Money, Taxes, Audits and the Underground Economy*

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Abstract

Economic literature on the underground economy is restricted to (i) the ratio of aggregate underground output to formal sector output. What insights can one draw about micro-level transactions, given what we know about published macro estimates? This paper introduces a two-sector monetary search environment to study the additional ratios: (ii) the quantity-per-trade ratio, (iii) aggregate private quantity ratio and (iv) price ratio between sectors, among others. A search framework is essential for separating (ii) from (iii), while bargaining is ideal for generating (iv). I then assess how monetary policy affects all of these ratios. Monetizing part of the government budget helps lower the tax burden on formal sector traders and hence increase this sector. Apart from this "seigniorage effect", I identify a residual "Tanzi effect", which acts in the opposite direction and partially reverses gains in the formal sector.

Keywords: Seigniorage Effect, Tanzi Effect, Underground Economy

JEL classification: E26, H26, L51, O23

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1 Introduction

Economic knowledge about the underground economy at the micro level is limited owing to non-reporting. Empirical studies are therefore confined to the ratio of aggregate underground output to aggregate formal sector output using latent variable analysis. Can we draw any insights about transactions by the representative firm, given what we know about published macro estimates? In this paper, I provide a theoretical contribution that achieves this. I extract the output-per-trade ratio and the price ratio in an underground sector trade relative to a formal sector trade, among others. I also show how these individual transactions collect into the aggregate ratios. These results present an opportunity to revisit many questions on the underground economy and to provide new answers on the micro level effects of economic policy. Considering monetary policy, I show that the effect of inflation on the aggregate underground economy ratios may be stronger than it's impact on the micro level ratios.

Despite the measurement handicap, overwhelming conclusions point to a significant and growing underground sector in most countries [see Schneider and Enste (2000)]. Rogoff (1998) estimates that notwithstanding the advent of the cashless society, currency circulating outside the banking system in the US in 1996 was just shy of $1500 per American, young and old inclusive.[1] It is particularly puzzling that each US resident is estimated to hold on average 24 $1 bills and $70 in coins. At least one way to give meaning to the evidence is to acknowledge the existence of a flourishing underground sector. These observations require that increased attention be paid to the underground economy and government policies that affect this sector.

On the subject of monetary policy, the literature is dominated by the suboptimality of Friedman rule and the quest for the optimal mix of taxes and seigniorage

[1]The corresponding figure for Canada is the equivalent of $611 US. Admittedly, a large portion of this money, especially high denomination US dollar notes, are held abroad. In the case of small denomination notes, a fair quantity may have been accidentally destroyed or lost.

that jointly maximize social welfare. Where there exists an underground sector, official sector taxation is distortionary since it shifts productive resources toward the unofficial sector. Alternative sources of government financing are thus useful. Seigniorage is one such alternative. Traditionally, inflation is considered to diminish the real value of government debt and this reduces the required tax hikes to service old bonds. By lowering taxes, seigniorage can indeed contribute to raising the relative size of the formal sector. Using an environment with the Walrasian auctioneer, Nicolini (1998) provides quantitative estimates of the optimal seigniorage rate for Peru, a country with underground output being about 40% of reported GDP, to be about 10% per annum. This outcome leaves a substantial gap between optimal and observed rates of inflation in many developing countries. Koreshkova (2003) reconciles the pervasiveness of high inflation in poor countries to the socially optimal choice by a benevolent government.

In the current paper, I reexamine the effect of money growth on sectoral output. Rather than add to the discussion concerning the optimal rate of seigniorage, I focus solely on the effect of money growth on the two sectors. The aim is to separate two important effects, namely the seigniorage effect - exactly as described above - and the Tanzi effect. Tanzi (1978) proposed a caveat to the traditional view on seigniorage by demonstrating that the longer the time lag between sending out the tax bill and receiving payments, the lower the value of the collection. This introduces a trade-off between the seigniorage motive for inflation and the erosion of outstanding government revenue. The mechanism of the Tanzi effect described in this paper acts similarly. Rather than using bonds, government and households are joint recipients of monetary transfers. We require both the government and households to hold reserves in order to transact in the next period.[2] Holding the time lag constant, the higher the inflation rate, the lower the value of these reserves. Depending on the rate of transfers to government vis-a-vis households, the tax rate can adjust in either direction, which in turn affects the relative size of

[2] One can reinterprete government reserves as outstanding government revenues.

the underground sector. To summarize our findings, seigniorage income benefits the formal sector, but these gains are dampened by the erosion of government reserves. While the total effect has long been recognized, its components have not been explicitly separated as done in this paper.

I employ a monetary search environment, since anonymity and imperfect information readily motivate a tax-evading sector.[3] The search framework also permits an endogenous role for money as the sole medium of exchange. A third advantage of a monetary search framework emanates from our ability to get price differentials between formal sector and underground sector goods, as observed in real economies. In Walrasian environments, even though the formal and underground markets may be physically separated, market clearing conditions ensure that prices are equal between sectors. If formal and underground goods are substitutes and equally weighted in the utility function, any sectoral differences in the unit price will cause excess demand or excess supply, requiring a reallocation of resources. If sales taxes are implemented as in Koreshkova (2003), the Walrasian auctioneer sets the same price for both sectors and firms internalize the tax in their supply decisions by *relatively* under-producing in the formal sector. When consumption taxes are used instead as in Nicolini (1998), prices remain equal, while consumers internalize the tax effect by under-consuming formal sector goods. Just as taxes in the formal sector, the detection and revenue-confiscation rate acts as a "duty" on underground sellers. In the same way, the detection and confiscation rate is internalized by underground sector producers and consumers. By using one-on-one matching and bargaining in this paper, we permit higher "duty-augmented prices" to arise in the formal market, echoing the observed reality.[4]

In the environment considered, households send a fraction of member buyers to

[3]Perfect information has its advantages, since it is easier to integrate credit, where it is considered important.

[4]With the Walrasian auctioneer, the inclusion of exogenous assumptions such as a cost for visiting the one of the markets can deliver output price differentials between sectors. As will differences in the quality of products in the two markets. Other non-Walrasian propositions such as monopolistic competition and pricing-to-market can also deliver price differences.

the formal market and others to the underground market. Formal and underground buyers may be allocated different sums of money per capita. The intensive margin, which describes what occurs within a representative transaction, can be fragmented into two parts. First, the quantity purchased by a representative buyer may differ between sectors because money brought into a match is different between sectors. Secondly, each unit of money acquires relatively higher quantities underground, since prices here are lower. The intensive margin is fully summarized by the resulting quantity-per-trade ratio. The extensive margin on the other hand considers the number of successful matches in each sector. In the Walrasian environments, only a single transaction is necessary to clear each sector market. As a result, the aggregate informal-to-formal output ratio and the quantity-per-trade ratio are one and the same. With search however, the aggregate ratio is the ratio of a sum of representative trades in each sector, which depends on the number of matches in each sector. If seigniorage policy helps reduce taxes, the duty-augmented price difference narrows between sectors. A unit of money buys more in the formal market compared to previously, and households substitute by sending more buyers to the formal market. Markets congestion worsens for buyers in the formal market and to compensate, each formal buyer is handed more money. Thus, the intensive margin unequivocally leads to relatively higher quantity-per-trade in the official sector. With more buyers in the formal market, the number of formal matches increase. Thus, the extensive margin reinforces the intensive margin and hence the aggregate ratio adjusts faster to inflation than the quantity-per-trade ratio.

The contributions of this paper are as follows. To the best of our knowledge, this is the first paper to break down the relative size of the underground economy into the relative output-per-trade, relative aggregate private output and endogenous relative price ratios. Also, the first to explicitly separate the seigniorage effect of money growth from the Tanzi effect on these ratios. Finally, a reasonable prognosis is that the relative price, which is a nominal ratio, will respond to changes in the inflation rate at least as fast as the response of the aggregate relative output ratios.

I show that in the a two-sector monetary search economy, the reverse may be the case.

This paper adds to the existing literature on the informal sector, which also include Chaudhuri (1989) and Fugazza and Jacques (2003). The next section presents a monetary search framework with households, a monetary authority, fiscal authority and a regulatory body. In section 3, I characterize the model and describe some properties of the equilibrium. Section 4 derives the price and output ratios and section 5 considers the effect of inflation. In section 6, I calibrate the model to data from the US and Nigeria and present quantitative estimates of the impact of money growth. Section 7 considers robustness and compares the results to some forerunners. I conclude in section 8.

2 Economic Environment

I extend the tractable framework introduced by Shi (1999) to allow for two sectors, formal and underground/informal. These are denoted by the subscripts f and i respectively. Goods are perishable between periods, irrespective of the sector in which they are produced. By this, we preclude the emergence of commodity money. Self-produced goods yield no utility and hence trade is essential for worthwhile consumption. These restrictions are standard in monetary search models, as they permit trade and an endogenous role for fiat money.

2.1 Agents

The economy is inhabited by a large number of anonymous and infinitely-lived agents who are either sellers or buyers. Agents visit one of two markets called the formal and underground markets. For now, the two markets are assumed to be on separate islands. Formal sector sellers pay taxes on income at the rate τ. Underground sellers risk detection at the rate a, upon which sales income is confiscated, $a \leq \tau$. A seller visiting island j enters the market with a production capability. This allows him to produce output, q_j, $j = f, i$, using the technology $q_f = Al_f$ or $q_i = l_i$ respectively. $A \geq 1$ is a constant, while l_j is labour input. A buyer entering market j carries $\widetilde{m}_j$ units of money.

Once in the market, agents match randomly and one-on-one. Anonymity forbids credit transactions and trade is *quid pro quo.* Whenever a buyer is matched with a seller, trade can occur if the offer is acceptable to both sides. Suppose an offer made by a buyer is the pair (q_j, x_j), where q_j is the quantity requested and x_j is the amount of monetary compensation. Such monetary payments cannot exceed the buyer's money holding on entering the match: $x_j \leq \widetilde{m}_j$, $j = f, i$. This feasibility constraint is intrinsic to the environment, given that trade is *quid pro quo.*

Let ω be the value of money and $\Phi(l_j) = l_j^{\phi}$ be the disutility of labour, $\phi > 1$. Then, for an offer to be accepted, it must satisfy the seller's individual rationality constraint. These are $(1-\tau)\, x_f \omega \geq \Phi(l_f)$ for the prospective formal seller and $(1-a)\, x_i \omega \geq \Phi(l_i)$ for the prospective underground seller. On both islands, we allow buyers to hold all the bargaining power and to make take-it-or-leave-it offers. Optimal offers ensure that these individual rationality constraints hold with equality. Combined with the feasibility constraints, we have:

$$(1-\tau)\,\widetilde{m}_f \geq \frac{\Phi(l_f)}{\omega} \text{ and} \tag{1}$$

$$(1-a)\,\widetilde{m}_i \geq \frac{\Phi(l_i)}{\omega}\ . \tag{2}$$

I name (1) and (2) the cash-and-carry constraints.

Sellers act as "offer takers", and take the quantity requested as given. Temporarily assume that the cash-and-carry constraints hold with equality in both sectors.[5] Then labour employment in each match is known and one can rewrite

[5] I later show that this is indeed the case in equilibrium.

the level of output per trade in each sector as:

$$q_f = A\left[(1-\tau)\,\widetilde{m}_f\omega\right]^{\frac{1}{\phi}} \text{ and} \tag{3}$$

$$q_i = \left[(1-a)\,\widetilde{m}_i\omega\right]^{\frac{1}{\phi}} . \tag{4}$$

With these quantities determined, we can proceed to find the quantity-per-trade ratio. I return to this later.

2.2 Households

Here, I collect agents into decision-making families or households.[6] From this stage, the focus is on the representative household, who's state and choice variables are in lower-case letters. Capital-case variables represent those of other households and the aggregate economy, which the representative household takes as given.

Time is discrete, denoted t. A household is constituted by a unit measure of "formal sector sellers", a unit measure of "underground sellers" and the measure b of "private buyers"; $b \in (0,1]$. Buyers are incapable of making sales and sellers do not buy goods. With this assumption, we disallow barter trades. The household chooses b_{ft} and b_{it}, being the number of member buyers to visit the formal and underground markets respectively. The allocation of buyers across sectors is constrained by:

$$b_{ft} + b_{it} \leq b . \tag{5}$$

There is no population growth; the number of households, sellers and b being exogenous constants.

