Are Price Matching Guarantees Anti-Competitive?*

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Abstract

This paper examines the incentives for price-matching guarantees in markets where information about prices is costly. Under some conditions the conventional explanation of price-matching as facilitating collusion finds support, and is even strengthened, but our model provides an additional explanation for the practice. A price-matching guarantee may be a credible and easily understood means of communicating to otherwise uninformed consumers that a firm is low-priced. The credibility of the signal is assured by the penalty that low-information-cost consumers would impose on a high-priced store that mimicked the price-matching policy.
1 Introduction

In many retail markets, sellers not only set prices but announce a guarantee that they will match the lowest advertised price that any customer can find in the market. A well-publicized example of such a guarantee is Gateway’s announcement on May 30, 2001 that it would match key competitors’ prices on comparable PC’s. The Gateway Guarantee promises customers that if they “present a current ad from Compaq, Hewlett-Packard, Dell, IBM, Sony or Toshiba for a new PC or server with specifications at least equal to Gateway’s specifications,” then Gateway will sell them a comparable PC “for as much as $1 less than the advertised price of its rivals”.1 The press release accompanying the announcement notes that the guarantee will be launched “with broadcast and cable TV advertising, as well as a full-page ad in USA Today and dozens of local daily newspapers across the country.” Price-matching is also observed in markets for sporting goods, books, house wares, cell phones, office products, consumer electronics, luggage, furniture, tires, toys, gasoline, eyewear, prescription drugs, and grocery products, among others.2

Price-matching guarantees, or meeting-competition clauses as they are sometimes called, would appear to be pro-competitive. Customers do not complain about getting guaranteed low prices.3 In the economics and antitrust literature, however, price-matching guarantees have a bad name. These guarantees are seen as a way to collude. The argument is that price-matching guarantees stifle price competition by removing the incentive to undercut (Hay 1982, Salop 1986). The firm offering a price-matching commitment to buyers is in fact guaranteeing its competitors that any lower price from them would be matched immediately—eliminating the gains from the price cut. A second theory explains price-matching as a means of price discriminating among consumers (Png and Hirschleifer 1988). Firms offering price-matching guarantees provide discounts selectively to customers who shop for and are aware of lower prices in the market while charging a high list price for non-searchers. Edlin (1997) uses this argument to suggest that price matching policies be prosecuted as an unfair method of competition in violation of the FTC Act or as a violation of the Robinson-Patman Act. In Edlin’s view, the market-wide impact of this practice

3Investment analysts viewed Gateway’s announcement of price-matching as a step up in the P.C. price war, not a reduction in price competition (Wall Street Journal, Ibid).
is to limit the disciplining power of active shoppers on market pricing. Whereas price searchers usually provide a positive externality to non-searching customers by driving the price down for everyone, in a market with price-matching guarantees, the pro-competitive benefits of active price shopping are limited to the active shoppers themselves. The theory of price-matching as anti-competitive is thus extended to markets with large numbers of consumers.

This paper investigates the incentives for price-matching guarantees in markets with consumer transactions costs. We are motivated by two types of empirical findings. First, in contrast to the consensus of views in the theoretical and policy literature (notable exceptions, however, are Hviid and Shaffer (1999) and Chen, Narasimhan and Zhang (2001), it is striking how little evidence there is that prices are higher when price matching guarantees are offered. Hess and Gerstner (1991) examine weekly supermarket prices in the Raleigh, North Carolina market, before and after the adoption of a price-matching guarantee by one of the supermarkets. They find a statistically significant increase in the average price of a basket of goods (relative to a basket of goods not covered by the price-matching policy) from before to after, but the price increase seems to be of the order of 2 percent.\(^4\) Arbatskaya, Hviid, and Shaffer (1999) examine advertised tire prices across the U.S., and do not find a statistically significant difference in the prices charged by price-matching firms and non-price-matching firms. If anything, price-matching firms seem to charge lower prices.\(^5\) The second type of evidence that casts doubt on the collusion effect is the simple finding that in experiments consumers believe that price-matching firms are lower priced than non-price-matching firms (Srivastava and Lurie 2001). Srivatava and Lurie exposed consumers to several simulated shopping environments in a controlled experiment and asked them questions about their perceptions of the price-matching and the non-price-matching stores. They find that subjects are more likely to stop searching, by as much as 25 percent, after they have been to a price-matching store than after they have been to a non-price-matching store. Consumers can be wrong, of course, but in general the observed behaviour

\(^4\)Moreover, even this finding is clouded by the possibility that cost and demand dynamics of the “covered” and “not covered” goods may have been different during this period (store brands figured prominently in the latter basket but not in the former).

\(^5\)The “share of low price guarantees” in a market increased advertised tire prices by a statistically significant amount in their study, but in economic terms, the maximum possible effect was 9–10 per cent (comparing markets with no firms offering low price guarantees to markets with all firms offering low price guarantees).
of economic agents deserves some weight.

We develop a model of a retail market that incorporates two kinds of transactions costs on the part of consumers: costs of shopping (travelling) among stores and the costs of information about the prices charged at various stores. The costs of price information are interpreted generally as the costs of obtaining, organizing and remembering information about prices offered at different retail stores. These include, for instance, the costs of obtaining and organizing newspaper ads, and memorizing the prices in them. The organization and memorization costs are nontrivial in today’s consumer economy given the enormous volume of advertising information to which consumers are subjected. This consumer heterogeneity is central to the model, and with respect to price information cost is captured in the simplest way: some consumers are fully informed about prices and other consumers are completely uninformed. Duopoly retailers are differentiated on the basis of location, and, in addition, they may have different costs following random draws from a known distribution of costs.

In this setting, two possible explanations or roles for price-matching guarantees emerge, depending on parameters and cost realizations. Under symmetric costs, the model supports and even strengthens the conventional theory of price-matching guarantees as facilitating collusion. In fact, with only an “epsilon” of uninformed consumers, the collusive price emerges as a dominant strategy equilibrium, instead of only one of a continuum of Nash equilibrium as in the conventional theory. This stands in sharp contrast to Hviid and Shaffer (1999) who observe that an arbitrarily small amount of transactions costs of a different type can destroy the ability of price-matching to facilitate collusion.

Finally, under asymmetric cost realizations in which cost differences are large, the low-cost firm offers a price-matching policy and the high-cost firm does not. The explanation of price-matching that is implied by this equilibrium is both simple and intuitive. The price matching guarantees emerge as a credible way of advertising that “we are a low-priced outlet.” Such a signal is valuable because a price-matching announcement is much easier for busy consumers to assimilate than detailed price information on many products in a dynamic pricing environment. High time-cost customers choose to shop at an outlet because they are aware of its price matching policy and because this signals to them that the outlet is relatively low-priced. These customers do not know the details of the exact prices that this chain charges at any given moment, and instead shop of the basis of inferences drawn from
the information embedded in the signal. What makes the signal credible is the vigilance of the low time-cost customers—the customers who monitor prices. Price matching allows the low prices elicited from the market by low-time-cost customers to be shared by busy customers, since the latter can just shop at price-matching stores, knowing that these stores offer good value, without going through the costs of detailed price comparisons. Thus price matching guarantees can facilitate the positive search externalities that active shoppers provide in markets with imperfect consumer information and transactions costs. This is the opposite of Edlin’s argument that price matching limits the extent of these externalities.

We develop the basic model in the next section of the paper, then examine the equilibria when price-matching is and is not allowed. We discuss evidence that allows one to distinguish among the competing explanations of price matching guarantees that emerge as possible equilibria in our model. In the conclusion we outline an extension to a model in which consumers decide endogenously whether to become informed about prices (and become “active shoppers”) or to shop on the basis of inferences drawn from stores’ price-matching policies (thus remaining “inference shoppers”). The extension reveals a Grossman-Stiglitz (1980) type paradox. If inference shopping is less costly, no consumers will want to become active shoppers; but the presence of some active shoppers is necessary for inference shopping to be possible in equilibrium.

2 The Model

2.1 Assumptions

We adopt the simplest model of a retail market in which consumers bear travel costs and costs of price information. The following are the essential assumptions.

1. Two firms, located at opposite ends of a unit line segment, compete in prices for the sale of an identical product.

2. Consumers are uniformly distributed along the line segment, with unit density. A consumer’s location is indexed by $s$.

