now q_i^1 can be both positive or negative because of the

buy-back option. Since $w_i \leq (1 + \theta)$, if extra input y is necessary, firm i prefers to source from firm c . On the other hand, if there is an excess of y , since $r_i \geq 0$, it is profitable to sell the extra back to firm c . Therefore, the strategies are:

$$q_i^{1*} = \begin{cases} x_i - y_i^0 - q_i^0 & \text{if } x_i > y_i^0 + q_i^0 \\ 0 & \text{if } x_i = y_i^0 + q_i^0 \\ -(y_i^0 + q_i^0 - x_i) & \text{if } x_i < y_i^0 + q_i^0 \end{cases}, \quad (27)$$

$$y_i^{1*} = 0. \quad (28)$$

Note that the buy-back option automatically implies that $y_i^0 + q_i^0 + q_i^{1*} = x_i$, i.e., firm $i = a, b$ always has y just enough to cover its realized demand.

We derive the optimal strategies for firm $i = a, b$ in period 0 first before deriving that of firm c . Firm i 's problem is to choose the level of in-house production y_i^0 and the order amount q_i^0 . Since $1 < w_i$, firm i should not place any order; therefore $q_i^{0*} = 0$. Its optimal production level is the solution to this problem:

$$\max_{y_i^0 \geq 0} \int_0^1 p_i x_i dF_i - y_i^0 - \int_{y_i^0}^1 w_i (x_i - y_i^0) dF_i + \int_0^{y_i^0} r_i (y_i^0 - x_i) dF_i, \quad (29)$$

where the first term is the expected revenue from its own sales; the second is the production cost in period 0; the third is the expected payment to firm c in period 1; the last is the expected revenue from selling back excess y to firm c in period 1. The solution is the solution to

$$F_i(y_i^{0*}) = \frac{w_i - 1}{w_i - r_i}. \quad (30)$$

Let us go back to the optimal strategy of firm c in period 0. To be consistent, it should hold a belief that firm i has produced y_i^{0*} as stated above, though it does not observe it. It expects firm i to order in period 1 as derived above and does not engage in any in-house production. Its problem is to choose a production level of y to maximize the expected profit. The optimal strategy is the

solution to this problem:

$$\begin{aligned} & \max_{y_c^0 \geq 0} \int_0^1 p_c x_c dF_c - y_c^0 - \int_{y_a^{0*} + y_b^{0*} + y_c^0}^3 (1 + \theta) (s - y_a^{0*} - y_b^{0*} - y_c^0) dG_s \\ & + \sum_{i=a,b} \left(\int_{y_i^0}^1 w_i (x_i - y_i^0) dF_i - \int_0^{y_i^0} r_i (y_i^0 - x_i) dF_i \right), \end{aligned} \quad (31)$$

where the first term is the expected revenue from its own sales; the second is the production cost in period 0; the third is the expected production cost in period 1; the last one is the summation of net revenue from transactions in period 1. The solution is the solution to

$$G_s (y_a^{0*} + y_b^{0*} + y_c^{0*}) = \frac{\theta}{1 + \theta}. \quad (32)$$

The aggregate period 0 production of y , therefore, is efficient. The aggregate period 1 production of y , which is equal to y_c^{1*} only because firm a and b would not produce in-house, is also efficient. This can be shown by substituting $y_i^{0*} + q_i^0 + q_i^{1*} = x_i$ into y_c^{1*} , and we will get

$$y_c^{1*} = \begin{cases} 0 & \text{if } \sum_{i=a,b,c} y_i^{0*} \geq s \\ s - \sum_{i=a,b,c} y_i^{0*} & \text{if } \sum_{i=a,b,c} y_i^{0*} < s \end{cases}. \quad (33)$$

□

Lemma 7. *Participation constraint (PC): given $1 < w_i \leq (1 + \theta) 1$ and $r_i \geq 0$, there is a non-empty set (w_a, r_a, w_b, r_b) such that the expected profit of each of the three firms is at least as high as that under individual optimization.*

Proof. It suffices to show that when $w_i = (1 + \theta)$, there is a pair of (r_a, r_b) that satisfies PC. First, the expected profit under a buy-back contract is $\pi_i^*(w_i, r_i)$ for firm $i = a, b$ and $\pi_c^*(w_a, r_a, w_b, r_b)$

for firm c , where

$$\begin{aligned} \pi_c^*(w_a, r_a, w_b, r_b) &= \int_0^1 p_c x_c dF_c - y_c^{0*} \\ &\quad - \int_{y_c^{0*} + y_a^{0*} + y_b^{0*}}^3 (1 + \theta) (s - y_c^{0*} - y_a^{0*} - y_b^{0*}) dGS \\ &\quad + \sum_{i=a,b} \left(\int_{y_i^{0*}}^1 w_i (x_i - y_i^0) dF_i - \int_0^{y_i^{0*}} r_i (y_i^0 - x_i) dF_i \right), \end{aligned} \quad (34)$$

$$\begin{aligned} \pi_i^*(w_i, r_i) &= \int_0^1 p_i x_i dF_i - y_i^{0*} - \int_{y_i^{0*}}^1 w_i (x_i - y_i^0) dF_i \\ &\quad + \int_0^{y_i^{0*}} r_i (y_i^0 - x_i) dF_i. \end{aligned} \quad (35)$$

One can easily check that the sum of the expected profit is equal to π^{**} . It implies that the mode of organization realizes all the surplus from risk-pooling.

For PC to hold, we now need $\pi_c^*(w_a, r_a, w_b, r_b) \geq \pi_c^*$ and $\pi_i^*(w_i, r_i) \geq \pi_i^*$, where, for $i = a, b, c$,

$$\pi_i^* = \int_0^1 p_i x_i dF_i - y_i^{0*} - \int_{y_i^{0*}}^1 (1 + \theta) (x_i - y_i^{0*}) dF_i. \quad (36)$$

Suppose $w_i = (1 + \theta)$, we set $\pi_i^*(w_i, r_i) = \pi_i^*((1 + \theta), \hat{r}_i) = \pi_i^*$. The solution is $\hat{r}_i = 0$.

Substitute (\hat{r}_a, \hat{r}_b) and $w_i = (1 + \theta)$ back to $\pi_c^*(w_a, r_a, w_b, r_b)$, we now check whether the constraint $\pi_c^*((1 + \theta), \hat{r}_a, (1 + \theta), \hat{r}_b) \geq \pi_c^*$ holds. Since the mode of organization realizes all the surplus from risk-pooling, therefore

$$\begin{aligned} \pi_c^*((1 + \theta), \hat{r}_a, (1 + \theta), \hat{r}_b) + \sum_{i=a,b} \pi_i^*((1 + \theta), \hat{r}_i) &= \pi^{**}, \\ \pi_c^*((1 + \theta), 0, (1 + \theta), 0) &= \pi^{**} - \sum_{i=a,b} \pi_i^* \geq \pi_c^*. \end{aligned}$$

Therefore the constraint holds. Therefore, if $w_i = (1 + \theta)$, there exists $\hat{r}_i = 0$ such that PC are satisfied. \square

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