There is a supply of perfectly divisible money, $M_t + M_{gt}$ per capita household, of which the representative household has m_t. The household allocates its stock of money into two folds, m_{ft} for formal sector buyers and m_{it} for underground

[6] A related environment that proceeds with agents - rather than households - is provided in Lagos and Wright (2005).

buyers. The allocation of money is constrained by:

$$m_{ft} + m_{it} \leq m_t \ . \tag{6}$$

After these allocations, a buyer bound to visit market j holds money to the sum of $\widetilde{m}_{jt} = \frac{m_{jt}}{b_{jt}}$, $j = f, i$. Only these resources are carried along to the market. There exists a government that sends a fixed and unmodelled number of "public buyers", $b_g \in (0, 1]$, to the formal market, each with money, $\frac{M_{gt}}{b_g}$. I return to discuss the government later.

I specify the timing of events next. Starting a period with money holdings m_t, the representative household makes decisions on the allocation of buyers and money. The household also instructs member buyers on (q_{jt}, x_{jt}), trade offers to make in the market and sellers on (Q_{jt}, X_{jt}), the offers to accept. The price of output in sector j is hence $\frac{x_{jt}}{q_{jt}}$, $j = f, i$.

t			$t+1$
Household decisions	Depart for market	Markets Open	Markets Close
$\longrightarrow$	$\longrightarrow$	$\longrightarrow$	$\longrightarrow$
b_{jt}, m_{jt}	Private Buyers $\rightarrow \frac{m_{jt}}{b_{jt}}$, $j = f, i$	Match, Bargain	Consumption
m_{t+1}	Public Buyers $\rightarrow \frac{M_{gt}}{b_g}$	Production	Transfers: T_t, T_{gt}
Trade instructions		Trade	
		Taxes, Audits	

Next, the markets open and matching occurs. Public and formal buyers visit only the formal market while informal buyers go to the underground market. After a bargain is reached, a successfully matched seller produces the desired output. Trade is then completed and taxes paid. Audits are conducted and a fraction of underground sellers have their incomes confiscated.

Markets close and agents go to their respective households. The monetary authority then delivers lump sum transfers of T_t to each household. Purchased goods and sales receipts are gathered. To take stock of the volume of pooled resources, we must account for the number of successful matches in each market. The effective matching rates depend on the density of buyers and sellers, as well as how these agents are distributed between sectors. First, the density of sellers in the formal market is $\frac{1}{1+B_{ft}+b_g}$ and we denote this by $\mathcal{S}_{ft}$. It also represents the matching probability for a formal sector buyer. Likewise, the probability of meeting an underground seller, formal buyer, underground buyer or public buyer are $\mathcal{S}_{it} = \frac{1}{1+B_{it}}$, $\mathcal{B}_{ft} = \frac{B_{ft}}{1+B_{ft}+b_g}$, $\mathcal{B}_{it} = \frac{B_{it}}{1+B_{it}}$ and $\mathcal{B}_{gt} = \frac{b_g}{1+B_{ft}+b_g}$ respectively. Now, suppose a households sends the measure b_{ft} of buyers and a unit measure of sellers to the formal market. The expected number of successful matches are respectively,

$$\int_{z=0}^{b_{ft}} \mathcal{S}_{ft} dz = b_{ft}\mathcal{S}_{ft} \ , \text{ and}$$

$$\int_{z=0}^{1} \mathcal{B}_{ft} dz + \int_{z=0}^{1} \mathcal{B}_{gt} dz = \mathcal{B}_{ft} + \mathcal{B}_{gt} \ .$$

The two terms in the second equation are expected successful matches of formal sector sellers with private formal buyers and public buyers. Similarly, the expected number of successful matches are $b_{it}\mathcal{S}_{it}$ for underground buyers, $b_g\mathcal{S}_{ft}$ for public buyers and $\mathcal{B}_{it}$ for underground sellers.[7]

The household's pooled resources therefore include goods of volume $b_{jt}\mathcal{S}_{jt}q_{jt}$ from sector j, $j = f, i$ and these are instantly consumed. Money, of volume $(1-\tau)\mathcal{B}_{ft}X_{ft}$ and $(1-a)\mathcal{B}_{it}X_{it}$, which were received from private buyers and $(1-\tau)\mathcal{B}_{gt}X_{gt}$ from public buyers are gathered. Here, X_{gt} is money paid per public buyer during trades.[8]

Household's Problem

Let $U(c_t)$ be utility from consuming c_t units of the consumption good, but zero for the household's domestically produced goods. Household members view

[7] It is easy to see that $\mathcal{B}_{ft} \equiv b_{ft}\mathcal{S}_{ft}$, $\mathcal{B}_{it} \equiv b_{it}\mathcal{S}_{it}$ and $\mathcal{B}_g \equiv b_g\mathcal{S}_{ft}$. Since it takes two to trade, one successfully matched seller implies a successfully matched buyer.

[8] The results do not depend on taxes being paid uniformly on all formal transactions, including those involving public buyers.

this utility as a common objective. $\beta \in (0,1)$ is the time discount factor. The household's problem involves choosing the optimal money holding, buyer allocation b_{jt}, money allocation m_{jt} and quantity demanded q_{jt}, $j = f, i$, so as to solve:

$$v(m_t) = \max U(c_t) - \mathcal{B}_{ft}\Phi(l_{ft}) - \mathcal{B}_{it}\Phi(l_{it}) - \mathcal{B}_{gt}\Phi(l_{gt}) + \beta E v(m_{t+1})$$

subject to (1), (2), (5), (6) and the liquidity constraint:

$$\begin{aligned} m_{t+1} - m_t \leq & \ (1-\tau)\left[\mathcal{B}_{ft}X_{ft} + \mathcal{B}_{gt}X_{gt}\right] + (1-a)\,\mathcal{B}_{it}X_{it} \\ & -b_{ft}\mathcal{S}_{ft}x_{ft} - b_{it}\mathcal{S}_{it}x_{it} + T_t \ . \end{aligned} \tag{7}$$

The first term on the right hand side is after-tax revenues from formal sector sales to private and public buyers. The second term is underground revenue, net of confiscations. The next two terms account for household purchases from both formal and underground markets. Incoming funds from sales, X_{gt} and X_{jt}, arrive simultaneously as outgoing funds x_{jt}, $j = f, i$ during purchases. Hence the former cannot be used to finance the latter.

The felicity function $U(c_t)$ is chosen to be CES:

$$U(c_t) = \left[\theta c_{ft}^{\sigma} + (1-\theta)\, c_{it}^{\sigma}\right]^{\frac{1}{\sigma}} \ , \ \sigma, \theta \in (0,1) \ .$$

σ is the elasticity of substitution. c_t is therefore a composite consumption comprising both sector goods. With $\sigma \in (0,1)$, formal and underground goods are imperfect substitutes [but see section 7]. Since goods are perishable, consumption is constrained by the feasibility condition:

$$c_{jt} \leq b_{jt}\mathcal{S}_{jt}q_{jt} \quad , \quad j = f, i \ .$$

The usual non-negativity requirements all apply:

$$m_{jt}, x_{jt}, c_{jt}, b_{jt} \geq 0 \ \ , \ \ j = f, i \ . \text{ and } m_t \geq 0 \ \forall t.$$

2.3 Government

The definition of a sector as "underground" suggests the existence of an authority that makes this distinction. Government consists of a monetary, fiscal and regulatory body. The monetary authority supplies money, $M_t + M_{gt}$, per capita household, of which the fiscal authority holds M_{gt}. Money supply grows at the gross rate γ per period. T_t of this new money goes to households and $T_{gt} = (\gamma - 1)(M_t + M_{gt}) - T_t$ goes to the fiscal authority. The regulatory authority conducts a financial audits per capita household per period, $a \in (0, 1)$. These audits are aimed at detecting underground sellers and confiscating their sales revenues. Confiscated revenues are readily transferred to the fiscal authority. The fiscal authority takes taxes in cash from formal sector sellers at the rate $\tau \in (0, 1)$.[9] M_{gt} is termed government reserves and are used to finance real expenditure during period t. As private agents depart for the market, public buyers join in, each with $\widetilde{M}_{gt} = \frac{M_{gt}}{b_g}$ units of money. The equivalent cash-and-carry constraint for the public buyer is:

$$(1-\tau)\,\widetilde{M}_{gt} \geq \frac{\Phi\left(L_{gt}\right)}{\Omega_t}\ , \tag{8}$$

where L_{gt} is the amount of labour required to produce for a public buyer. Therefore, $q_{gt} = AL_{gt}$.

After markets close, public purchased goods to the amount of $b_g \mathcal{S}_{ft} q_{gt}$ are collected and these are used to pay for government spending. The budget constraint summarizing the real fiscal position is:

$$G = \frac{b_g \mathcal{S}_{ft} q_{gt}}{b_{ft} \mathcal{S}_{ft} q_{ft} + b_g \mathcal{S}_{ft} q_{gt}}\ , \tag{9}$$

where $G \in (0, .5)$ is a constant. Note that the denominator represents reported

[9] In a monetary search environment with anonymous agents, a possible way to implement taxation is by sending τ government agents (per capita household) to roam the market and collect sales revenues from formal sector firms. Although we do not take a stand on exactly how these taxes are collected, we assume zero cost of collection.

GDP. Alternatively put, public spending is a constant percentage of reported private spending (that is $b_g \mathcal{S}_{ft} q_{gt} = \frac{G}{1-G} b_{ft} \mathcal{S}_{ft} q_{ft}$).

Since government revenues are nominal while expenditure is real, the government faces a liquidity constraint much like private households. The equation that summarizes the evolution of government reserves looks as follows:

$$M_{gt+1} + b_g \mathcal{S}_{ft} X_{gt} \leq M_{gt} + \tau \left[\mathcal{B}_{ft} X_{ft} + \mathcal{B}_g X_{gt} \right] + a \mathcal{B}_{it} X_{it} + T_{gt}. \tag{10}$$

Given the timing of events, incoming public funds arrive simultaneously as outgoing funds and the former cannot be used to finance the latter in any given period. Hence, government spending in period t can only be financed out of reserves: $\frac{M_{gt}}{b_g} \geq X_{gt}$.[10] The fiscal authority also engages in pooling of nominal tax receipts, comprising $\tau \mathcal{B}_{ft} X_{ft} + \tau \mathcal{B}_{gt} X_{gt}$ from formal sector firms and $a \mathcal{B}_{it} X_{it}$ from the regulatory authority. Reserves are carried forward after deducting current spending: $b_g \mathcal{S}_{ft} X_{gt}$.

Note that the money growth rate is endogenous. Suppose that τ is reduced with a held constant. The government's liquidity constraint goes into deficits as consistent with optimal regions of the Laffer curve. This requires an adjustment in transfers T_{gt} to supply the funds necessary to alleviate the fiscal position, which in turn changes γ. In summary, (9) and (10) emphasize the inherent interaction between the fiscal and monetary policy variables τ and γ (and the regulatory instrument a, where variable).

3 Characterizing the equilibrium

In this section, I examine the euler conditions that characterize an equilibrium. Let λ_{jt}, $j = f, i$, be the Lagrange multiplier on the cash-and-carry constraints in each successful match. m_{jt} is chosen such that the cash-and-carry constraint binds to an equal extent in expectation in each sector: $(1-\tau) \mathcal{S}_f \lambda_{ft} = (1-a) \mathcal{S}_i \lambda_{it}$. The

[10] This constraint was earlier eluded to in inequality (8).

implied euler equations for money are:

$$\frac{\omega_t}{\beta} = \omega_{t+1} + (1-\tau)\,\mathcal{S}_{ft}\lambda_{ft+1} \quad \text{and} \tag{11}$$

$$\frac{\omega_t}{\beta} = \omega_{t+1} + (1-a)\,\mathcal{S}_{it}\lambda_{it+1}\ . \tag{12}$$

Money kept between periods delivers its discounted value in the next period as well as helps alleviate the cash-and-carry constraint in future trade matches. From (11) and (12), it can be shown that both cash-and-carry constraints bind in all successful matches in equilibrium if $\gamma > \beta$. From this point on, we assume this to be the case.