3. Consumers bear a common travel cost, $t$, per unit distance, that is independent of the quantity purchased.
4. Consumers have a common, quasi-linear utility, \( u(q) + e \) where \( q \) is the amount of the product consumed and \( e \) is expenditure on other goods. \( u(\cdot) \) is strictly increasing and concave. Travel costs are independent of \( q \), so the net surplus for a consumer at \( s \) travelling to, say, firm 1 and purchasing at price \( p \) is \( v(p) - s\theta \) where \( v(\cdot) \) is the indirect utility function corresponding to \( u(\cdot) \). We assume that \( v(0) \) is finite. A consumer’s demand function upon reaching a firm is \( q(p) = -v'(p) \).

5. A fraction \( \alpha \) of the consumers at any location are uninformed about the prices charged at the firms.\(^6\)

6. Firms face random, independent draws on costs: \( c_L \) with probability \( \lambda \) and \( c_H \) with probability \( (1 - \lambda) \); \( c_L < c_H \).

7. After the simultaneous realization of costs, observed by both firms, the firms simultaneously decide whether to announce price-matching guarantees. A price-matching guarantee means that any consumer of a firm who has information as to the price charged by the other firm, can obtain the same price at the firm where the purchase is being made. After the price-matching guarantee decisions, firms simultaneously decide on prices.

8. After prices are decided upon, consumers each decide from which firm to purchase. Each informed consumer observes the prices and purchases where the consumer surplus net of transportation costs is higher.

9. Price-matching guarantees are observed by all consumers. Uninformed consumers condition their expectations as to the prices at the two firms on the firms’ decisions on whether or not to announce price-matching, and purchase where their expected consumer surplus net of transportation costs is higher.

We look for the perfect Bayesian equilibrium in the game described by the assumptions above. The equilibrium consists of (1) a “price-match decision” (to announce a price-matching guarantee or not) contingent upon the pair of cost realizations, and a

\(^6\)We emphasize that we are exploring the consequences of costly price information, but do not explain in the model why this information is costly. In reality, there are thousands of retail prices to keep track of at each outlet and price information costs are largely the time costs of organizing and retaining this information; in our model, however, each store sells a single product at a single price.
price on the part of each firm, contingent upon the history of cost realizations and policy decisions; (2) purchase decisions by each consumer; and (3) an expectational distribution on the part of each uninformed consumer, as to the price set by each firm, given the firm’s observable decision on price-matching, that satisfy: (i) rationality of each consumer’s purchase decision, given that consumer’s expectations if the consumer is uninformed; (ii) profit-maximization on the part of each firm given the mapping from its policy decision into consumer expectations and the strategy by its rival; and (iii) consumer expectations that are rational, i.e. that satisfy Bayes’ law (along the equilibrium path of the game) given the equilibrium policy decisions by firms and the probability λ.

A full characterization of the set of equilibria, including mixed strategy equilibria, for all values of the model’s parameters is complex. Our interest is in showing sufficient conditions for each theory of price-matching to be supported as an outcome of the rational interaction of agents in a market with transactions costs. We restrict attention to a set of model parameters based on three considerations. First, assumptions 8 and 9 contain the assumption that net surplus is always positive, even for a consumer located at $s = 1/2$. Assumption 9 contains a second simplifying assumption: that it does not pay an uninformed consumer, having discovered a high price at one outlet, to travel to the other outlet in search of a lower price.\footnote{One could replace this assumption with a more basic assumption that travel costs are linear up to a distance of 1, then become prohibitive. This is similar to the common assumption in price dispersion models that uninformed consumers simply buy from the first store they visit. Similarly, the assumption that consumers always purchase could be justified with a more basic assumption that the first unit of consumption yields a very high level of marginal utility.}

Our third restriction is to parameters of the model where a pure strategy equilibrium exists (at least for actions along the equilibrium path of actions; mixed strategies are unavoidable off the equilibrium path). For $t$ sufficiently small, for example, a only mixed strategy equilibria exists.\footnote{In terms of the pseudo-dynamic story associated with Bertrand equilibria, the two outlets could compete price down in an attempt by each to capture more of the price-informed, low-search-cost consumers; then, when prices became too low, an outlet would “give up” on capturing informed consumers and resort to a high price because of the profits that this would generate from uninformed consumers. The cycle of competing price down would then begin again. More formally, the failure of profit functions to be quasi-concave means that pure strategy Bertrand equilibria are not assured.} The economic analysis and application of spatial models has a long tradition of acknowledging the possibility of non-existence of pure strategy
equilibria but focusing on parameters for which this problem does not arise.\footnote{Virtually all address models of price competition on a circle or a line share the property that for some parameters, payoff functions are discontinuous. Most spatial models adopt, implicitly or explicitly, a restriction on endogenous strategies ("no mill-price undercutting") to avoid non-existence of pure strategy equilibrium. See d’Aspremont, Gabsewicz and Thisse (1979) and Eaton and Lipsey (1986).} We defer the statement of the relevant regularity assumption on the model’s parameters (Assumption 10) until after a description of the demand functions in the model. Finally, as is common in games with imperfect information we impose a mild symmetry restriction on beliefs: we look for equilibria in which the expectations of uninformed consumers as to the price at a particular firm depend on the firm’s observable past actions (the price-match decision) but not on the identity of the firm.

The equilibrium that we have described cannot be solved via backwards induction, since there are no proper subgames (not linked by consumers’ information sets). However, the simple demand structure of the model provides an approach to solving the game, which involves finding expectations that are self-realizing. Consider an arbitrary set of price expectations on the part of uninformed consumers, i.e. an expected price at each outlet for each combination of price-match decisions. Let $\mathbf{b} = (b_{00}, b_{01}, b_{10}, b_{11})$ represent the surplus expected from Firm 1 by each uninformed consumer, gross of transportation costs, in each of the consumer’s information sets: $v_{01}$, for example, is the consumer’s expected surplus following the consumer’s observation that Firm 1 has not announced price-matching ($i = 0$) and if Firm 2 has announced price-matching ($j = 1$).\footnote{Expected surplus from Firm 2 is given symmetrically.} These expectations determine a partition of the set of uninformed consumers, which in turn determines for each firm a measure of “captive” uninformed consumers, each with demand $q(P)$, over which the firm has monopoly power: uninformed consumers have rational price expectations in equilibrium but do not respond, in their decisions of where to purchase, to (off-equilibrium) changes in a firm’s price. The impact of the game’s history at the point of simultaneous decisions on prices is summarized by these two amounts of captive consumers, as well as by which of the two firms are competing under the constraint of a binding price-matching agreement. We refer to the pricing game that is induced by an arbitrary set of expectations, or an arbitrary allocation of uninformed consumers to the two firms, as an induced price game.

We proceed by starting with an arbitrary set of expectations $\mathbf{b} = (b_{00}, b_{01}, b_{10}, b_{11})$. We consider the equilibrium in the pricing decisions induced by the expectations $\mathbf{b}$ and...
each pair of price matching decisions. The payoffs from the set of subsequent induced price games will determine optimal price-matching policy decisions at each cost pair drawn by Nature. Finally, the optimal price-match decisions and the equilibrium prices of the induced price games determine a set of “actual” expected surpluses conditional upon each pair of policy decisions, \( V = (v_{00}, v_{01}, v_{10}, v_{11}) \), which are determined by the equilibrium policy and pricing decisions and the probability \( \lambda \). This procedure defines an operator \( \Phi \), via \( \Phi(\mathbf{p}) = V \), on the possible set of surplus expectations in \( \mathbb{R}^4 \). A fixed point of \( \Phi \), \( V^* \), yields a Bayesian Perfect equilibrium as the equilibria of the associated induced price games, and the equilibrium policy decisions that follow from these induced price games.