Next, we turn to the optimal quantity that is demanded in each trade match. The associated first order conditions are derived as:

$$\theta c_t^{\frac{1}{\sigma}-1} c_{ft}^{\sigma-1} = \lambda_{ft}\frac{\Phi\left(L_{ft}\right)}{\Omega_t}\frac{\phi}{q_{jt}} + \omega_t \frac{dx_{ft}}{dq_{ft}} \quad \text{and} \tag{13}$$

$$(1-\theta)\, c_t^{\frac{1}{\sigma}-1} c_{it}^{\sigma-1} = \lambda_{it}\frac{\Phi\left(L_{it}\right)}{\Omega_t}\frac{\phi}{q_{it}} + \omega_t \frac{dx_{it}}{dq_{it}}\ , \tag{14}$$

which are similar to Shi (1999). Demanding a higher quantity yields marginal utility from the additional units. The marginal cost is incurred at two levels. At the buyer level, buying a larger quantity requires of the buyer to pay more money, thus making the corresponding cash-and-carry constraint more binding. The rate at which this constraint becomes more binding depends on how much is required to motivate the seller to deliver the additional quantity, which in turn depends on the seller's labour disutility costs on the margin. Secondly, as buyers purchase higher quantities from the market and need more money to do so, the household is pressured to deliver more money to its buyers. This causes the liquidity constraint (7) to become more binding.

The first order condition for b_{ft} is given as:

$$\begin{aligned} 0 &= \mathcal{S}_{ft}\left[\theta c_t^{\frac{1}{\sigma}-1}c_{ft}^{\sigma-1}q_{ft}-\lambda_{ft}\frac{\Phi\left(L_{ft}\right)}{\Omega_t}-\omega_t x_{ft}\right] \\ & -\mathcal{S}_{it}\left[\left(1-\theta\right)c_t^{\frac{1}{\sigma}-1}c_{it}^{\sigma-1}q_{it}-\lambda_{it}\frac{\Phi\left(L_{it}\right)}{\Omega_t}-\omega_t x_{it}\right]. \end{aligned} \tag{15}$$

Allocating more buyers to the formal sector generates more formal sector purchases and yields the associated marginal benefits in consumption utility. All things being equal, as more buyers visit the formal sector, $\frac{m_{ft}}{b_{ft}}$ declines and the cash-and-carry constraint binds further in this sector. The household is pressured to deliver more money to formal sector buyers, causing the liquidity constraint to become more binding as well. A similar effect pertains to the underground sector. For the marginal buyer, the net benefits must be equal between sectors in expectation.

All households are alike except for preferences, and so I apply symmetry as usual. To focus on the interaction between γ and τ, the value of a is fixed at a constant. The only state variable is money. To proceed to describe an equilibrium therefore, it is essential to ensure that this variable evolves at a constant rate. Assuming fixed money growth rate γ, the euler condition for money holding in steady state reduces to:

$$\lambda_{ft} = \frac{\gamma-\beta}{\left(1-\tau\right)\beta\mathcal{S}_{ft}}\omega_t \quad \text{and} \tag{16}$$

$$\lambda_{it} = \frac{\gamma-\beta}{\left(1-a\right)\beta\mathcal{S}_{it}}\omega_t\ . \tag{17}$$

Next, monetary policy needs to be conducted in a consistent manner. Apart from γ being constant, I define a parameter $\pi\in[0,1]$ such that $T_{gt}=\pi\left(\gamma-1\right)\left(M_t+M_{gt}\right)$ and $T_t=\left(1-\pi\right)\left(\gamma-1\right)\left(M_t+M_{gt}\right)$. These two assumptions enable us state the tax rate as a function of the money growth rate, $\tau\left(\gamma\right)$.

3.1 The Equilibrium

Definition of Equilibrium

A symmetric monetary search equilibrium is defined as the tax rate τ, the sequence of government money holding $(M_{gt})_{t=0}^{\infty}$, the set of household choices $(b_f, m_{ft})_{t=0}^{\infty}$ and the implied value of money $(\Omega_t)_{t=0}^{\infty}$ such that given a constant money growth rate γ, the following requirements are met: (i) each household solves its optimization problem; (ii) the representative household's decisions replicate the aggregate decisions $(B_f, M_{ft})_{t=0}^{\infty}$; (iii) prices are positive, though bounded (the value of money is positive and bounded); and (iv) the government budget balances.

In particular, an equilibrium involves a solution to a system of five equations for B_f, M_{ft}, Ω_t, τ and M_{gt}.[11] From the households side of the economy, optimal choices by the representative household and symmetry among households require that the following first three be satisfied:

$$\theta C^{\frac{1}{\sigma}-1} C_f^{\sigma-1} = \frac{\gamma - \beta\left(1-\mathcal{S}_f\right)}{\beta \mathcal{S}_f} \Omega_t \frac{M_{ft}}{B_f} \frac{\phi}{Q_f} \ , \tag{18}$$

$$\left(1-\theta\right) C^{\frac{1}{\sigma}-1} C_i^{\sigma-1} = \frac{\gamma - \beta\left(1-\mathcal{S}_i\right)}{\beta \mathcal{S}_i} \Omega_t \frac{M_{it}}{B_i} \frac{\phi}{Q_i} \ , \tag{19}$$

$$\frac{\frac{M_{it}}{B_i}}{\frac{M_{ft}}{B_f}} = \frac{\gamma - \beta\left(1-\mathcal{S}_f\right)}{\gamma - \beta\left(1-\mathcal{S}_i\right)} \ . \tag{20}$$

From the government's side of the economy, equilibrium involves the additional two:

$$G = \frac{1}{\frac{B_f}{b_g}\frac{Q_f}{Q_g} + 1} \ , \tag{21}$$

$$M_{gt} = \frac{\tau\left(\gamma\right) \mathcal{S}_f M_{ft} + a \mathcal{S}_i M_{it} + \pi\left(\gamma-1\right) M_t}{\left(1-\pi\right)\left(\gamma-1\right) + \left[1-\tau\left(\gamma\right)\right] \mathcal{S}_f} \ . \tag{22}$$

Variables without the time subscript represent equilibrium real values. Those with

[11] All other variables can be derived as functions of the five in the definition: B_f, M_{ft}, Ω_t, τ and M_{gt}. See appendix for details.

time subscripts are nominal values, which depend on the money stock at date t.[12]

Given values for G, a and γ, there exists an equilibrium. (18), (19) and (20) deliver a values for B_f, M_{ft} and Ω_t, all in terms of τ. Next, the government sector adds two variables, τ and M_{gt}, which are derived from (21) and (22).

4 Size and Prices

In this section, I shed light on the implications of the model for the quantity ratios $\widetilde{R}_W = \frac{Q_i}{Q_f}$, $\widetilde{R}_S = \frac{C_i}{C_f} = \frac{B_i \mathcal{S}_i Q_i}{B_f \mathcal{S}_f Q_f}$ and $\widetilde{R} = \frac{B_i \mathcal{S}_i Q_i}{B_f \mathcal{S}_f Q_f + b_g \mathcal{S}_f Q_g}$, as well as relative prices $\frac{p_{ft}}{p_{it}}$. $\widetilde{R}$ is the aggregate size of the underground economy relative to total reported formal output, when measured in quantities. $\widetilde{R}_S$ on the other hand is the quantity of underground output relative to private formal output only. $\widetilde{R}_W$ is the ratio of quantity-per-trade in private matches.

4.1 Relative Quantities

Since the cash-and-carry constraint binds in each sector, (3) and (4) fully describe output per trade in each case. Using these outcomes together with (20), the equilibrium quantity-per-trade ratio becomes:

$$\widetilde{R}_W = \frac{1}{A}\left[\frac{\gamma - \beta\left(1 - \mathcal{S}_f\right)}{\gamma - \beta\left(1 - \mathcal{S}_i\right)} \frac{1 - a}{1 - \tau\left(\gamma\right)}\right]^{\frac{1}{\phi}} . \tag{23}$$

The above ratio completely describes the intensive margin, which can be fragmented into two parts. The first has to do with what a unit of money can do in each market. Recall that the tax rate is higher than the confiscation rate. Thus, of each unit of money brought into a match in each sector, the underground seller takes home more (after expected confiscations) than do the formal seller (after taxes). If $A = 1$, it follows that a unit of money buys more output underground than it does in the formal sector, implying a high $\widetilde{R}_W$. The second concerns how

[12] (22) can also be derived using the household liquidity constraint (7), since they are two sides of the same monetary flow circle.

much money is actually taken into a match in each sector. Equation (20) outlines a clear relationship between the amount of money given to a buyer and the market congestion for buyers in his sector relative to the other sector. Buyers sent to the sector with the worse market congestion are always handed more money per capita. Suppose market congestion is worse for formal buyers than for underground buyers. Then households tend take advantage of each successful formal match to acquire large quantities. They do so by giving more money per capital to formal buyers than to underground buyers. In other word, high market congestion for formal buyers ($\mathcal{S}_f < \mathcal{S}_i$) contributes to reduce $\widetilde{R}_W$. Finally, superior technology in the formal sector means that even with equal financial rewards and equal market conditions, formal sector sellers can deliver higher quantities within each match.[13]

Next, (18) and (19) imply that household choice of q_{jt}, $j = f, i$, must satisfy the condition $\frac{C_i}{C_f} = \left[\frac{1-\theta}{\theta}\frac{\mathcal{S}_i}{\mathcal{S}_f}\frac{Q_i}{Q_f}\right]^{\frac{1}{1-\sigma}}$. This readily delivers the aggregate private trades equivalent as:

$$\widetilde{R}_S = \left[\frac{1-\theta}{\theta}\frac{\mathcal{S}_i}{\mathcal{S}_f}\widetilde{R}_W\right]^{\frac{1}{1-\sigma}} \quad . \tag{24}$$

Comparing with the $\widetilde{R}_W$, it becomes clear that $\widetilde{R}_S$ stresses the effect of the matching rate on aggregate market outcomes. Suppose $\widetilde{R}_W$ is given. Then for the representative buyer sent to each island, the tightness of the underground market relative to the formal market, $\frac{\mathcal{S}_i}{\mathcal{S}_f}$, determines the quantity of expected purchases by an underground buyer relative to a formal buyer: $\frac{\mathcal{S}_i}{\mathcal{S}_f}\widetilde{R}_W$. Preference parameters are also reflected in $\widetilde{R}_S$ because households are mindful of the effect of their buyer allocation decisions on the eventual mix of goods that they consume. On one hand, a high θ causes households to send more buyers to the formal sector, thus lowering $\widetilde{R}_S$. On the other hand, the higher the degree of substitutability between goods, the more households even out the mix of formal and underground goods in the consumption basket. They do so by allocating member buyers so

[13] One can consider the effect of technology as a third dimension of the intensive margin. I ignore this by assuming A to be a constant, unaffected by policy.

as to dampen the quantity-per-trade ratio, the relative market tightness and the weights placed on the two goods. Given the bargaining outcome and market conditions, households employ their buyer allocation decision to edge closer to their preferred mix of goods. The allocation of buyers between sectors and its effect on market congestion and aggregate trade outcomes is termed the extensive margin. This margin is conclusively captured by $\widetilde{R}_S$. A search framework is essential for separating $\widetilde{R}_S$ from $\widetilde{R}_W$.

Government spending is a constant fraction of private purchases: $b_g \mathcal{S}_f Q_{gt} = \frac{G}{1-G} C_{ft}$. Substituting into $\widetilde{R}$ above, we have:

$$\widetilde{R} = (1 - G)\, \widetilde{R}_S \ . \tag{25}$$

Since government agents only buy from the formal sector, this has a dampening effect on the informal ratio.

4.2 Relative Price and the Price-Weighted Ratios

Price in each transaction is $p_{jt} = \frac{X_{jt}}{Q_{jt}} = \frac{M_{jt}}{B_j} \frac{1}{Q_j}$, $j = f, i$ in equilibrium. Again, using (20), the relative price ratio in private trades reduces to $\frac{p_{ft}}{p_{it}} = \frac{\frac{M_{ft}}{B_f}}{\frac{M_{it}}{B_i}} \widetilde{R}_W$, or:

$$\frac{p_{ft}}{p_{it}} = \frac{1}{A} \left[\frac{\gamma - \beta (1 - \mathcal{S}_i)}{\gamma - \beta (1 - \mathcal{S}_f)} \right]^{1-\frac{1}{\phi}} \left[\frac{1-a}{1 - \tau(\gamma)} \right]^{\frac{1}{\phi}} \ . \tag{26}$$

The larger the difference between the tax and audit rates, the larger the difference between the duty-augmented prices. With relatively high market congestion for formal buyers, each brings more money into a match and this further increases the price difference between sectors. However, high technological know-how in the formal sector reduces this price ratio. Clearly, equal prices is the exception rather than the norm, unlike in Walrasian models with homogeneous goods. Secondly, this outcome does not depend on homogeneity between formal and underground goods [see section 7]. In the Walrasian equivalent heterogenous goods, price differences are solely an outcome of preference parameters, since sellers internalize the tax and audit effects in their production decisions.[14]

With the price ratio determined, we proceed to restate these quantity ratios in price-weighted terms as follows:

$$R_W = \frac{p_{it}}{p_{ft}}\widetilde{R}_W = \frac{\gamma - \beta\left(1 - \mathcal{S}_f\right)}{\gamma - \beta\left(1 - \mathcal{S}_i\right)} \ ,$$

$$R_S = \frac{p_{it}}{p_{ft}}\widetilde{R}_S \ \text{ and}$$

$$R = \left(1 - G\right) R_S \ .$$

This final ratio, R, has been the subject of virtually all of what is known about the underground economy, both in the theory and empirical literature. The current paper allows us to utilize their results and work backwards to gain insights on the other five ratios and the price ratio as established above. In environments with the Walrasian auctioneer, only a single transaction is necessary in each trading period in each sector. There is therefore no clear distinction between R_W and R_S. Further, since prices are often equated between sectors, the previous two are also not differentiated from quantity equivalents, $\widetilde{R}_W$ and $\widetilde{R}_S$. Some of these results are particularly useful since empirically, micro level data may not be forthcoming in studies on the underground economy.

5 Monetary Policy

5.1 Same Island

For simplicity, I start by assuming that both sectors are on the same island, however formal buyers do not buy from underground sellers and vice versa. Money growth will have intensive margin effects through adjustments in the tax rate. However,

[14] The above price ratio (26) also includes these preference parameters, since they affect buyer allocation, which in turn affects $\mathcal{S}_i$ and $\mathcal{S}_f$.

market congestion for buyers remains equal between sectors,[15] and so will be money allocation per buyer. Although market congestion is unchanged, extensive margin effects persist because the number of buyers sent to each sector changes.

By definition, seigniorage is a tax on money balances. Given that the government must hold monetary reserves in order to transact, money growth also taxes government balances. By implication, $\pi > 0$ only guarantees that government is a gross recipient of transfer income, but not necessarily a net beneficiary of inflationary policy. In fact, for some small values of π, government reserves can be eroding to the relative benefit of households. We therefore have the following proposition.

Proposition 1 *Suppose inflation is zero ($\gamma = 1$) and the economy is in equilibrium, characterized by buyer allocation B_f^*, B_i^* and policy variables a and τ. If there is a move to positive inflation, there exists a minimum threshold $\widehat{\pi}$:*

$$\widehat{\pi} = \frac{\tau \frac{B_f^*}{b} + a \frac{B_i^*}{b}}{1 - \tau + \tau \frac{B_f^*}{b} + a \frac{B_i^*}{b}} > 0 \ , \tag{27}$$

for which government becomes a net beneficiary of inflationary policy iff $\pi > \widehat{\pi}$.

Proof. If $\mathcal{S}_f = \mathcal{S}_i$, then (20) implies that all buyers hold the same amount of money: $\frac{m_{ft}}{b_{ft}} = \frac{m_{it}}{b_{it}}$. Hence, $\frac{M_{jt}}{M_t} = \frac{b_{jt} \frac{m_{jt}}{b_{jt}}}{b_{ft} \frac{m_{ft}}{b_{ft}} + b_{it} \frac{m_{it}}{b_{it}}} = \frac{b_{jt}}{b_{ft} + b_{it}} = \frac{B_j^*}{b}$, $j = f, i$. Substituting in (22) and rearranging gives:

$$\left[(1 - \pi) (\gamma - 1) + (1 - \tau) \mathcal{S}_f \right] \frac{M_{gt}}{M_t} = \tau \mathcal{S}_f \frac{B_f^*}{b} + a \mathcal{S}_i \frac{B_i^*}{b} + \pi (\gamma - 1)$$

Define $\widehat{\pi}$ as the value of π for which the fiscal body and households both receive transfers proportional to their respective reserves: $\frac{M_{gt}}{M_t} = \frac{T_{gt}}{T_t} \left(= \frac{\widehat{\pi}}{1 - \widehat{\pi}} \right)$. Substituting into the above and simplifying gives (27). The same result can be obtained using (7)$\cdot$ ■

[15] In particular, when both sectors are on the same island, we mean $\mathcal{S}_f = \mathcal{S}_i = \frac{1}{2 + b + b_g}$. A match between a formal buyer and an underground seller - or vice versa - is classified as an unsuccessful match. Substituting in (20) means all buyers hold same sums of money.

This proposition seeks to explain the nature and implication of lump sum transfers T_t and T_{gt}. Intuitively, both households and the fiscal authority need to hold reserves in order to buy goods. Only when $\frac{M_{gt}}{M_t} < \frac{T_{gt}}{T_t}$ will household reserves erode to the benefit of government. In other words, when $\pi > \widehat{\pi}$, the tax rate can adjust downwards in the face of positive seigniorage income. When $\pi \in (0, \widehat{\pi})$, monetary transfers received by the government help reduce taxes, but such reductions are overturned by the erosion of M_{gt}.

From the ratios above, the effect of inflation on the relative size of the informal sector is given by:

$$\frac{d\widetilde{R}}{d\gamma} = \frac{1}{\phi(1-\sigma)} \frac{\widetilde{R}}{1-\tau} \frac{d\tau}{d\gamma} \,. \tag{28}$$

In section 2, I described the interaction between τ and γ. Money growth affects the tax rate and this in turn affects the relative size of the sectors. Using the nominal government budget constraint (22), the effect on the tax rate can be summarized as:

$$\frac{d\tau}{d\gamma} = \varphi(M_{gt}) - \frac{\pi}{\mathcal{S}_f} \frac{M_t + M_{gt} + (\gamma - 1)\frac{dM_{gt}}{d\gamma}}{M_{ft} + M_{gt}} \,, \tag{29}$$

where $\varphi(M_{gt})$ measures the required change in the tax rate resulting solely from the erosion of value of government reserves.[16] Higher money growth rate impacts the tax rate on two fronts. The first term is the positive impact through the Tanzi effect. The second term is the effect of transfers in alleviating the tax rate when $\pi > 0$. The first effect persists even with $T_t = 0$, $\forall t$, as shown in the following proposition:

Proposition 2 *In an economy with positive money growth* $(\gamma > 1)$, *the inflation effect is positive even if government is the sole beneficiary of monetary transfers. In particular,* $\varphi(M_{gt}) > 0$ *even if* $\pi = 1$.

[16] The dependence of $\varphi(\cdot)$ on other variables has been suppressed. See appendix.

Proof. See appendix· ■

Intuitively, government always needs to hold reserves in order to buy goods in each period. The erosion of these reserves remain, even when $\pi \in (\widehat{\pi}, 1]$.

The Tanzi and seigniorage effects are respectively:

$$\frac{d\widetilde{R}_{tanzi}}{d\gamma} = \frac{1}{\phi(1-\sigma)} \frac{\widetilde{R}}{1-\tau} \varphi_t(M_{gt}) \quad \text{and}$$

$$\frac{d\widetilde{R}_{seig.}}{d\gamma} = -\frac{1}{\phi(1-\sigma)} \frac{\widetilde{R}}{1-\tau} \frac{\pi}{\mathcal{S}_f} \frac{M_t + M_{gt} + (\gamma - 1)\frac{dM_{gt}}{d\gamma}}{M_{ft} + M_{gt}} .$$

That is, with lump sum transfers to households, the response of the relative size may be opposite to what has been suggested in the literature. The seigniorage effect is consistent with findings by other authors. To sum our findings, while seigniorage spending helps alleviate the tax rate and increases the formal sector, such gains are dampened by the erosive impact on government reserves.

A somewhat puzzling implication of the model concerns to the responsiveness of the aggregate quantity ratios and the relative price to changes in the inflation rate, when both sectors are on the same island. Since the latter is a nominal ratio, one will expect that this ratio would respond at least as fast as the quantity ratios in equilibrium. However, we have the following corollary:

Corollary 3 *Suppose both sectors are on the same island* ($\mathcal{S}_f = \mathcal{S}_i$) *and that formal and underground goods are substitutes* ($\sigma \in (0,1)$)*, the aggregate quantity ratios are more responsive to inflation than the relative price.*

Proof. From (24) and (26), elasticities can be compared as:

$$\frac{d\frac{p_{ft}}{p_{it}}}{d\gamma} \frac{\gamma}{\frac{p_{ft}}{p_{it}}} = (1-\sigma) \frac{d\widetilde{R}_S}{d\gamma} \frac{\gamma}{\widetilde{R}_S} .$$

Since $\widetilde{R} = (1-G)\widetilde{R}_S$, the same result applies in comparing the responsiveness of $\widetilde{R}$ and $\frac{p_{ft}}{p_{it}}$. However, the price ratio and the quantity-per-trade ratio $\widetilde{R}_W$ both respond at the same rate· ■

To explain, assume without loss of generality that $\pi = 1$. Higher inflation therefore translates unequivocally into lower taxes. Recall that in this simplified environment, all buyers hold equal amounts of money at any given date. Therefore, with higher inflation and lower taxes, this given sum of money can buy higher quantities in the formal sector, leading to a lower quantity-per-trade ratio: $\widetilde{R}_W$. The same argument suggests a relative reduction in prices in the formal sector, thus a fall in $\frac{p_{ft}}{p_{it}}$. Compared to each other, to what extend do these two ratios decline? Since $\mathcal{S}_f = \mathcal{S}_i$, (23) and (26) are identical, and hence both intensive margin ratios decline at the same rate. However, as the tax-augmented price lowers in the formal sector, households send more buyers to this sector. This extensive effect reinforces the real ratio only, resulting in a larger response of the aggregate ratio than the micro level ratios. That is, $\widetilde{R}_S$ (and $\widetilde{R}$) tend to fall faster than $\frac{p_{ft}}{p_{it}}$ when goods are substitutes.

Next, I consider the effect on the price-weighted ratios. Notice that in the reduced environment, $R_W = 1$ and unaffected. The extensive effect persists, as buyers move to the formal market and cause R_S (and R) to fall. If both goods are close substitutes ($\sigma > .5$), these also decline faster than the price ratio.[17]

Another interesting result is established by comparing the quantity ratios and the price-weighted ratios:

$$\frac{dR}{d\gamma}\frac{\gamma}{R} = \sigma\frac{d\widetilde{R}}{d\gamma}\frac{\gamma}{\widetilde{R}}\ .$$

If seigniorage policy leads to a documented increase in R (hence R_S), this increase is only a fraction of that in actual goods consumed, $\widetilde{R}$ (hence $\widetilde{R}_S$). This is because the fall in $\frac{p_{it}}{p_{ft}}$ - which is an inherent component of any empirical estimate of the underground economy - partially dampens the actual rise in the underground output ratio when measured in quantities.

[17]Specifically, $\frac{d\frac{p_{ft}}{p_{it}}}{d\gamma}\frac{\gamma}{\frac{p_{ft}}{p_{it}}} = \frac{1-\sigma}{\sigma}\frac{dR_S}{d\gamma}\frac{\gamma}{R_S}$.

5.2 Separate Islands

When the sectors are separated, this restores two dimensions. Buyers now hold different sums of money depending on their sector of assignment. Also, market congestion is no longer equal between sectors. Again, I restrict the discussion by assuming a high value for π, such that inflation lowers taxes.