In fact, if the game’s history at the point of the pricing decisions includes symmetric price-match decisions, then the marginal uninformed consumer is at \( s = 1/2 \), and the induced pricing game includes equal endowments of captive uninformed consumers. If the history includes asymmetric price-match decisions, then the effect of expectations can be summarized by the marginal uninformed consumer, \( s_u \). If the history includes price-matching by Firm 1 only, for example, the marginal consumer given the expectations, satisfies \( v_{10} - s_u t = v_{01} - (1 - s_u)t \), i.e. \( s_u = \frac{1}{2} + \frac{(v_{10} - v_{01})}{2t} \). (The marginal consumer given a history of price-matching by Firm 2 only must satisfy \( 1 - s_u \) given the symmetry assumption on expectations.) The entire history of the game at the stage of price competition, as it affects the outcome of the various induced pricing games, can thus be summarized by a single expectations parameter, \( s_u \), as well as by which of the firms are competing under a price-match constraint.\(^{11}\)

Starting with an arbitrary parameter \( s_u \), we examine the induced price games following all possible histories of the game up to the pricing decisions. The payoffs from the induced price games then determine the payoffs from the price-match decisions. Suppose that we find that only one realization of costs results in price-matching by Firm 1 only – and similarly for Firm 2 – and that the equilibrium prices in the induced price game following this history result in a marginal informed consumer, \( s_1(p_1, p_2) = \frac{1}{2} + \frac{[v(p_1) - v(p_2)]}{2t} \), that satisfies \( s_1(p_1, p_2) = s_u \). Then we have found an equilibrium: Assigning price expectations to the uninformed consumer that are the actual prices following this history will maintain the marginal uninformed consumer at \( s_u \), since uninformed and informed consumers have the same travel costs, and the price expectations will be self-realizing following

\(^{11}\)The summary of expectations by a single parameter, in the search for self-realizing expectations, allows us to use the simplest of all fixed-point theorems: the intermediate value theorem.
this history. Assigning the symmetrically-defined price expectations following a history of price-matching by Firm 2 only, and assigning the actual distribution of equilibrium prices in the games induced by \( s_u = .5 \) following symmetric price-match decisions, yields an entire set of expectations that are self-realizing.

The proposed methodology for characterizing the Bayesian equilibrium of our price-matching game may seem cumbersome, in light of the large number of distinct price games that must be considered. Even after invoking symmetry, there are three possible cost histories, \{\((c_L,c_L),(c_H,c_L),(c_H,c_H)\)\}, three possible price-match decision histories, \{\((0,0),(1,0),(1,1)\)\} to consider after each of the two symmetric cost realizations \((c_L,c_L)\) and \((c_H,c_H)\), and all four distinct price-match histories to consider after the cost realization \((c_H,c_L)\). This yields 10 distinct induced price games. We find, however, that only three classes of equilibria emerge in the induced pricing games: (1) the Bertrand equilibrium prices with unrestricted prices given the demand functions derived from informed consumers and arbitrary measures of uninformed captive consumers (for the cases of symmetric and asymmetric costs); (2) equilibria in which each firm sets the monopoly price corresponding to the demand function \(q(P)\) and its own cost; and (3) a mixed strategy equilibrium when the supports of the equilibrium strategies lie in the interval between the Bertrand equilibrium price and the monopoly price.

With the aid of Figure 1, we can preview the equilibrium characterization. Nature draws costs at the top of Figure 1, which are either symmetric (at \(c_L\) or \(c_H\)) or asymmetric (say, \(c_1 = c_L\) and \(c_2 = c_H\)). Policy choices following symmetric cost realizations involve the possibilities of neither firm choosing price matching ("PM"), both firms choosing PM or only one firm (say, Firm 1) choosing PM. Similarly for asymmetric costs, on the right hand side of the Figure. The lemmas indicated characterize the equilibria in the induced price games given arbitrary expectations, i.e. an arbitrary endowment of captive consumers for each firm. Following symmetric costs and "no-PM", is a standard Bertrand equilibrium; following "both PM" the equilibrium of the induced price game is collusion at the monopoly price; and following "PM by Firm 1 only" is a mixed strategy equilibrium in which the expected payoff to Firm 1 exceed the payoff from the induced game following "no PM" (holding constant the endowment of captive consumers) and the expected payoff to Firm 2 is less than the payoff from the induced game following "both PM". Following asymmetric costs, the induced price game depends only on whether the high-cost firm
(Firm 2) has invoked PM since the PM constraint is never binding on a low-cost firm in the Bertrand equilibrium for large cost differences. If the high-cost firm has not invoked PM, then the standard Bertrand equilibrium follows, given the demands and the costs faced by the two firms. If the high-cost firm has invoked PM, then both firms charge their respective monopoly prices corresponding to the demand function $q(P)$ and their realized costs. Moving one stage up the tree, we characterize the equilibrium PM decisions given expectations in $E$, showing that PM by both firms is an equilibrium action following a symmetric cost realization and PM by the low-cost firm only is an equilibrium action following an asymmetric cost realization.

The critical parameter separating the case of PM emerging as a strategy signalling a low price from the role of PM as a cartel-facilitating device is the cost difference in the two firms. The logic of this proposition hinges on the delegation aspect of PM: when invoked by a high-cost firm, PM essentially delegates to the low-cost firm the decision of which price both firms should charge to the informed consumers. Delegation under PM of the prices for all firms to a single firm has the attraction that the price will be set closer to the cooperative level than the Bertrand price of the low-cost firm. When cost differences are large, however, this advantage is more than offset by the damage imposed on the high-cost firm by a policy of PM, arising from the delegation of its pricing decision to a firm whose costs and optimal price are very different from its own. In this case, uninformed – but rational – consumers know that the behaviour of informed consumers in invoking their PM rights would penalize the high-cost firm if the latter offered a PM guarantee. These consumers know, therefore, that the announcement of PM must therefore signal a relatively low-cost and low-price firm.

3 Equilibrium When Price-Matching Is Not Allowed

We previewed our results on the incentives for price matching in the previous section. An additional issue is the impact that price matching has on equilibrium prices in the market. As a benchmark, we develop in this section the model in the case where price matching is not allowed. This allows us subsequently to identify the impact of price matching.

Demand and Equilibrium in an Arbitrary Induced Price Game: We first characterize the firm demand functions and Bertrand equilibria for an price game induced by an arbitrary marginal uninformed consumer $s_u$. Given actual prices $(p_1, p_2)$, the demand
facing Firm 1 is given by

\[ D_1(p_1, p_2, s_u) = [(1 - a)s_1(p_1, p_2) + as_u] \cdot q(p_1) \]  

(1)

and similarly for Firm 2. Note that the elasticity of the demand flowing from the uninformed consumers is the only elasticity of \( q(p_1) \), which is the \textit{market} elasticity of demand. All consumers who purchase from a particular firm purchase the same amount. We denote profits as \( \pi_i(p_1, p_2, s_u; c_i) = (p_1 - c_i)D_i(p_1, p_2; s_u) \) depending on the context. It is important to note that at the stage of the game when the two firms compete in prices, each is taking its endowment of uninformed customers over which it has monopoly power as exogenous. The greater the size of this endowment the lower its demand elasticity.

We now make a regularity assumption on the demand functions sufficient to ensure that pure strategy exist for the induced price games, and that the equilibrium prices of these games are continuous in \( s_u \). More particularly, the following assumption is sufficient for the best-response functions of each firm to be increasing (strategic complementarity), contraction mappings (Vives 1999).

10 Given the parameters \( t, c_L, a \) and \( q(\cdot) : \) for all \( s_u \in (0,1) \) and all \( p_1, p_2 \in (c_L, 1) \), \( D_1 \) satisfies: (1) \( \frac{\partial^2 \ln D_1(p_1, p_2; s_u)}{\partial p_1 \partial p_2} > 0 \), (2) \( -\frac{\partial \ln D_1(p_1, p_2; s_u)}{\partial p_1} > -\frac{\partial \ln D_1(p_1, p_2; s_u)}{\partial p_2} \), and (3) \( \frac{\partial^2 \ln D_1(p_1, p_2; s_u)}{\partial p_1^2} > \frac{\partial^2 \ln D_1(p_1, p_2; s_u)}{\partial p_1 \partial p_2} \) and similarly for \( D_2 \).

We have verified these regularity conditions on \( D_1 \) and \( D_2 \) for the case of linear demand \( q(p) = 1 - p \) and \( c_L \in (XX, 1) \) \textit{[in progress]} but recognize that the conditions are not satisfied for all possible values of the model’s parameters. In fact, as in many spatial economic models, the conditions are violated and a pure strategy equilibrium does not exist when \( t \) is sufficiently small. \(^{12}\)

For arbitrary \( s_u \), given realized costs \( c_1 \) and \( c_2 \), the Bertrand equilibrium of the induced price game is defined in the usual way to be the set of prices \( (p_1^*, p_2^*) \) satisfying \( p_1^* = \arg \max_{p_1} \pi_1(p_1, p_2^*; s_u) \) and \( p_2^* = \arg \max_{p_2} \pi_2(p_1^*, p_2; s_u) \).

\textbf{Lemma 1} Under the regularity conditions of Assumption 10, the Bertrand equilibrium of the induced price game is unique and continuous in \( s_u \).

\(^{12}\)It turns out that where the violation of the assumption matters for the outcome of the model, only mixed strategy equilibria obtain. Our adoption of the conventional focus on the equilibria with pure strategies (along the equilibrium path) thus already essentially contains the regularity conditions.
Lemma 1 follows directly from the fact that under the regularity conditions, the best-response functions of the two firms cross and cross only once (Vives (1999)). For reference below, denote the monopoly price corresponding to $c_L$ as $p_{L}^m = \max(p - c_L)q(p)$ and similarly for $p_{H}^m$.

Equilibrium When Price-Matching Is Not Allowed: When price matching is not allowed, then our assumption of symmetry on expectations means that an induced price game is contingent upon $s_u = \frac{1}{2}$.

Proposition 1 When price-matching is not allowed, a Perfect Bayesian equilibrium exists. The equilibrium price following a symmetric cost realization is the Bertrand price corresponding to that cost realization and $s_u = \frac{1}{2}$, which is less than the collusive price but greater than the price that would prevail if all consumers were informed. The equilibrium prices charged by each firm following an asymmetric cost realization are less than that firm’s monopoly price but greater than the price that firm would charge if all consumers were informed.

Appendix A derives the algebraic form of the demand functions and contains a parametric example of this equilibrium. (Proofs not outlined in the text are contained in Appendix B.) Proposition 1 is clear. The price charged by a firm, given its competitor’s price, must reconcile two forces. On the one hand, it has monopoly power on the uninformed consumers because they don’t observe prices until arriving at the firm; the entire elasticity of demand for these consumers is the elasticity of $q(p)$. Changing price will not reduce the firm’s number of captive from them because these consumers make decisions on where to shop based on expected prices, not actual prices. With respect to the uninformed consumers, the monopoly price is profit-maximizing. On the other hand, the two firms compete with each other for the informed consumers. The profit-maximizing price from these consumers is less than the monopoly price but greater than marginal cost (because the firms are differentiated). The interaction of these two forces brings the equilibrium price down to a Bertrand price that is below the monopoly price but above the Bertrand price that would prevail if all consumers were informed.
Figure 1: Equilibrium in the induced price games given large \((c_H - c_L)\) and arbitrary expectations

4 Equilibrium When Price-Matching is Allowed

Recall that when price-matching is allowed, the timing of the game is as follows: (1) nature draws the costs for each firm; (2) firms simultaneously decide whether to offer price-matching guarantees or not; (3) firms simultaneously set prices; and (4) consumers choose if and where to shop and how much to buy. The game tree when price matching is allowed is provided in a very summarized form in Figure 1. We proceed by characterizing the equilibria in the induced pricing games as summarized in the figure.
4.1 Induced Price Games Following Symmetric Cost Realizations

Consider first the pricing game following symmetric cost realizations and no price-matching by either firm. This situation is identical to the pricing equilibrium when price-matching is not allowed, which was considered in Proposition 1.

**Lemma 2** In the induced pricing game following a symmetric cost realization and no price-matching by either firm, and conditional upon \( s_u = .5 \), a unique Bertrand equilibrium exists with firms setting identical prices greater than the price that would obtain if all consumers were informed and strictly less than the monopoly price corresponding to the realized cost.

Now consider the pricing game following price-matching by both firms, which is the pricing game studied in the existing literature on price-matching. As a benchmark for our model, we consider as well the case where the informational transactions costs are zero. We have the following lemma.

**Lemma 3** In the induced pricing game following a symmetric cost realization and price-matching by both firms, and conditional upon \( s_u = .5 \): (a) if all consumers are informed, then any price between the unit cost and the collusive price is an equilibrium; (b) in this case only the collusive price satisfies normal-form trembling hand perfection; (c) if some consumers are uninformed, then the collusive price charged by both firms is a dominant strategy equilibrium.

The case of all consumers being informed is the standard model of price matching as supporting collusion. As part (a) of this lemma indicates, however, under horizontal differentiation some strategic uncertainty is introduced by price-matching: the set of Nash equilibrium prices include not just prices above the Bertrand price but prices below. If we view price-matching as the choice among different pricing sub-games, then product differentiation introduces the risk that price-matching will lead to a lower equilibrium price. The set of best responses to any price between unit cost and the collusive price, if both firms have earlier announced price-matching guarantees, is the price itself or any higher price. Each firm knows that any decision on its part to cut price below the current price would automatically be matched by its rival, so there is no incentive to cut price below the rival’s price. Any price increase beyond the rival’s price has no effect, since under price-matching the price paid by informed consumers is the lower of the two prices in the market. It follows
from this characterization of best-response sets that any price between the Bertrand price and the collusive price is a Nash equilibrium.\textsuperscript{13}

Part (b) of the lemma shows that the coordination problem associated with the continuum of Nash equilibrium prices in the sub-game is resolved if one is willing to accept the “normal-form trembling-hand perfect” refinement. Any “tremble” or uncertainty as to the rival’s strategy choice leads to a unique equilibrium which is also the Pareto optimum among the continuum of equilibria. This is guaranteed by the fact that under the price game following price-matching by both players, the collusive price is the only price that is not weakly dominated. To see this, for the case of Firm 1, let $p_2$ be an arbitrary price for Firm 2. Suppose first that $p_2 < p_m$. Any price $p_1 \geq p_2$ yields the same outcome for Firm 1 as $p_m$, since in this case $p_2$ determines the price charged to customers by both firms. A price $p_1 < p_2$, however, yields a lower profit for Firm 1 than $p_m$ since it lowers price below $p_2$ for both firms and under the regularity conditions of Assumption 10, this must reduce profit. A similar argument in the case of $p_2 \geq p_m$ shows that $p_m$ weakly dominates any other price for Firm 1.

In discussing cartel pricing and cartel formation, economists categorize the cartel’s problems as (1) agreeing on a price; (2) monitoring adherence to the agreement; and (3) punishing deviants from the agreement (Jacquemin and Slade (1986)). Under price-matching with no informational frictions in the market, punishment is immediate and automatic. Monitoring is also costless, being effectively delegated to the buyers’ side of the market. The problem of agreeing on a price is resolved by (b) of the lemma – but solved only by reliance on the trembling-hand refinement of Nash equilibria.

Even an “epsilon” of uninformed consumers, however, means that price-matching provides a stronger solution to the agreement problem. The collusive price is a dominant strategy: it is the best response to any price set below the collusive price because it allows the responding firm to collect collusive profits from each of its uninformed customers while not affecting the price that it charges to its informed customers (who are able to pay the lower price by showing evidence of the low price charged by the rival). The collusive price is the best response to any price greater than the collusive price because it results in the

\textsuperscript{13}Note that there is another class of equilibria: one firm sets the collusive price and the rival sets any higher price. The property (a) of price matching was first noted (for the case of identical products) by Z.Chen (1995). Chen argued that forward induction will narrow the equilibrium outcome to collusive pricing. We prefer to invoke the refinement of trembling-hand perfection (part (b) of the lemma).
responding firm earning half the collusive profits in the market, which is clearly the best that it can do. Subject to our confirmation below that price-matching by both players is an equilibrium pair of strategies following a symmetric cost realization, the friction of informational transactions costs strengthens the role of price-matching in facilitating cartel pricing.

This result contrasts sharply with Hviid and Shaffer (1999). Hviid and Shaffer argue that even an epsilon amount of consumer transactions costs in redeeming a price-matching offer can move a price equilibrium from collusive pricing to marginal cost pricing. When firms sell undifferentiated products even a penny of consumer transactions cost in redeeming a price-matching offer is enough to allow a firm profitably to undercut the prevailing price, making the entire collusive arrangement unravel. This is because consumer, having observed an undercutting firm, would prefer to buy from the undercutting firm rather than incur the transactions cost of invoking his or her rights under the price-matching guarantee at the higher priced store. This argument depends upon firms being completely identical from the perspective of each consumer, as Edlin (1997) has noted, and hence is ruled out in our model by the model's recognition that consumers find some stores more convenient than others. The Hviid-Shaffer argument about the non-robustness of collusion-through-price-matching to the introduction of small transactions costs in one dimension thus depends on an assumption of zero transactions costs in another dimension. The mere presence of an epsilon of high-transactions cost consumers in our model serves to strengthen, not weaken, the cartel-facilitating role of price-matching. The impact of even a small amount of transactions costs on market organization is very sensitive to the nature of the transactions costs.

Finally, consider the situation where one firm chooses price-matching and the other firm chooses no-price-matching. In this case we can state the following lemma.