I earlier fragmented the intensive margin into two parts: how much money is sent into a match and what a unit of money can do inside a match. Suppose that γ increases. Under mild conditions, relative prices decline in the formal market, and each unit of money buys more goods in this sector. Households substitute by sending more buyers to the formal market. Since buyers inflict negative externality on each other during matching, market congestion worsens for buyers in the formal market - $\mathcal{S}_f$ declines - and improves for underground buyers - $\mathcal{S}_i$ rises. To compensate, each household hands more money to its formal buyers relative to its underground buyers, as demonstrated by (20). On both accounts therefore, the intensive margin indicates a fall in $\widetilde{R}_W$ as inflation increases for high values of π. Since $\frac{p_{it}}{p_{ft}}$ has increased after the tax reduction, the adjustment in R_W ($= \frac{p_{it}}{p_{ft}}\widetilde{R}_W$) depends on which of the two components respond faster. As far as taxes decline, R_W also declines. Unlike the conclusion from the previous subsection, the quantity-per-trade ratio $\widetilde{R}_W$ also responds faster than the price ratio.

Next, we turn to the aggregate ratios and examine the effect on $\widetilde{R}_S$ ($= \frac{b_{it}}{b_{ft}}\frac{\mathcal{S}_i}{\mathcal{S}_f}\widetilde{R}_W$). As identified earlier, the buyer ratio $\frac{b_{it}}{b_{ft}}$ declines, reinforcing $\widetilde{R}_W$. Again, as noted, higher inflation and lower taxes cause market congestion in the formal market, leading to a rise in $\frac{\mathcal{S}_i}{\mathcal{S}_f}$. Clearly, buyers constitute only one component of the matching paradigm and hence $\frac{b_{it}}{b_{ft}}$ responds faster to inflation than $\frac{\mathcal{S}_i}{\mathcal{S}_f}$. As a result, $\widetilde{R}_S$ and $\widetilde{R}$ both decline, indeed at a faster rate than the micro ratio $\widetilde{R}_W$. The same analysis shows that R_S and R decline faster than R_W. Since $\frac{p_{it}}{p_{ft}}$ has increases, the quantity ratios $\widetilde{R}_S$ and $\widetilde{R}$ decline faster than their price-weighted equivalents, R_S and R. Schneider and Enste (2000) observe sustained increase in the size of the underground economy (R) in most countries in the past few decades. An implication of the results in this paper is that in terms of actual commodity quantities $\left(\widetilde{R}\right)$, the increase in the underground sector may be even higher than documented.

6 Calibration and Results

In this section, I calibrate the base model with two islands to match data from the US and Nigeria in 2001 and conduct two policy experiments. The first experiment is to vary γ, with $\pi = 0$. This enables us document the Tanzi effect. In the second experiment, we set $\pi = 1$ and then vary γ, thereby ascertaining the total effect of money growth.

The model period is set to one month, with $\beta = 0.997$. ϕ is set to 2, while σ is chosen to be 0.9, reflecting a high degree of substitutability. Using time diary data, Juster and Stafford (1991) estimate that US residents spend on average 23.9 hours on paid work and 6.8 hours shopping per week. We choose b to equal $\frac{6.8}{23.9}$.[18] The parameters chosen are:

Table 1:	Model Parameters				
	R	G	τ	a	$\gamma - 1$
US	8.8%	16.665%	10.73%	.154%	2.8262%
Nigeria	76%	22.6%	19.8%	.008%	18.786%

First, published estimates of R are taken from Schneider and Enste (2000) for each country for the period 1989 to 1990. The assumption is that these values are approximately still relevant for 2001. Next, I retrieve US data on general government final consumption expenditure as a percentage of GDP and tax revenue as a percentage of GDP for 2001 from the World Development Indicators database of the World Bank. These are used to approximate US values for G and τ respectively. Also collected from the same database are annual rates of CPI inflation

[18] For now, this figure is adapted for both countries considered here, as well as all other countries simulated in the appendix. Also see the appendix for sensitivity analysis on ϕ and σ. The value of A plays a role similar to θ in pinning down the relative size, R. Hence, we calibrate only one, θ, and set $A = 1$.

for both countries in 2001, which are used to represent $\gamma - 1$. For Nigeria, τ is taken to equal average government revenue excluding grants as a percentage of GDP from 1997 to 2001 as reported in the regional economic outlook by the IMF. The corresponding figure for G in Nigeria - from the same source and same period - is taken to be government expenditure as a percentage of GDP. I also retrieve annual data on fines and forfeits in both countries from the IMF's Government Finance Statistics. The audit rate a is then calculated as the ratio of these revenues to underground output in each country.[19] In the calibration, we assume that taxes and confiscations are sufficient to finance each government, thus $\pi = \widehat{\pi}$. Actual household money stock, M_t, is normalized to 1, while M_{gt} is endogenously determined by the nominal government budget (22).

The values of b_g, $\widehat{\pi}$ and θ are the most challenging to calibrate, given no direct equivalents from economic data. The model is simulated to simultaneously satisfy three conditions. First, a value for b_g such that government purchases are a fraction, G, of official GDP. In other words, we choose b_g to balance the real government budget. Notice that nominal revenues from τ and a alone do not guarantee a balance budget. The number of public buyers spending this revenue is important in determining the aggregate goods successfully purchased using this revenue. The second criterion is a value for $\widehat{\pi}$ such that $\widehat{\pi} = \frac{M_{gt}}{M_t + M_{gt}}$. Third, a parameter θ such that the relative size of the underground economy, R, is consistent with estimates in Schneider and Enste (2000) for each country. The values of θ and b_g derived from this exercise for a given country are then retained for all other simulations for that country. However, π is varied to ascertain when we conduct experiments to the effects of inflation. For the sake of comparison, we also calibrate a hypothetical economy (HE) in which G, τ, a and γ are set the the US

[19] For example, for every dollar of output produced in the official sector, there is \$0.76 of output underground in Nigeria [Schneider and Enste (2000)]. GDP values were retrieved from the International Financial Statistics database, from which underground output was derived as 76%. I then used the fines and forfeits data to find the yearly percentages and then average. Unreported years were omitted.

economy, but with $\theta = 0.5$. We allow b_g and $\widehat{\pi}$ to adjust as above.[20] Simulating all three economies, we document the resulting characteristics of the equilibrium. The results for the benchmark economy are summarized in Table 2.

Table 2:	Calibrated Results		
	USA	Nigeria	HE
θ	0.5984	0.5307	.5
$\frac{b_i}{b}$	0.0936	0.5081	0.6100
$\frac{m_{it}}{m_t}$	0.0748	0.4937	0.6182
$\frac{p_{ft}}{p_{it}}$	1.2081	1.1491	1.0512
R_W	0.7836	0.9442	1.0352
R_S	0.1034	1.0342	1.5638
R	0.0880	0.7600	1.3327
$\widetilde{R}_W$	0.9467	1.0850	1.0881
$\widetilde{R}_S$	0.1249	1.1884	1.6439
$\widetilde{R}$	0.1063	0.8733	1.4009
$\frac{b_g}{b}$	0.1903	0.2592	0.0791
$\frac{M_{gt}}{M_t}$	0.1348	0.1250	0.0566
$\widehat{\pi}$	0.1188	0.1111	0.0536

First, we consider the hypothetical economy. This economy is characterized by (i) equal weights on both goods as well as (ii) lower duty-augmented prices in the underground sector. Not only do households allocate more buyers underground than to the formal sector $\left(\frac{b_i}{b} > .5\right)$; they also give each underground buyer relatively more money per capita $\left(\frac{m_{it}}{m_t} > \frac{b_i}{b}\right)$. Each unit of money buys more in goods underground. Combined with the fact that each underground buyer holds more money, the intensive margin gives $\widetilde{R}_W > 1$. With more buyers underground, the extensive margin reinforces the intensive margin with $\widetilde{R}_S > \widetilde{R}_W$. Since $\frac{p_{it}}{p_{ft}} < 1$, the price-weighted ratios are less than the quantity ratios, such that $R_W < \widetilde{R}_W$ and $R_S < \widetilde{R}_S$. Since in this economy the formal sector is relatively small, the values of b_g and M_{gt} required to match government spending are also small, which accounts for the values of $\frac{b_g}{b}$, $\frac{M_{gt}}{M_t}$ and $\widehat{\pi}$.

[20] For a full outline of a suggested algorithm, see the appendix.

Turning to the real economies, higher taxes than audits contribute to the price ratio $\frac{p_{ft}}{p_{it}} > 1$ in both countries. This ratio is even higher in the US because of much higher weights placed on formal goods. Despite low underground prices, relatively high distaste for them means very few buyers are sent underground. Market congestion is low for underground buyers and hence each need little money. The two dimensions of the intensive margin thus act in opposite directions. The net intensive margin is such that $\widetilde{R}_W < 1$. With few buyers underground, the extensive margin reinforces the intensive margin and hence $\widetilde{R}_W > \widetilde{R}_S$. Again, since prices are lower underground, the price-weighted ratios are lower than their quantity equivalents in the US.

In Nigeria, low underground prices provide adequate incentives for slightly higher buyer allocation to this sector. Nevertheless, buyer congestion is higher in the formal sector due to the presence of government buyers. As a result, households give each formal buyer more money per capita than to underground buyers. Here again, the two dimensions of the intensive margin work in opposite direction but the price differences is stronger with $\widetilde{R}_W > 1$. Again, the extensive margin reinforces the intensive margin: $\widetilde{R}_S > \widetilde{R}_W$.

The model performs poorly in predicting a significant portion of US currency outstanding, with the underground sector accounting for only 7.48% of currency outstanding. For Nigeria, this percentage climbs to almost 50%, but there is no data to compare this figure with. After adjusting the quantity ratios for prices, we find that ratio of actual goods consumed underground $\left(\widetilde{R}\right)$ is about 1.83% higher in the US and over 10% higher in Nigeria than documented.

When $\pi = 0$, the seigniorage effect is eliminated, leaving the Tanzi effect. With increased inflation, the Tanzi effect leads to increases in the tax rate. The underground sector increases in relative size. The opposite is the case when $\pi = 1$. It is worth noting that the real ratios adjust faster to inflation than the price-weighted equivalents. This is because the effect of policy on the price ratio is in the opposite direction of the impact on relative output. We conclude that if

Table 3: Monetary Policy (Nigeria)

γ^{12}	τ	$\frac{b_i}{b}$	$\frac{p_{ft}}{p_{it}}$	R_W	R_S	R	$\widetilde{R}_W$	$\widetilde{R}_S$	$\widetilde{R}$	$\frac{M_{it}}{M_t}$	$\frac{M_{gt}}{M_t}$
						$\pi = 0$					
β^{12}	0.1977	0.5069	1.1499	0.9425	1.0281	0.7555	1.0838	1.1822	0.8688	0.4921	0.1257
1.19	0.2007	0.5105	1.1504	0.9452	1.0442	0.7674	1.0874	1.2012	0.8828	0.4964	0.1238
1.8	0.2068	0.5176	1.1517	0.9504	1.0763	0.7910	1.0946	1.2396	0.9110	0.5049	0.1199
						$\pi = \widehat{\pi} = 0.1111$					
1.18786	0.198	0.5081	1.1491	0.9442	1.0342	0.7600	1.0850	1.1884	0.8733	0.4937	0.1250
						$\pi = 1$					
β^{12}	0.2027	0.5115	1.1523	0.9446	1.0472	0.7696	1.0884	1.2066	0.8867	0.4973	0.1233
1.19	0.1773	0.4897	1.1396	0.9360	0.9608	0.7061	1.0666	1.0949	0.8046	0.4731	0.1350
1.8	0.1304	0.4521	1.1165	0.9225	0.8293	0.6094	1.0300	0.9259	0.6804	0.4322	0.1562

monetary policy leads to an observed change in the sectoral distribution of the economy (R), real relative output $\left(\widetilde{R}\right)$ may actually have adjusted more than documented. If seigniorage helps reduce the tax rate, then it narrows the price difference between the sectors and brings the price-weighted ratios closer to the quantity ratios.

7 Discussion

In the previous sections, price differences come directly from take-it-or-leave-it offers. This outcome is robust since take-it-or-leave-it offers are a special case of bargaining. Using generalized Nash bargaining instead, price differences still arise. In this case, the price ratio depends not only on the policy variables τ and a, but also on preference parameters θ and σ, as well as the bargaining parameter. Admittedly, there are other ways to achieve interesting relative prices, some of which are common in the open economy macro literature.

Next, I defend the choice of $\sigma \in (0,1)$. In the current set-up, this parameterization plays a considerable role in getting significantly large demand for both sector's goods in all policy states. However, setting $\sigma = 1$ does not necessarily lead to a single-sector equilibrium. This is because as the number of buyers visiting a particular sector approaches zero, the matching success rate improves substantially. This provides adequate incentives for households to allocate more buyers to that sector. Since our purpose is not to explain the existence of the underground sector, we simply calibrate parameters that help us match the data and allow us to proceed with policy experiments. Previous authors, including Nicolini (1998) and Koreshkova (2003), have on the other hand used supply-side restrictions that directly forbid corner solutions. The demand-side approach employed in the current paper is therefore comparable.

We fixed the number of sellers, s_f and s_i exogenously to avoid multiple equilibria and also set $s_f = s_i$ solely for simplicity. All the results stand, in general, for the case with $s_f \neq s_i$. In the calibration exercise, choosing $\theta > \frac{1}{2}$ contributes to deliver an aggregate private formal sector output that is larger than underground sector output ($R_S < 1$), although this could alternatively be achieved with $s_f > s_i$. If we endogenize the allocation of sellers between sectors however, sellers are likely to move in the same direction as buyers, further strengthening our results on the extensive margin. Introducing capital into the environment compels one to take a stand on which good(s), formal or underground, can be accumulated into capital, if not both. How exactly are they combined in the constitution of a uniform stock of capital? Are underground goods just as good as formal sector goods in capital formation? These dimensions are avoided by the labour approach employed in this paper.

A familiar assertion in the literature concerns the importance of credit to formal sector agents, which is simply not available to those underground. For instance, a considerable part of Koreshkova (2003) sheds light the potential reaction of households when their money stocks lose value owing to seigniorage spending. Her approach allows households to choose between cash and credit as means of payment. When inflation increases, it becomes more worthwhile to buy on credit.[21] Household demand for credit services strictly increases on the margin and since these services are counted as part of formal sector activity, the erosion of value further promotes this sector. Although interesting, the alternative approach adopted in this paper is motivated by concern that although credit may play a role, it may not be as pivotal as previously thought in the literature. Credit services may not be exclusively produced by - nor accessible to - formal sector agents. Rotating Saving and Credit Associations (ROSCAs) have taken root in many developing countries allowing informal business owners access to financial intermediation.[22]

[21] This is dependent on the assumption that no interest is applied on credit balances, once they are settled by the end of the period. Although the interest rate on bonds rise with inflation, this has no negative impact on the quantity demanded of credit services.

[22] A sample of these include "partner" in Jamaica, "chit" in India, "susu" in Ghana, "kye" in Korea, "hui" in China and Taiwan, "tanda" in Mexico, "altin gunu" in Cyprus, "ekub" in Ethiopia, "hagbad" in Somalia, "pasanakus" in Bolivia, "arisan" in Indonesia, and "tontines" in Senegal. For ways to include credit in models with anonymous agents, see Berentsen, Camera

Participation rates are as high as 45% among high income groups in Thailand, which is significant relative to participation rates in underground production [see Besley and Levenson (1996)].

Regarding our results on relative price and quantity-per-trade ratio, a somewhat related paper in the literature is McLaren (1998). He considers a non-monetary economy with markets for imported goods. There are several markets, each for a specific class of imported goods. Depending on the tax rate and concentration of tax inspectors in a given market, traders decide either to import legally and pay the associated taxes or to smuggled at a risk of detection. Quantity per importer is fixed and only the choice of sector is endogenously influenced by policy. In equilibrium, traders in the market for a particular class of good are all simultaneously legitimate importers or all smugglers. Although separate prices can be derived for the two sectors, only one is operational for each commodity class. This outcome is a direct result of price-taking behaviour by all market participants. He then studied the optimal tax and audit rates in a Ramsey-type equilibrium. The current paper on the other hand endogenizes production quantities, prices and buyers' sector choice, and these depend on fundamentals as well as economic policy, including money.

8 Conclusion

Seigniorage remains an important source of financing in many countries. If government faces a cash flow constraint similar to private households, the usefulness of inflation in alleviating the tax burden is compromised. Although seigniorage can help raise the relative size of the formal sector, such gains are restrained by the diluted adjustment in the tax rate. In this paper, I provide quantitative estimates of the counteracting seigniorage and Tanzi effects of inflation, as well as the impact on the underground economy relative to the formal sector.

and Waller (2005).

Walrasian models of the underground economy are particularly useful due to the ease of incorporating credit. One weaknesses however is that only a single transaction is necessary to clear each market. There is no clear separation of the underground-to-formal sector quantity-per-trade ratio from the aggregate ratio. In other words, only the intensive margin matters in predicting the effect of policy on both sectors. The extensive margin, peculiar to the search models, allow us to separate micro level transactions from the aggregate counterparts. In this case, the aggregate ratio is the ratio of the sum of several individual transactions, dependent on the number of matches in each sector in each trade period. The number of matches is affected by the agent allocation to each sector, which in turn affects the degree of market congestion given the matching technology.

In response to higher inflation and lower taxes, households send more buyers to the formal market and less to the underground sector. Market congestion improves for buyers in the underground market, but the aggregate number of underground matches strictly decrease. With improving market congestion, underground buyers are handed less money per capita, thereby reducing relative underground output on the intensive margin. Since the number of matches has decreased in the underground sector, the extensive margin reinforces the intensive margin. This extensive margin reinforcement affects the aggregate ratios only. In summary, we find that seigniorage policy has a larger impact on the aggregate ratios than on the micro level equivalents.

Suppose we contend that the relative size of the underground sector described in the empirical literature are the price-weighted values. Then it follows that in terms of actual quantities, the underground sector is much higher than previously thought because prices are known to be higher in the formal sector. Through matching and bargaining, the model considered in this paper generates higher prices in the formal sector than underground. Using there prices, we find the underground sector in terms of quantities to be 1.83% higher in the US and over 10% higher in Nigeria than previously reported. Since inflation reduces the tax rate, it narrows the price difference between the sectors and brings the price-weighted ratios closer to the quantity ratios.

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Appendix

The household solves:

$$\begin{aligned} v(m_t) = & \max_{b_{jt},m_{jt},q_{jt},m_{t+1},j=f,i} \left[\theta c_{ft}^{\sigma} + (1-\theta)\, c_{it}^{\sigma}\right]^{\frac{1}{\sigma}} - \mathcal{B}_{ft}\Phi(l_{ft}) - \mathcal{B}_{it}\Phi(l_{it}) - \mathcal{B}_{gt}\Phi(l_{gt}) \\ & + \beta E v(m_{t+1}) + b_{ft}\mathcal{S}_{ft}\lambda_{ft}\left[(1-\tau)\frac{m_{ft}}{b_{ft}} - \frac{\Phi(L_{ft})}{\Omega_t}\right] \\ & + b_{it}\mathcal{S}_{it}\lambda_{it}\left[(1-a)\frac{m_{it}}{b_{it}} - \frac{\Phi(L_{it})}{\Omega_t}\right] + \omega_t\{m_t + (1-\tau)\left[\mathcal{B}_{ft}X_{ft} + \mathcal{B}_g X_{gt}\right] \\ & + (1-a)\,\mathcal{B}_{it}X_{it} - b_{ft}\mathcal{S}_{ft}x_{ft} - b_{it}\mathcal{S}_{it}x_{it} + T_t - m_{t+1}\} \end{aligned}$$

Labour required to produce q_{jt} is: $L_{it} = q_{it}$, $L_{jt} = \frac{q_{jt}}{A}$, $j = f, g$. Also, $\Phi(l_{jt}) = l_{jt}^{\phi}$, $j = f, i, g$.

The Euler conditions for money (11) and (12) are derived by using the condition for optimal allocation of money between sectors: $(1-\tau)\,\mathcal{S}_f\lambda_{ft} = (1-a)\,\mathcal{S}_i\lambda_{it}$. The FOCs for output and buyers, (13) (14) and (15) follow direct from the above set up. Now, we proceed to show how we arrived at (18), (19) and (20). First, note that if money is valued, $\lambda_{jt} \geq 0$, $j = f, i$ and hence $(1-\tau)\,x_{ft} = \frac{\Phi(L_{ft})}{\Omega_t}$ and $(1-a)\,x_{it} = \frac{\Phi(L_{it})}{\Omega_t}$. Thus, $\frac{dx_{ft}}{dq_{ft}} = \frac{1}{1-\tau}\frac{\Phi(L_{ft})}{\Omega_t}\frac{\phi}{q_{ft}}$ and $\frac{dx_{it}}{dq_{it}} = \frac{1}{1-a}\frac{\Phi(L_{it})}{\Omega_t}\frac{\phi}{q_{it}}$. These substituted into the first order conditions yield:

$$\theta c_t^{\frac{1}{\sigma}-1} c_{ft}^{\sigma-1} = \left[\lambda_{ft} + \frac{\omega_t}{1-\tau}\right] \frac{\Phi(L_{ft})}{\Omega_t} \frac{\phi}{q_{ft}} \ ,$$

$$(1-\theta) c_t^{\frac{1}{\sigma}-1} c_{it}^{\sigma-1} = \left[\lambda_{it} + \frac{\omega_t}{1-a}\right] \frac{\Phi(L_{it})}{\Omega_t} \frac{\phi}{q_{it}} \ ,$$

$$\begin{aligned} 0 \ &= \ \mathcal{S}_f \left[c_t^{\frac{1}{\sigma}-1} c_{ft}^{\sigma-1} q_{ft} - \left\{ \lambda_{ft} + \frac{\omega_t}{1-\tau} \right\} \frac{\Phi(L_{ft})}{\Omega_t} \right] \\ &\quad -\mathcal{S}_i \left[c_t^{\frac{1}{\sigma}-1} c_{it}^{\sigma-1} q_{it} - \left\{ \lambda_{it} + \frac{\omega_t}{1-a} \right\} \frac{\Phi(L_{it})}{\Omega_t} \right] . \end{aligned}$$

To simplify these three FOCs further, consider the following two properties of the equilibrium. First, with a constant money growth rate $m_{t+1} = \gamma m_t$, the value of money declines at the growth rate of money: $m_{t+1}\omega_{t+1} = m_t\omega_t$. Thus the euler for money gives $\gamma\omega_t m_t = \beta\omega_{t+1}m_{t+1} + \beta(1-\tau)\mathcal{S}_f\lambda_{ft+1}m_{t+1}$. Rearranging,

$$(1-\tau)\mathcal{S}_f\lambda_{ft} = (1-a)\mathcal{S}_i\lambda_{it} = \frac{\gamma-\beta}{\beta}\omega_t \ . \tag{30}$$

Second, due to the restriction $\gamma \geq \beta$, $\lambda_{jt} \geq 0$, $j = f, i$ and the cash and carry constraints bind in all transactions:

$$(1-\tau)\frac{m_{jt}}{b_{jt}}\omega_t \ = \ L_{jt}^{\phi} \quad , \ j = f, g \tag{31}$$

$$(1-a)\frac{m_{it}}{b_{it}}\omega_t \ = \ L_{it}^{\phi} \tag{32}$$

Substituting (30), (31) and (32) in the FOCs for output and imposing symmetry ($\omega_t = \Omega_t$ and $l_{jt} = L_{jt}$, $j = f, i$ etc), we have:

$$\theta c_t^{\frac{1}{\sigma}-1} c_{ft}^{\sigma-1} = \frac{\gamma - \beta(1-\mathcal{S}_f)}{\beta\mathcal{S}_f}\omega_t \frac{m_{ft}}{b_{ft}} \frac{\phi}{q_{ft}} \tag{18}$$

$$(1-\theta) c_t^{\frac{1}{\sigma}-1} c_{it}^{\sigma-1} = \frac{\gamma - \beta(1-\mathcal{S}_i)}{\beta\mathcal{S}_i}\omega_t \frac{m_{it}}{b_{it}} \frac{\phi}{q_{it}} \tag{19}$$

Again, substituting (30), (31) and (32) in the first order condition for buyers and imposing symmetry gives:

$$\begin{aligned} 0 &= \mathcal{S}_f\left[\theta c_t^{\frac{1}{\sigma}-1} c_{ft}^{\sigma-1} q_{ft} - \frac{\gamma-\beta\left(1-\mathcal{S}_f\right)}{\beta \mathcal{S}_f} \omega_t \frac{m_{ft}}{b_{ft}}\right] \\ & -\mathcal{S}_i\left[(1-\theta)\, c_t^{\frac{1}{\sigma}-1} c_{it}^{\sigma-1} q_{it} - \frac{\gamma-\beta\left(1-\mathcal{S}_i\right)}{\beta \mathcal{S}_i} \omega_t \frac{m_{it}}{b_{it}}\right]. \end{aligned}$$

Simplifying using (18), (19) gives:

$$\left[\gamma-\beta\left(1-\mathcal{S}_f\right)\right] \frac{M_{ft}}{B_f} = \left[\gamma-\beta\left(1-\mathcal{S}_i\right)\right] \frac{M_{it}}{B_i} \ . \tag{20}$$

At the government side (21) follows easily from (9). Next, note that $\mathcal{B}_{jt} = b_{jt}\mathcal{S}_j$, $j = f, i$, $\mathcal{B}_g = b_g \mathcal{S}_f$, $T_{gt} = \pi\left(\gamma-1\right)\left(M_t + M_{gt}\right)$ and $M_{gt+1} = \gamma M_{gt}$ in steady state. Substituting these into (10) and simplifying gives (22). The derivation of all the ratios are explained in the paper. $\frac{dR}{d\gamma}$ follows directly.