**Lemma 4** In the Bertrand pricing game following a symmetric cost realization and price-matching by only one firm, and conditional upon arbitrary expectations on the part of the uninformed consumers: (a) there is no pure strategy equilibrium; (b) a mixed strategy equilibrium exists and the support of any equilibrium mixed strategy is bounded above by the collusive price and bounded below by the Bertrand equilibrium price corresponding to the realized cost and the given expectations; (c) the payoff to the non-price-matching firm is strictly less than its payoff if both firms charged the collusive price; (d) the payoff to the
price-matching firm is strictly better than its payoff in the Bertrand equilibrium following no price-matching by either firm.

4.2 Induced Price Games Following Asymmetric Cost Realizations

If the possible cost realizations $c_L$ and $c_H$ in our model are sufficiently close, then the equilibrium of the model is, not surprisingly, identical to the equilibrium with no cost uncertainty. Price-matching plays the role of facilitating collusive pricing; in fact, as in the symmetric cost realization of the previous section, price-matching implements collusive pricing as a dominant strategy equilibrium. There can be no other role for price-matching. To explore additional roles for price-matching, we analyse in this section the predictions of the model where the difference in costs, $c_H - c_L$, is large.

With asymmetric cost realizations, four possibilities arise for asymmetric policy choices, as illustrated in Figure 2: neither firm chooses PM; both do; only the low-cost firm chooses PM and only the high-cost firm chooses PM. The following lemmas parallel Lemmas 2 and 3 respectively.

**Lemma 5** In the induced pricing game following an asymmetric cost realization and no price-matching by either firm, or price-matching by the low-cost firm only, given arbitrary $s_u$, the Bertrand prices are an equilibrium. The prices $p_1$ and $p_2$ are increasing and decreasing in $s_u$, respectively, and strictly less than the corresponding monopoly prices. The marginal informed consumer, $s_I$, is strictly decreasing in $s_u$. The profits earned by Firm 1 are strictly increasing in $s_u$.

**Lemma 6** In the induced pricing game following an asymmetric cost realization and price-matching by both firms, and conditional upon arbitrary $s_u$, $p_L^m$ and $p_H^m$ is a dominant strategy equilibrium. The low-cost firm determines the price charged to informed consumers by both firms.

The low-cost firm knows, after observing the cost realizations, that it has essentially been delegated the task of setting the price for the informed consumers of both firms. That is, taking a strategy of $p_H^m$ on the part of the high-cost firm as given, the low-cost firm chooses its price $p_1$ to solve $\max_p \pi_1(p, p; s_u; c_1)$ since over the range of prices below $p_H^m$ the price-matching guarantee of Firm 2 will be in effect. This maximization problem is solved by $p_L^m$. Firm 2, on the other hand, will respond to $p_L^m$ with $p_H^m$ since it has no control over
the price it sets to informed consumers (except for the option of setting a price below $p^m_L$ for both itself and its rival, which is unprofitable) and therefore maximizes profit obtained from uninformed customers, which is achieved at $p^m_H$.

The final induced pricing game to consider is that following an asymmetric cost realization and price-matching by only the high-cost firm. If only the high-cost firm has announced price-matching, then the same logic as in the last lemma applies to establish $p^m_L$ as a best response to $p^m_H$. Almost the same logic applies to establish $p^m_H$ as a best response to $p^m_L$. The one option for the high-cost firm that affects its price to informed consumers is still undercutting the price $p^m_L$ of its rival, but now this option does not induce an automatic matching response on the part of its rival. Nonetheless, if the difference in the costs of the two firms is sufficiently large, then $c_H$ will be greater than $p^m_L$ and the high-cost firm would earn negative profits by undercutting $p^m_L$, compared with positive profits from remaining at $p^m_H$.

**Lemma 7** In the equilibrium of the induced pricing game following an asymmetric cost realization and price-matching by only the high-cost firm, and conditional upon arbitrary price expectations, if $(c_H - c_L)$ is sufficiently large, then each firm sets its own collusive price.

4.3 Equilibrium

We now use the characterizations of the induced price games to characterize the equilibrium.

**Proposition 2** If $\alpha$ is sufficiently small and $(c_H - c_L)$ is sufficiently large, then an equilibrium exists in which both firms adopt price-matching if the cost realization is symmetric and only the low-cost firm adopts price-matching when the cost realization is asymmetric. Following symmetric costs, both firms set the monopoly price corresponding to the realized cost; following an asymmetric cost realization, the firms set Bertrand prices.

Lemmas 2 to 4 show that for a fixed partition of uninformed consumers, it never pays to forego price-matching if the cost realization is symmetric. Under the expectations that are rational given the actions dictated by this lemma, foregoing price matching would in addition decrease a firm’s share of uninformed consumers. If $\alpha$ is sufficiently small, and $(c_H - c_L)$ is sufficiently large, then the cost to the high-cost firm exceeds the collusive
price of the low-cost firm by a large enough amount that it does not pay to adopt price-matching: With an asymmetric cost realization, it does not pay the high-cost firm to adopt price matching under these parameters, since this would entail a loss for every informed consumer that would be sufficiently large as to offset any benefit from attracting more uninformed consumers at its own collusive price. (If $\alpha$ is very close to 1, however, even giving away the product to informed consumers would be worth the benefits of attracting half of the uninformed consumers.) The low-cost firm has no incentive to avoid price-matching since price-matching has no effect on equilibrium prices or profits for a given partition of consumers, and given the equilibrium expectations, price-matching can only increase the firm’s set of uninformed customers. Starting with consumer expectations in $E$, the resulting price game generates expected (with respect to Nature’s draws on costs) surpluses conditional upon each pair of price-match decisions that preserve the inequalities of $E$. The impact of the expectations on equilibria across the set of induced price games can be summarized by the marginal uninformed consumer, $s_u \equiv \frac{1}{2} + (v_1 - v_2)/2t$, following the price-match decisions in which only one firm (say, Firm 1) announces price-matching. An equilibrium involves a value for $s_u$ that induces a price game following price-matching by Firm 1 only in which the marginal informed consumer, $s_I(p_1, p_2)$ is equal to $s_u$. At this equilibrium, since the low-cost firm sets a strictly lower price than the high-cost firm following an asymmetric cost realization, consumers rationally expect a strictly greater surplus from a price-matching firm than from a non-price-matching firm.

Now that we have established that price-matching can play the role of signalling a low price, the question is raised as to the impact of price-matching on the equilibrium in the market. The following proposition shows that, surprisingly, the impact of price-matching when it is used to signal a low-price is to increase the price of the firm adopting price-matching.

**Proposition 3** *Compared to the equilibrium of the game in which price-matching is not allowed, the effect of price-matching, when it is adopted in equilibrium by the low-cost firm only (following an asymmetric cost realization) is to increase the price of the price-matching firm and to decrease the price of the non price-matching firm.*

The proposition describes a second surprising effect of price-matching. The impact of adopting the practice is to decrease a rival’s price. In the conventional model of price-matching as a facilitating device the very point of a price-match announcement by a firm
is to induce its competitors to maintain high prices. The proposition, follows from the reallocation of uninformed, captive consumers to the price-matching firm in the move from the non-price-matching game to the price-matching game. This leads to the convergence of the two prices under price-matching relative to the non-price-matching equilibrium because of the direct effect of the respective changes in price elasticity of demand caused by changes in the quantities of captive consumers (see equation (1)). The direct effect of the change in the own-elasticity of demand for each firm is mitigated, but not completely offset, by the change in the rival’s price under strategic complementarity.

The allocative effects of price-matching in the market are two-fold: the effect of the information conveyed by the price-matching announcement on uninformed consumers’ decisions of where, and if, to buy; and the effect of the price-changes on the purchases of all consumers. The first of these impacts can only be a positive impact on overall welfare since consumers shop under full information. The second impact on welfare is mixed – but being a price impact on welfare is only of second order. As the parametric example in the following subsection illustrates, the overall welfare impact of price-matching in this model is positive for a wide range of parameters. We caution, however, that a third allocative effect of price matching, its impact on consumers’ incentive to invest in information about prices (i.e. to price shop) is missing in the simple model. This effect is discussed in the conclusion.

4.4 Example

In progress. This section has two aims: (1) to delineate the range of $a$ and cost differences $c_H - c_L$ that yield the signalling explanation; and (2) to investigate the overall impact on total surplus.