We turn to Proposition 2. Again, from (22), $\left[(1-\pi)\ \left(\gamma-1\right) + (1-\tau)\,\mathcal{S}_f\right] M_{gt} = \tau \mathcal{S}_f m_{ft} + a\mathcal{S}_i m_{it} + \pi\left(\gamma-1\right) M_t$. Differentiating with respect to the money growth rate gives $\left[(1-\pi) - \frac{d\tau}{d\gamma}\mathcal{S}_f\right] M_{gt} = \frac{d\tau}{d\gamma}\mathcal{S}_f m_{ft} + \tau \mathcal{S}_f \frac{dm_{ft}}{d\gamma} + a\mathcal{S}_i \frac{dm_{it}}{d\gamma} + \pi M_t$.[23] Collecting sides,

$$\frac{d\tau}{d\gamma} = \varphi\left(M_{gt}\right) - \pi \frac{M_{gt} + M_t + \left(\gamma-1\right)\frac{dM_{gt}}{d\gamma}}{\mathcal{S}_f\left[m_{ft} + M_{gt}\right]}$$

where $\varphi\left(M_{gt}\right) = \frac{M_{gt} + \left[\ (\gamma-1) + (1-\tau)\mathcal{S}_f\right]\frac{dM_{gt}}{d\gamma} - \tau \mathcal{S}_f \frac{dm_{ft}}{d\gamma} - a\mathcal{S}_i \frac{dm_{it}}{d\gamma}}{\mathcal{S}_f\left[m_{ft} + M_{gt}\right]}$.

[23]Note that M_t is a stock variable and $\frac{dM_t}{d\gamma} = 0$.

A Suggested Numerical Procedure

1. Collect published estimate on R. Name this R_{publ}. This estimate is our final target. Get estimates of G, τ, a and γ from data. Estimate b as proportion of time spend shopping. Set $A = 1$ and $m_t = 1$.

2. Guess $\theta \in [0, 1]$.

3. Guess $b_g \in [0, b]$.

4. Guess ω_t

5. Guess $m_{ft} \in (0, m_t)$. This also implies $m_{it} = m_t - m_{ft}$.

6. Guess $b_{ft} \in (0, b)$. This also implies $b_{it} = b - b_{ft}$.

7. Use (20) to verify b_{ft}.

8. Guess $\widehat{\pi}$. Based on this guess, find m_{gt} using (22).

9. Use $\frac{m_{gt}}{m_t} = \frac{\widehat{\pi}}{1-\widehat{\pi}}$ to verify $\widehat{\pi}$. Based on these guessed values, we have enough to compute q_{ft} $\left(= A\left[(1-\tau)\,\omega_t \frac{m_{ft}}{b_{ft}}\right]^{\frac{1}{\phi}}\right)$, q_{it} $\left(= \left[(1-a)\,\omega_t \frac{m_{it}}{b_{it}}\right]^{\frac{1}{\phi}}\right)$, c_{ft}, c_{it}, c_t and q_{gt} $\left(= A\left[(1-\tau)\,\omega_t \frac{m_{gt}}{b_g}\right]^{\frac{1}{\phi}}\right)$.

10. Use (19) to verify the guess of m_{ft} in step 6.

11. Use (18) to verify the guess of ω_t in step 5.

12. Use (21) to verify the guess of b_g in step 4. Now find R.

13. If $R < R_{publ}$, increase θ.

14. The values of θ and b_g obtained from this exercise are maintained for experiments.

Data

1. Data on the relative size of the underground economy (R) was culled from Schneider and Enste (2000). Other ratios were derived using these values of R and formulas outlined in section 4.

2. For Nigeria, data on G, τ and inflation were taken from the IMF's African Department database for 2001 as described in section 6. For each country considered in Tables 4 & 5, I collect data on general government final consumption expenditure as a percentage of GDP and tax revenue as a percentage of GDP from the World Development Indicators database of the World Bank. The data period is 1965 to 2004 and unreported years were omitted. The averages are used to approximate G and τ respectively for each country. Annual rates of CPI inflation, are also collected from the same source for 2000 to 2005 and are used to represent $\gamma - 1$ for each of these countries. However, US results in Table 2 use G, γ and τ values for 2001 only.

3. I retrieve annual data on fines and forfeits (F_t) from the IMF's Government Finance Statistics. The audit rate, a, is then calculated as the ratio of these revenues to underground output (U_t) in each country. For example, according to Schneider and Enste (2000), for every dollar of output produced in the official sector, there is \$0.088 worth of output produced underground in the US. GDP values were retrieved from the International Financial Statistics database, from which U_t was derived as 8.8% in the case of the US. The audit ratio is then estimated as:

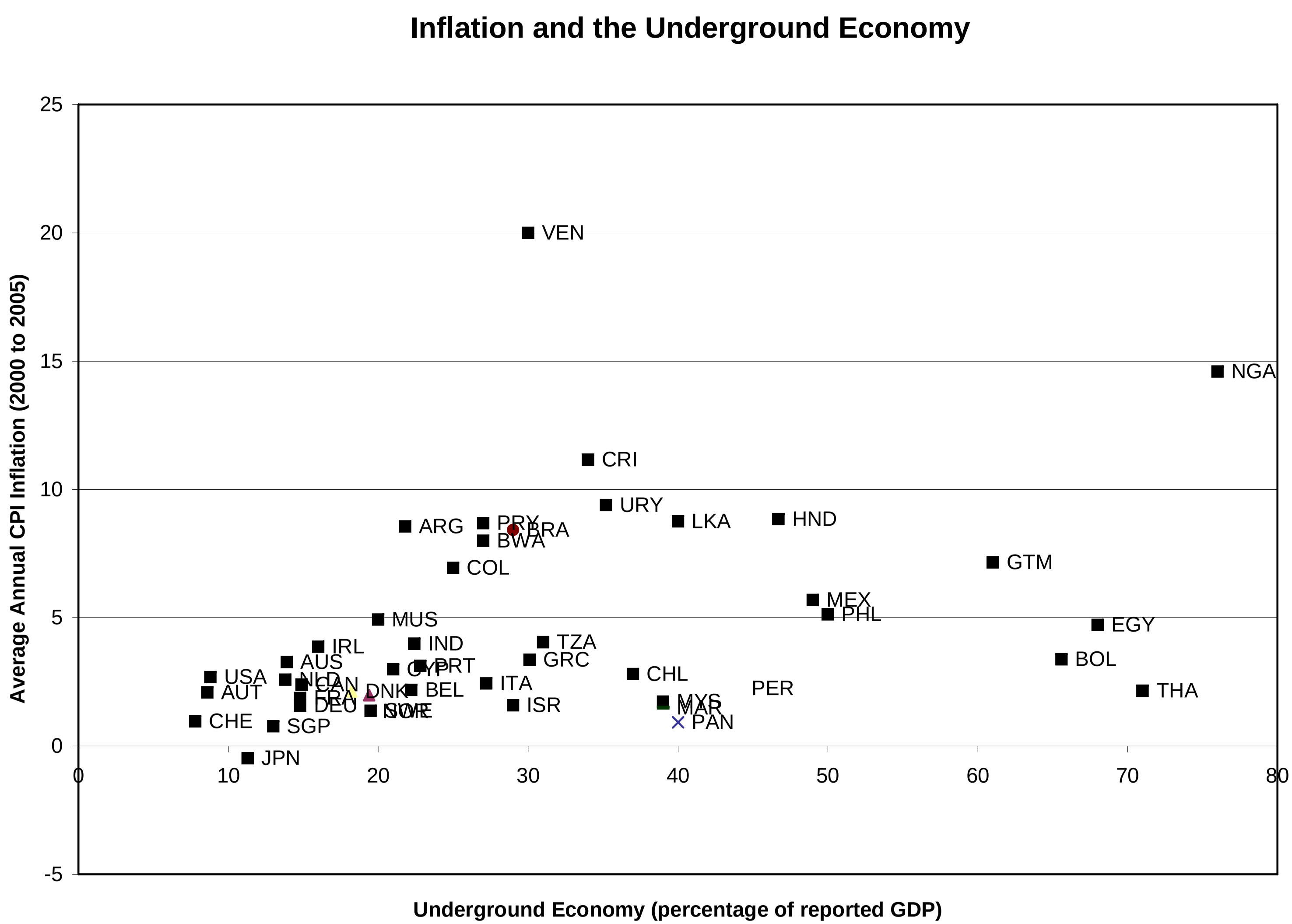
$$a^{US} = \frac{1}{T}\sum_t \frac{F_t^{US}}{U_t^{US}} \times 100\% \ ,$$

   where t is the year. F_t and U_t are both in local currency units. The data period ranged from 1965 to 1995. Again, unreported years were omitted.