5 Empirical Implications

This section offers some guidance for the determination of which theory – collusive, price discrimination, or signalling - is most plausible in specific cases of price matching, by comparing the testable implications of the three theories. We discuss seven implications distinguishing the collusive theory, the price discrimination theory and the information-conveyance or signalling theory. These implications are summarized below.
<table>
<thead>
<tr>
<th>Implied Implication</th>
<th>Collusion</th>
<th>Price-dis-</th>
<th>Price-</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
<td>crimination</td>
<td>matching</td>
</tr>
<tr>
<td>1 Significant percentage of customers invoke PM rights</td>
<td>Yes</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>2 PM adopted by all firms in market</td>
<td>Yes</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>3 PM firms are lowest priced in market</td>
<td>Yes</td>
<td>X</td>
<td>Yes</td>
</tr>
<tr>
<td>4 PM increases prices of PM-adopting firms</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>5 PM increases prices of non-PM-adopting firms</td>
<td>Yes</td>
<td>?</td>
<td>X</td>
</tr>
<tr>
<td>6 PM profitable with large number of firms</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>7 PM policy terminates consumer search</td>
<td>?</td>
<td>X</td>
<td>Yes</td>
</tr>
<tr>
<td>8 PM adopted only by firms with small firms</td>
<td>X</td>
<td>Yes</td>
<td>Yes</td>
</tr>
</tbody>
</table>

- **Number of customers invoking rights**: If PM is used for the purpose of price discriminating, then a significant number of customers must exercise their rights under the guarantee to obtain lower prices from the price-matching firm. Gateway, for example, would not incur the expenses of an advertising campaign in order to offer selective discounts (via the price-matching program) to, say, 2 or 3 percent of its customers. On the other hand, if price-matching is a cartel coordinating device then the guarantee of matching discounts is a credible threat which the theory predicts is not exercised in equilibrium and if price-matching is a signal of low prices then its purpose is solely as a credible signal that is costly for higher-priced (and higher cost) firms to duplicate. Since it is offered by only the lowest-priced firms in the market, the rights under the guarantees are not exercised. In short, the observation that very few customers invoke the guarantee to obtain lower prices from the price-matching firm is consistent with the cartel theory and the signalling theory but not with the price discrimination theory.

- **Universality of price-matching within the market**: The cartel theory as it has been developed in the literature predicts that all firms in the market offer price-matching. In the absence of some asymmetries in the incentives for firms to cheat on a cartel price, there is no reason for the device to be adopted by only some firms within a given (product and geographic) market. An extension to the theory would appear to allow a single firm to act as a cartel ringmaster in announcing, via a price-matching agreement, to all other, smaller firms – smaller firms generally have more incentive to cheat on cartels – that it will automatically match any price cuts. In an oligopolistic
market, the use of price-matching only by the rivals of a particularly aggressive firm would be consistent with the cartel theory. The signalling theory predicts that only some price firms adopt PM; and the price-discrimination theory is also inconsistent with all firms adopting PM since a firm can use PM as a price discrimination device only if some other firm charges a lower price in equilibrium. In sum, the observation of PM by all firms in a market is consistent only with the cartel theory.

• **Which firms, among a subset of firms in a market, adopt PM?** This evidence can distinguish the price discrimination theory from the signalling theory. The lowest-priced firms adopt PM under the signalling theory. These firms do not adopt PM under the price discrimination theory because that theory requires that consumers at a PM firm be able to locate lower-priced firms elsewhere in the market.

• **Impact of PM on prices of PM-adopting firms:** If one is examining prices in a given product market across a variety of geographic markets or areas, or comparing prices in a given region before and after PM is adopted, then the prediction of the cartel theory is that PM causes the prices of the PM-adopting firms to increase. The purpose of the instrument is to protect cartel pricing in this theory. Under the price discrimination theory, the list prices of a firm are will also rise with the adoption of PM as the firm avails itself of the opportunity to charge higher prices to consumers with more inelastic demands. (The average transactions price, net of PM refunds, may rise or fall.) In the signalling theory, where the purpose of PM is to signal a low price, the impact of PM on the prices of the firms adopting the practice is, ironically, to increase prices (Proposition 3). The direction of price change by PM firms offers no test for distinguishing among the theories (although it does offer a test of the theories collectively). The magnitude of the price increase, however, may offer a distinguishing test. Hess and Gerstner (1991) examine weekly supermarket prices in the Raleigh, North Carolina market, before and after the adoption of a price-matching guarantee by one of the supermarkets. They find a statistically significant increase in the average price of a basket of goods (relative to a basket of goods not covered by the price-matching policy) from before to after, but the price increase appears to be of the order of 2 percent.\(^\text{14}\) This is too low to represent a plausible move from

\(^{14}\text{Moreover, even this finding is clouded by the possibility that cost and demand dynamics of the “covered” and “not covered” goods may have been different during this period (store brands figured prominently in}\)
competitive to cartel pricing.

- **Impact of PM on prices of rival firms:** The signalling theory predicts that in a market where a single firm adopts price-matching, the impact is to lower the prices of other firms (Proposition 3). On the other hand, in the cartel theory (if plausibly extended to a model with asymmetries in incentives to invoke PM), the very purpose of the instrument is to provide rivals with the incentive to set or maintain high prices. The price discrimination theory would appear to offer only ambiguous predictions as to the effect on non-PM rivals’ prices.

- **Profitability in a market with many firms:** Notwithstanding the theory that PM facilitates cartel pricing, cartels are realistic only for markets with specific structure features: a small number of firms, some barriers to entry, relatively stable costs, and transparent prices. The price discrimination theory does not require a small number of firms (Edlin (1997)). The signalling theory is developed in this paper in a model with only two firms, but this is not necessary for the theory. The explanation could plausibly be extended to the case of a large number of firms in a market with price dispersion, arising because of variation in costs or entirely as a response to variation in consumer search costs or in tastes with respect to the trade-off between lower prices and higher quality or service. Firms at the lower end of the price distribution could offer PM as a signal of low prices.

- **Impact of PM on consumer search:** In the price discrimination theory, the discovery by a consumer that a store has a PM policy is evidence to the consumer that other stores are charging even lower prices. This evidence should, if anything, encourage the consumer to search further. In the cartel theory, a consumer might search further in response to the discovery of a PM policy if the consumer believed that the practice was being invoked by a “pocket” of colluding firms selling close substitutes within a broader, differentiated product market; or a consumer having read the literature might infer that all prices are cartelized and that further search would be futile. The impact on consumer search is ambiguous. In the signalling model, generalized to a market in which even information about price-matching policies is costly to obtain, a PM announcement would tell the consumer that the firm was out of the latter basket but not in the former).
relatively low-priced and that further search was unlikely to be optimal. Srivastava and Lurie (2001) found experimental evidence that consumer’s incentive to continue searching was, as predicted by this model, reduced by a PM policy. Srivatava and Lurie exposed consumers to several simulated shopping environments in a controlled experiment and asked them questions about their perceptions of the price-matching and the non-price-matching stores. They find that subjects are more likely to stop searching, by as much as 25 percent, after they have been to a price-matching store than after they have been to a non-price-matching store.

- **Adoption of PM only by firms with small market shares**: If one observed PM being adopted only by a firm with, say, a 5 percent share of a market in which there were large rivals, one could be confident that the cartel theory, even generalized to the idea that PM is invoked to inhibit price cutting by rivals, were not at work. A large firm will not be significantly deterred by a guaranteed match, on the part of a small rival, of its own price cuts because the fraction of its demand that is diverted by the small rival’s price match is unlikely to be significant. On the other hand, this observation is consistent with either of the other two theories.

A literal reading of our model would yield another prediction: that PM is adopted for signalling purposes only by firms that have been endowed by nature with lower costs in the market. In many if not most retail markets, firms have access to very similar technologies. This prediction, however, is an artifact of our simplifying assumption that the only variation among firms that could be reflected in price differences is cost variation. In reality, firms may choose to adopt a low-cost distribution strategy or sell a product at the low-price, low-quality end of the market in order to attract customers with particular preferences. Many models with consumer heterogeneity predict vertical product differentiation even where firms face identical costs.\(^{15}\) We believe that the theory of price-matching policies as signals of low prices could be extended to such models.

### 6 Conclusion

In a model of differentiated retail competition that incorporates consumer heterogeneity in information, location, and travel costs, we investigate price matching policies. We show that

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\(^{15}\)E.g., Iyer (2000).
two types of equilibria can arise with respect to price-matching policies. Under symmetric cost structures, price-matching by both firms is an equilibrium, and it leads to collusive pricing by both firms. Under large asymmetries, however, we find that only the low-cost firm would use a price-matching policy in equilibrium, and the resulting pricing is not collusive. Instead, it involves Bertrand pricing with the low-cost firm setting a lower price than the high-cost firm.