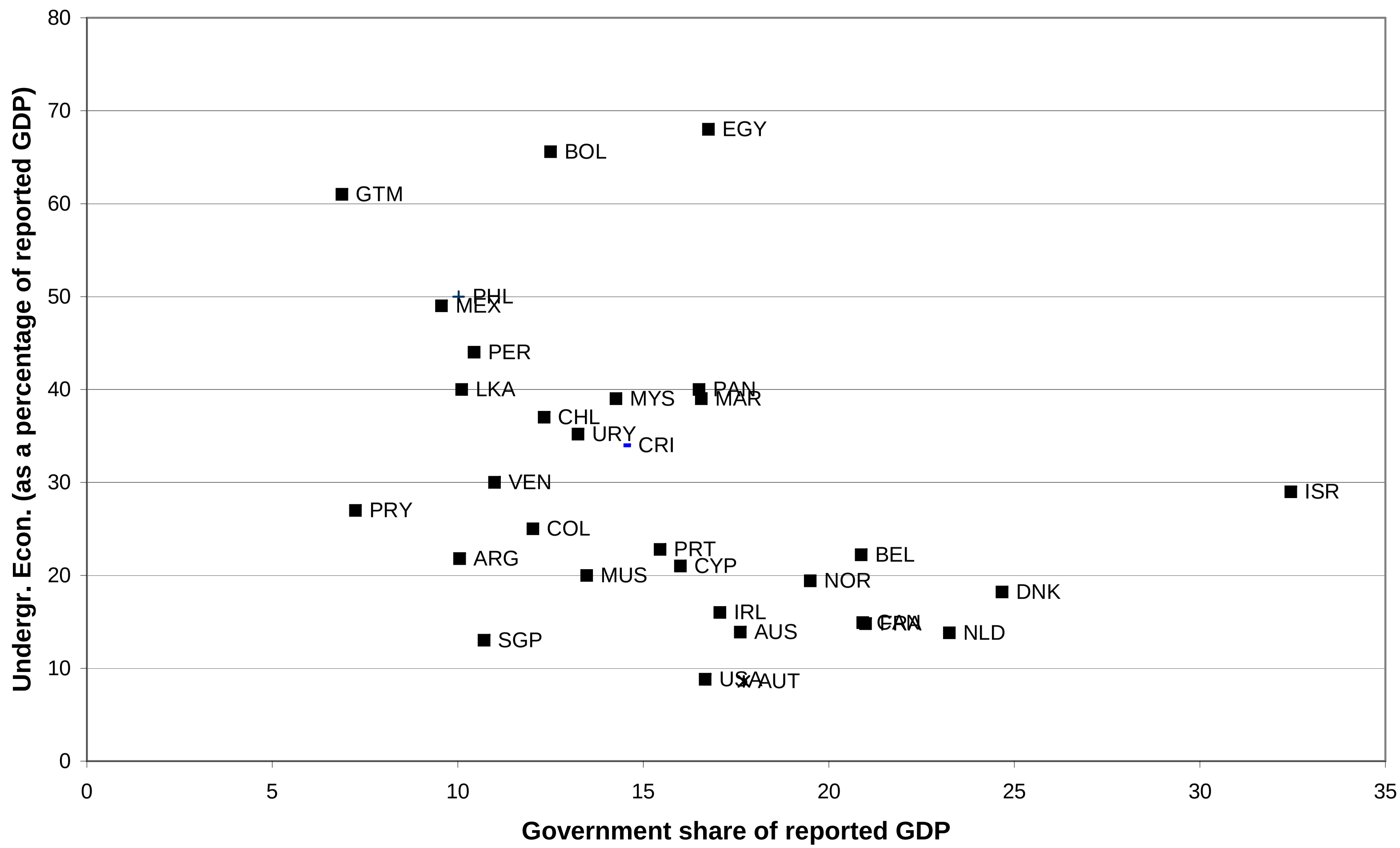
Table 4: OECD Countries

Country	θ	$\frac{b_i}{b}$	$\frac{M_{it}}{M_t}$	$\frac{p_{ft}}{p_{it}}$	R_W	R_S	R	$\widetilde{R}_W$	$\widetilde{R}_S$	$\widetilde{R}$	$\frac{b_g}{b}$	$\frac{M_{gt}}{M_t}$	$\widehat{\pi}$
Australia	0.5931	0.1436	0.1251	1.2382	0.8527	0.1687	0.139	1.0558	0.2089	0.1721	0.1272	0.2693	0.2121
Austria	0.5908	0.0942	0.0795	1.1636	0.8303	0.1045	0.086	0.9662	0.1216	0.1001	0.158	0.2444	0.1964
Belgium	0.5707	0.2185	0.1965	1.1865	0.8751	0.2806	0.222	1.0383	0.3329	0.2634	0.1325	0.3299	0.2481
Canada	0.582	0.1577	0.1338	1.192	0.8249	0.1884	0.149	0.9832	0.2246	0.1776	0.3412	0.1496	0.1301
Denmark	0.5939	0.1939	0.1711	1.2965	0.8578	0.2416	0.182	1.1122	0.3133	0.236	0.1915	0.3743	0.2724
France	0.587	0.1572	0.1362	1.2211	0.845	0.1873	0.148	1.0319	0.2287	0.1807	0.1943	0.2644	0.2091
Ireland	0.5857	0.1607	0.1412	1.2142	0.8583	0.1929	0.16	1.0421	0.2342	0.1943	0.1324	0.2304	0.1872
Mexico	0.5367	0.3504	0.3355	1.0946	0.9359	0.5418	0.49	1.0244	0.593	0.5364	0.0573	0.0842	0.0776
Netherlands	0.5922	0.1516	0.1299	1.243	0.8353	0.1798	0.138	1.0383	0.2235	0.1715	0.2563	0.2642	0.209
Norway	0.588	0.1936	0.1725	1.2624	0.868	0.241	0.194	1.0958	0.3042	0.2449	0.1228	0.3188	0.2417
Portugal	0.479	0.2116	0.1922	0.7835	0.8863	0.2697	0.228	0.6944	0.2113	0.1786	0.0542	0.3928	0.282
USA	0.593	0.0949	0.0784	1.1739	0.8116	0.1056	0.088	0.9527	0.124	0.1033	0.3007	0.1109	0.0999

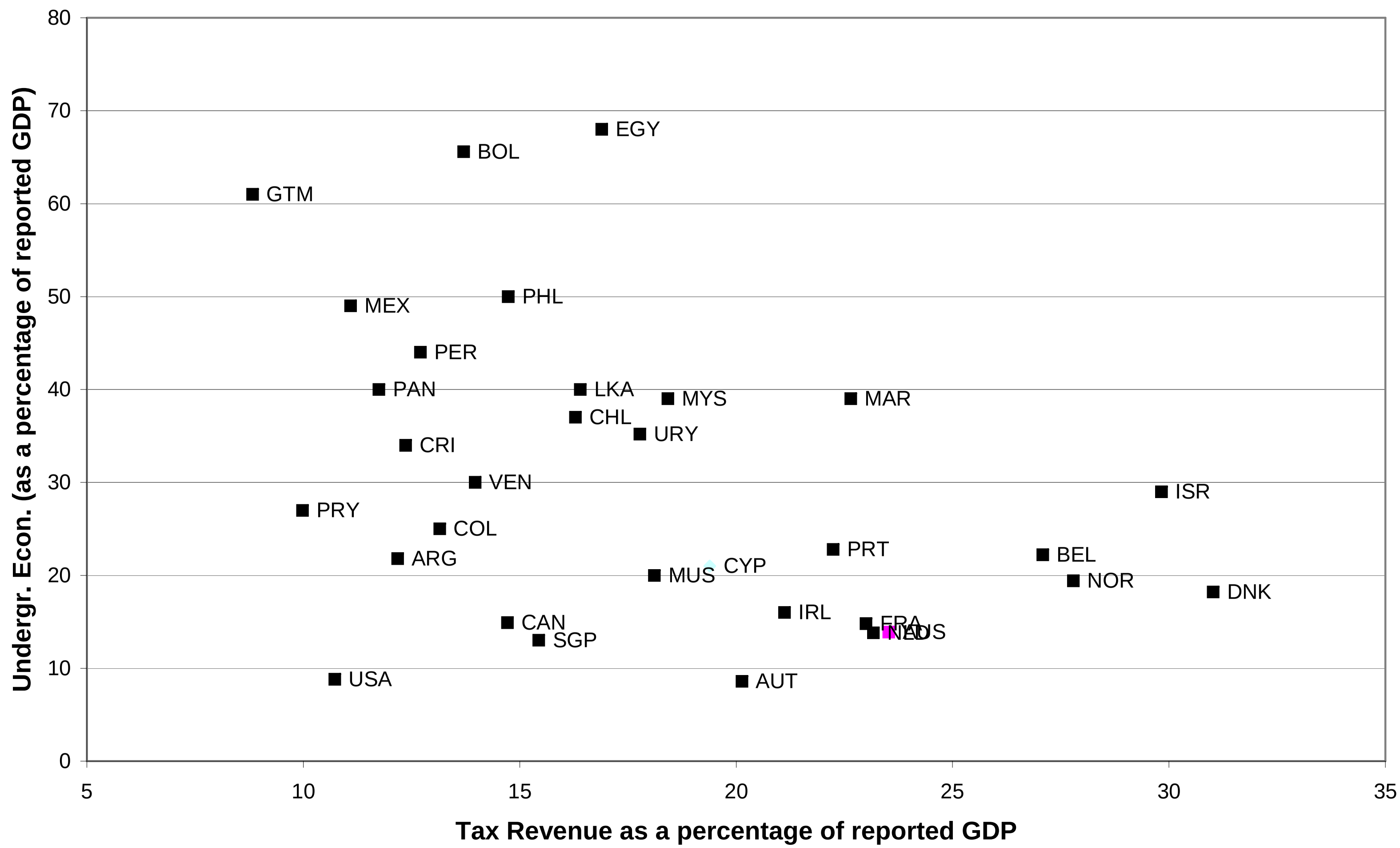
Table 5:						Other Countries							
Country	θ	$\frac{b_i}{b}$	$\frac{M_{it}}{M_t}$	$\frac{p_{ft}}{p_{it}}$	R_W	R_S	R	$\widetilde{R}_W$	$\widetilde{R}_S$	$\widetilde{R}$	$\frac{b_g}{b}$	$\frac{M_{gt}}{M_t}$	$\widehat{\pi}$
Argentina	0.5666	0.1932	0.1746	1.1352	0.8836	0.2424	0.218	1.003	0.2751	0.2475	0.0726	0.1145	0.1028
Bolivia	0.5288	0.428	0.418	1.099	0.9595	0.7497	0.656	1.0545	0.8239	0.7209	0.0734	0.0925	0.0847
Chile	0.5519	0.2961	0.2774	1.141	0.9125	0.422	0.37	1.0412	0.4816	0.4222	0.0705	0.1427	0.1248
Colombia	0.5625	0.2196	0.1997	1.1374	0.8869	0.2842	0.25	1.0087	0.3232	0.2843	0.0955	0.1222	0.1089
Costa Rica	0.5521	0.2818	0.2618	1.1221	0.9037	0.3976	0.34	1.0141	0.4462	0.3815	0.1449	0.1051	0.0951
Cyprus	0.5738	0.1992	0.1782	1.1865	0.8718	0.25	0.21	1.0343	0.2966	0.2492	0.1192	0.2004	0.167
Egypt	0.5308	0.449	0.4393	1.1188	0.9613	0.8169	0.68	1.0755	0.9139	0.7608	0.1097	0.1141	0.1024
Guatemala	0.5269	0.3949	0.3842	1.0709	0.9562	0.6551	0.61	1.0239	0.7015	0.6532	0.034	0.0598	0.0564
Israel	0.577	0.2997	0.2699	1.2808	0.8637	0.4293	0.29	1.1062	0.5499	0.3714	0.3771	0.313	0.2384
Malaysia	0.5526	0.3122	0.2935	1.156	0.915	0.4549	0.39	1.0577	0.5258	0.4508	0.0838	0.1606	0.1384
Mauritius	0.5752	0.1866	0.1671	1.1796	0.8747	0.2311	0.2	1.0318	0.2727	0.2359	0.0888	0.185	0.1561
Morocco	0.558	0.3181	0.2993	1.188	0.9155	0.4674	0.39	1.0876	0.5553	0.4633	0.0916	0.2054	0.1704
Panama	0.5446	0.3236	0.3007	1.1222	0.8991	0.479	0.4	1.009	0.5376	0.4489	0.1978	0.0934	0.0854
Paraguay	0.5572	0.2236	0.2055	1.112	0.898	0.2911	0.27	0.9985	0.3237	0.3002	0.0426	0.0882	0.0811
Peru	0.5419	0.3289	0.312	1.1112	0.9251	0.4913	0.44	1.0279	0.5459	0.4889	0.0619	0.1013	0.092
Philippines	0.5405	0.3563	0.3421	1.1173	0.9395	0.5557	0.5	1.0496	0.6208	0.5586	0.0461	0.1138	0.1021
Singapore	0.5822	0.1269	0.1102	1.1646	0.8525	0.1456	0.13	0.9928	0.1695	0.1514	0.0673	0.1659	0.1423
Sri Lanka	0.5508	0.3063	0.29	1.1361	0.925	0.445	0.4	1.051	0.5055	0.4545	0.0445	0.14	0.1228
Uruguay	0.5562	0.2866	0.2686	1.1499	0.9141	0.4057	0.352	1.0511	0.4665	0.4048	0.0758	0.1604	0.1382
Venezuela	0.5594	0.2481	0.2312	1.1287	0.9114	0.337	0.3	1.0288	0.3804	0.3386	0.0704	0.1251	0.1112


Inflation and the Underground Economy
Average Annual CPI Inflation (2000 to 2005)
Underground Economy (percentage of reported GDP)
25
20
15
10
5
0
-5
0
10
20
30
40
50
60
70
80
VEN
NGA
CRI
URY
LKA
HND
ARG
PRY
BRA
BWA
COL
GTM
MEX
PHL
MUS
EGY
IND
TZA
IRL
AUS
GRC
BOL
USA
NLD
CYP
PRT
CHL
CAN
ITA
PER
THA
AUT
FRA
DNK
BEL
MYS
DEU
SWE
NOR
ISR
MAR
CHE
SGP
PAN
JPN

Government Spending and Underground Economy



Taxation and the Underground Economy



Fines and Forfiets as a percentage of Underground Economy (Audit Rates)