Our results simultaneously strengthen and weaken the traditional view that price-matching are collusive devices. On the one hand, the collusive pricing result under symmetric and somewhat asymmetric cost structures arises not just as a Nash equilibrium, but as a dominant strategy equilibrium. Moreover, in contrast to Hviid and Shaffer’s (1999) results with transactions costs, we find that the mere presence of uninformed consumers, strengthens the collusion in the sense that instead of a multiplicity of equilibria ranging from Bertrand pricing to monopoly pricing—which poses the inevitable coordination issue—we get a unique dominant strategy equilibrium involving monopoly pricing. These results strengthen the theoretical basis for price-matching as a facilitating device.

On the other hand, the result that only a low-cost firm would use price-matching under conditions of significant cost disparities, and that the resulting pricing is more competitive than collusive, weakens the collusion argument. It accords more with the “common-sense” view of price-matching as a way to compete. It also agrees with the empirical literature that says that consumers view price-matching stores as more competitive than non-price-matching stores.

This result shows that price-matching policies do not offer unalloyed positive benefits to the firms that adopt them. For a firm with a cost disadvantage, adopting a price-matching policy amounts to delegating the pricing decision to its low cost competitor. Such delegation cannot be advantageous in the presence of a cost disadvantage. By not offering a price-matching policy a high cost firm keeps control of its pricing.

The presence of uninformed consumers is crucial to our results. Without uninformed consumers, there would be no positive incentive for the low cost firm to adopt a price-matching policy. It is the possibility that these consumers may take price-matching as a signal of lower costs that allows the low cost firm to adopt price matching. Of course, this same possibility might motivate a high cost firm not to cooperate in providing the signal. With a large enough cost difference, however, the disadvantage of delegating the
pricing decision to the low-cost firm—a disadvantage that has to borne both in the informed segment of the market and in the uninformed segment—is greater than the share penalty in the uninformed segment from the adverse signal.

Early in the paper we commented on the paucity of evidence regarding the prices actually charged by price-matching and non-price-matching firms. In this draft of the paper, we have outlined various empirical tests that can distinguish the three theories. In the next version, we will systematically review the evidence of prices and the incidence of price-matching in the examples listed in the introduction.

We will also offer an extension, to a model in which consumers decide endogenously whether to become informed about prices (and become “active shoppers”) or to shop on the basis of inferences drawn from stores’ price-matching policies (thus remaining “inference shoppers”). The extension reveals a Grossman-Stiglitz paradox. If inference shopping is less costly, no consumers will want to become active shoppers; but the presence of some active shoppers is necessary for inference shopping to be possible in equilibrium. This paradox can be resolved (in the now standard way) in a “noisy rational expectations” framework, in which the price-matching policy conveys some but not all information about prices.\(^{16}\)

\(^{16}\)Specifically, we will assume that price-matching is decided after the realization of “long-term” costs, but a short-term cost shock (such as the amount of excess inventory) is realized after the price-match decision but before prices are set.
Appendix A: Example

[In progress]: describes linear demand example, as in MathCad simulation program.

Appendix B

Proof of Proposition 1:

Under assumption 10, the demand functions \( D_i(p_1, p_2, 5) \) yield Best-Response functions that are upward-sloping, symmetric and cross only once, at some price \( p^B \). The pair of prices \((p^B, p^B)\) clearly constitutes an equilibrium. Letting \( \varepsilon_1(p_1, p_2, s_u, a) \) be the absolute value of the elasticity of \( D_1(p_1, p_2, s_u) \) for given \( a \), the first-order condition for Firm1 under following any cost realization yields the standard equation

\[
\frac{p_1 - c_1}{c_1} = \frac{1}{\varepsilon_1(p_1, p_2, s_u, a)} (2)
\]

and similarly

\[
\frac{p_2 - c_2}{c_2} = \frac{1}{\varepsilon_2(p_1, p_2, s_u, a)} (3)
\]

Totally differentiating these equations, and using the facts that that for each \( i, j = 1, 2 \)

(i) \( \varepsilon_i \) is decreasing in \( a \) (easily shown); (ii) \( \varepsilon_i \) is increasing in \( p_i \) (implied by (1) and (3) of assumption 10); (iii) \( \varepsilon_i \) is increasing in \( p_j \) (strategic complementarity, (1) of assumption 10), shows that both prices are increasing in \( a \). The proposition then follows directly: \( a = 0 \) is the case of all consumers being informed, and the price at \( a = 1 \) is the monopoly price.

Proof of Lemma 2: parallels the proof of Proposition 1.

Proof of Lemma 3: outlined in text.

Proof of Lemma 4:

(a) To prove that a pure strategy equilibrium does not exist following PM by one firm only (say, Firm 1), note first that in this price game, Firm 2’s best response function satisfies \( BR_2(p) = p \) for all \( p \in [p_1^B, p^m] \) where \( p_1^B \) is the Bertrand equilibrium price that would set in a price game induced by the same \( s_u \) but without price-matching, and \( p^m \) is the monopoly price for the demand curve \( q(\cdot) \) at the common realized cost. This is because (a) Firm 2 knows that any price cut below \( p \) will be automatically matched by Firm 1, and since \( \pi_2(p, p) \) is decreasing in \( p \) in this range such undercutting does not pay, and (b) under the regularity conditions on demand, \( \pi_2(p, p_2) \) is decreasing in \( p_2 \) for \( p_2 \geq p \geq p^B \) so responding with a price \( p_2 > p \) would not be profitable. (To summarize (a) and (b), were it not for the price-matching guarantee, it would pay Firm 2 to undercut any price
strategy subsets \([0\ B \ pi\ m] \) by Firm 1.) Firm 1’s profits are always higher by setting \(p^m\) than by matching any price \(p\) less than \(p^m\) since by raising its price to \(p^m\) Firm 1 collects maximum profits from the captive uninformed consumers without affecting its price \(p\) (obtained under the price-matching guarantee) to informed consumers. It follows that the only possible pure strategy equilibrium is \((p^m, p^m)\). But this pair is not an equilibrium: \(BR_1(p^m) < p^m\) because the elasticity of \(D_1\) at \((p^m, p^m)\) exceeds the elasticity of the demand curve \(q(p)\). (b) A mixed strategy equilibrium exists per Glicksberg’s (1952) theorem since the payoff functions of the firms are continuous. Using a conventional argument, it is easily shown that under the regularity assumptions on demand, the strategy subsets \([0, p^m_1]\) and \([0, p^m_2]\) can be eliminated through iterated strict dominance.\(^{17}\) Moreover for Firm 2, \((p^m, \infty)\) is strictly dominated by \(p^m\) since \(\pi_2(p_1, p_2)\) is decreasing in \(p_2\) for any \(p\) and any \(p_2 > p^m\) and similarly for Firm 1. Thus only the strategy subsets \([p^m_1, p^m]\) and \([p^m_2, p^m]\) survive \(^{17}\)To prove that the strategy subsets \([0, p^m_1]\) and \([0, p^m_2]\) can be eliminated through iterated strict dominance under the price game following price-matching by Firm 1 only, note first that these subsets can be eliminated through iterated strict dominance of the unconstrained pricing game because under the regularity conditions in assumption 10, this game has a unique Bertrand equilibrium and satisfies strategic complementarity. The results of Milgrom and Roberts (1990) imply that in any strategic complementarity (more generally, supermodular) game with a unique equilibrium, the equilibrium can be solved via iterated elimination of strictly dominated strategies. Next, we argue that the same iterated elimination procedure eliminating the strategy subsets \([0, p^m_1]\) and \([0, p^m_2]\) in the unconstrained pricing game continue to hold in the game following price-matching by Firm 1 only. For Firm 2, if a particular interval \([0, P_2]\) is eliminated from Firm 2’s strategy set because any \(p \in [0, P_2]\) is dominated by some \(p > P_2\), then \(p\) continues to dominate \(p\) in the price-game following price-matching by Firm 1, because raising price from \(p\) to \(p\) now not only increase’s Firm 2’s own price but possibly its rival’s price as well. To make the analogous argument for Firm 1, let \(p \in [0, P_1]\) is dominated by some \(p \geq P_1\). Raising price from \(p\) to \(p\) is still dominant against any \(p_2 < p\) since this would increase price for the uninformed consumers while leaving the price for the informed consumers unchanged; if raising price to \(p\) were profitable in the unconstrained game, an increase to this price must be profitable for only the uninformed consumers. Raising price from \(p\) to \(p\) is dominant against any \(p_2 > p\) is profitable since over this range the profit of Firm 1 is unaffected by the price-match constraint. Finally, consider any \(p_2 \in [p, \hat{p}]\). Raising price from \(p\) to \(p_2\) is profitable for Firm 1 in the unconstrained game because assumption 10 ensures the concavity of profits in price for Firm 1 in this game; the same increase is profitable in the price-match game since over this range Firm 1’s profits are unaffected by the constraint. Raising price further from \(p^m\) to \(p\) is profitable since this increase has no effect on the price paid by informed consumers but does increase the price paid by uninformed consumers. Thus \(p\) continues to dominated by \(p\) in the price-match game. Since the iteration of strictly dominant strategies eliminates \([0, p^m_1]\) and \([0, p^m_2]\) in the unconstrained game, it eliminates these sets in the pricing game following price-match by Firm 1 only.
iterated strict dominance. The supports of mixed strategy equilibrium strategies are always contained within strategy sets surviving iterated strict dominance, proving (b).

(c) At any realization \((p_1, p_2)\) of the two equilibrium mixed strategies, either (i) \(p_1 \leq p_2\) or (ii) \(p_1 > p_2\). In the first case, the payoff to Firm 2 is less than its payoff at \((p^m, p^m)\), since the price-matching guarantee is irrelevant and \(\pi_2(p_1, p_2) < \pi_2(p^m, p^m)\). In case (ii), the realized profits for Firm 2 are \(\pi_2(p_2, p_2)\) since the prices to informed consumers for both firms is given by \(p_2\) under the price-match guarantee of Firm 1. \(\pi_2(p_2, p_2) < \pi_2(p^m, p^m)\). Thus, whatever the realization of the mixed strategies, Firm 2’s realized profits are less than \(\pi_2(p^m, p^m)\).

(d) This proof parallels the proof of (c).

**Proof of Lemma 5:**

The only part of this lemma that does not follow the proof of Proposition 1 is the monotonicity of the prices \(p_1\) and \(p_2\) in \(s_u\). In the case of asymmetric costs and no price-matching, the two first-order conditions defining the equilibrium of the price game can be written as

\[
\frac{p_1 - c_1}{p_1} = \frac{1}{\varepsilon_1(p_1, p_2, s_u)} \tag{4}
\]

\[
\frac{p_2 - c_2}{p_2} = \frac{1}{\varepsilon_2(p_1, p_2, s_u)} \tag{5}
\]

where \(\varepsilon_i(p_1, p_2, s_u)\) is the absolute value of the elasticity of demand \(D_i\) with respect to \(p_i\). It follows directly from equation (1) that \(\partial \varepsilon_1/ds_u < 0\) and \(\partial \varepsilon_2/ds_u > 0\). Totally differentiating equations (4) and (5), and using the three regularity conditions in assumption 10 yields \(dp_1/ds_u > 0\) and \(dp_2/ds_u < 0\). The extension within this lemma of the Bertrand prices (of the no-price-matching price game) to the case of price-matching by only the low-cost firm follows because price-matching has no impact on the price game induced by a fixed set of expectations when it is offered only by the low-cost firm: The only effect of price-matching is to make under-cutting by the high cost firm of its rival’s price even more unprofitable than without price-matching in that any such price-cutting is now automatically matched by its rival. Since this price undercutting is not profitable for the high cost firm at the Bertrand equilibrium following no price-matching, the Bertrand equilibrium is sustained following a price-matching announcement by the low-cost firm alone.

**Proof of Lemma 6:**

To show that \((p_L^m, p_H^m)\) is a dominant equilibrium following a realization \((c_L, c_H)\) of costs
for Firms 1 and 2 respectively and price-matching by both firms, consider the best response by Firm 1 to a price $p_2$ by Firm 2. If $p_2 < p^m_L$, then Firm 1 could affect the price it charges to informed consumers only by undercutting $p_2$. The profits it obtains from informed consumers are increasing in $p_1$ for $p_1 < p^m_L$ because of the concavity of profits, ensured by assumption 10, meaning that such undercutting would decrease profits from both informed consumers and uninformed consumers. The best response by Firm 1 to price $p_2 > p^m_L$ is $p^m_L$, since over the range of prices $p_1$ below such a value for $p_2$, Firm 1’s profit is equal to $\pi(p_1, p_1 s_u)$, since Firm 1 is effectively setting the informed consumers’ price for both itself and its rival. This is maximized at $p^m_L$. (Setting $p_1$ above $p_2 > p^m_L$ is clearly unprofitable.)

The proof of dominance for Firm 2 is similar.

**Proof of Lemma 7:**

Following a cost realization $(c_L, c_H)$ for Firms 1 and 2 and price-matching by Firm 2 only, $P^m_L$ is a best response by Firm 1 to $P^m_H$ by an argument essentially the same as in the proof of Lemma 6. Turning to Firm 2, over the range $p_2 \geq P^m_L$, $\pi_2$ is maximized at $P^m_H$ since over this range, Firm 2 is setting a price only for the uninformed consumers. The remaining possibility to exclude is that Firm 2 profits by lowering price to some $p_2 < P^m_L$.

This is unprofitable providing that

$$\{(1-a)[1-s_I(p^m_L, p_2)] + as_u\}q(p_2)(p_2 - c_H) < \{(1-a)[1-s_I(p^m_L, p^m_H)] + as_u\}q(p^m_H)(p^m_H - c_H)$$

which condition is met for sufficiently large $c_H$, e.g. for $c_H > p^m_L > p_2$ which makes the left-hand side negative.

**Proof of Proposition 2:**

Following a cost realization $(c_L, c_H)$ for Firms 1 and 2 and price-matching by Firm 1 only, the induced price game conditional upon an arbitrary $s_u$ yields a unique marginal informed consumer $s_I$. Denote this mapping as $s_I = \Phi(s_u)$. \(\Phi\) is a non-negative and, from Lemma 5, decreasing function and therefore has a unique fixed point $s^*_u$ from the intermediate value theorem. Conditional upon $s^*_u$ for the

Assign the uninformed consumers the following expectations over prices: conditional upon observing price-matching by only one firm, consumers expect the equilibrium prices of the game induced by $s^*_u$ and price-matching by the low-cost firm; conditional upon price-matching by both firms, consumers expect prices $(p^m_L, p^m_L)$ with probability $\lambda^2 / [\lambda^2 + (1-\lambda)^2]$
and \((p_m^H, p_m^H)\) with probability \((1 - \lambda)^2 / [\lambda^2 + (1 - \lambda)^2]\); conditional upon price-matching by neither firm consumers expect equilibrium prices of the induced games described in the lemmas as following symmetric cost realizations and corresponding to \(c_L\) and \(c_H\) with the probabilities just described. Assign the firms the following actions: equilibrium prices as described in the lemmas in each induced price game (induced by \(s_u = \frac{1}{2}\) in the price game following no price-matching by either firm); price-matching announcements by both firms following a symmetric cost realization; and price-matching by only the low-cost firm following an asymmetric cost realization. The price actions are rational by construction; and the expectations are rational by construction providing that the assigned price-match decisions are rational. A symmetric cost realization, price-matching by both firms is rational, since a decision to deviate by not announcing price-matching would have two impacts: (1) it would reduce the share of the uninformed consumers shopping at the deviating firm, by the construction of consumer expectations; and (2) it would lead to a mixed strategy equilibrium in which, from Lemma 4, the deviating firm would earn less profits than in a price game following price-matching by both firms even at the same share of uninformed consumers. Price-matching by only the low-cost firm is rational for both firms, following an asymmetric cost realization with sufficiently large \(c_H\), ceteris paribus: if the low-cost firm were to drop price-matching it would decrease its share of the uninformed customers by construction of the expectations and this would lead to a decrease in its profits from Lemma 5. If the high-cost firm were to adopt price-matching, it would be forced to charge \(p_m^L\) to the informed consumers by Lemma 7. The change in profits to the high-cost firm (say, Firm 2) from moving into the price game of Lemma 7 from the price game of Lemma 5 would be negative providing:

\[
\{(1-a)(1-s_1(p_B^1, p_B^2)) + a \frac{1}{2} \}q(p_B^2)(p_B^2 - c_2) > (1-a)(\frac{1}{2})q(p_m^L)(p_m^L - c_H) + a(1-s_u^*)q(p_m^H)(p_m^H - c_L) \]

(7)

The two terms on the right-hand side represent the profits from selling to the informed consumers at a price of \(p_m^L\) and to the uninformed consumers at a price of \(p_m^H\). For sufficiently small \(a\) and sufficiently large \(c_H\), this inequality is satisfied.
7 References

References


