Essays on Money, Search and the Underground Economy

by

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Abstract

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This thesis examines the effect of household level decisions on the emergence of commodity money. It also studies how micro level decisions and macroeconomic variables interact to impact optimal monetary policy, as well as the extent of tax evasion.

In the first chapter, I show that when goods are perfectly divisible, the fundamental and speculative equilibria of Kiyotaki and Wright (1989) can coexist. I compare these two equilibria in terms of welfare. The speculative equilibrium is always a better lubricated economy with a higher quantity of commodity money circulating. When goods with high storage costs start to circulate, they crowd out the circulation rate of goods with lower storage costs, resulting in a version of Gresham’s Law. A direct consequence is that the speculative equilibrium is not Pareto superior.

The second chapter studies the optimal rate of seigniorage in an economy characterized by bilateral trade and a tax-evading underground sector. Optimal inflation depends on which sector, formal or underground, is more congested with buyers. Calibrated to the economy of Peru, the model predicts that the optimal inflation
rate is about 42.6% per annum. This offers a possible motivation for the high rates of inflation observed in that country in the 1980s. A policy that returns this economy to the Friedman rule delivers a welfare loss that is equivalent to a 14% drop in permanent consumption for the representative household. If the formal sector is more congested however, optimal inflation falls to 1.48%, close to the rate observed in 2005.

In a progressive tax system, earnings inequality implies differences in marginal tax rates, hence, differences in the marginal incentive of each agent to evade those taxes. Although individual differences in tax evasion incentives can be eliminated by a simple flat tax regime, little is known about the potential effect of such a policy on the tax-evading sector. In the third chapter, I study quantitatively the effect of a revenue-neutral flat tax reform on the degree of tax evasion in the United States. I find that although a flat tax reform brings fundamental changes in the distribution of wealth and formal sector earnings, the relative size of the tax-evading sector is hardly affected.
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Chapter 1

Commodity Money with Divisible Goods

Economists have traditionally imposed ad hoc restrictions that assign value to intrinsically worthless objects as money. In the early economy however, money took the form of commodities that had value on their own account. Since not all goods circulated as money, what are the properties of those that did? This paper studies an economy in which individual households act in self interest in choosing the types of goods that they use to trade. I find that once goods with bad intrinsic properties begin to circulate, they drive out the circulation rate of those with good intrinsic properties, a version of Gresham’s Law. When multiple equilibria exist, I compare their welfare properties.
1.1 Introduction

The aim of this paper is to identify commodity money when goods are perfectly divisible. That is, what economic conditions can explain how certain goods became exalted to the status of media of exchange in the early economy?¹

An important progress in the literature is the contribution by Kiyotaki and Wright (1989) (KW). Their paper was the first to attempt an answer to this question using rational agents in a dynamic economy with random pair-wise matching, and without fiat money. In deciding on the goods to accept and those to reject in trade, agents compare storage costs and liquidity (marketability) properties of goods.² The authors show that goods with the lowest storage costs always act as money. If a steady state shows trade patterns in which only one commodity emerges as money, then that commodity must also be the cheapest to store. They term this first steady state the fundamental equilibrium. However, high marketability can entice agents to speculate and temporarily accept the high storage cost goods, though such goods are not ultimately desired for consumption by those agents. The rationale for accepting these goods is that they offer better trade opportunities in subsequent periods, during which the agent hopes to trade them for his desired consumption units. Specifically, if exactly two types of goods circulate as money, then the second is the commodity with the highest storage costs (assuming a world of only three goods). The authors term this second steady state as the speculative equilibrium. Furthermore, they find that no other trade patterns can be supported as steady states. Finally, the two equilibria identified cannot coexist. That is, there is no set

¹The focus of this paper is on the role of money as a medium of exchange. Other mechanisms that assign objects value as money are built on the overlapping generations and spatial separation environments, following Wallace (1980) and Townsend (1980), as well as the cash-in-advance environment, as in Lucas (1980). Some of these mechanisms emphasize other roles of money.

²By marketability, the authors capture the probability of finding other agents willing to take this commodity as acceptable means of payment during trade.
of economic fundamentals that can simultaneously support both equilibria.

One of the restrictions imposed in KW is that goods are assumed to be indivisible. This has caused many to question whether some of their results are direct consequences of this assumption. In this paper, I allow for fully divisible goods and test whether the above two equilibria can be identified. I also ask whether the above pair are the only equilibria, and whether they can coexist.

Once I allow for divisibility, the analysis extends in two major dimensions, the first being bargaining. In KW, trade is reduced to one-for-one exchange due to indivisibility. With divisible goods however, agents can bargain over fractions of commodities. Hence, I apply an appropriate bargaining solution. Secondly, tractability becomes a challenge. Agents of a given type can hold a wide distribution of different commodities, with each agent’s stocks depending on his unique history of matches and trades. I employ the concept of households, each with a continuum of members, similar to Shi (1997, 1999). By the law of large numbers, the state variable of a household type becomes degenerate in steady state, once the optimal policy decisions of that household type are determined. Thus, the model proposed in this paper is tractable in equilibrium and one can derive closed-form solutions for state variables.\(^3\) Off equilibrium, households can hold any mix of goods they wish. That is, the off-equilibrium distribution of state variables is non-tractable. Nevertheless, confirming an equilibrium requires the evaluation and comparison of all off-equilibrium trade strategies, with their associated non-tractable state variables. Hence, the equilibria are confirmed numerically. This process involves first

\(^3\)The roles of the large household assumption are (i) for tractability and (ii) to permit a closed form solution for the inventory distribution. See Lagos and Wright (2005) for a related tractable framework in which agents rebalance their portfolios in centralized markets after rounds of decentralized trade. Other generations of tractable monetary search models assume divisible goods, indivisible fiat money, unit storage capacity and perishability (services, in place of commodities). This other approach, as employed in Trejos and Wright (1995) and Shi (1995), eliminates any possible role for commodity money.
assuming a steady state, say the fundamental equilibrium, and evaluating the associated inventory distribution. Next, the numerical simulation seeks to confirm that the sum of discounted instant returns to each household from any deviation strategy, permanent or transitory, be less than that from continuing with the strategy consistent with the steady state.\textsuperscript{4}

I confirm both the fundamental and speculative equilibria, leading to the conclusion that the existence of these two steady states in KW is not reliant on the assumption of indivisibility. I find that with divisible goods, the two steady states can coexist. To explain how coexistence arises, I follow two main steps. The first step is to note that there is a pivotal household type whose accept or reject decision for units of commodity money tilts the balance between the two equilibria. This household type makes two additional forms of trade in the speculative equilibrium, which they do not in the fundamental equilibrium. One of these trades is acquiring units of commodity money (the commodity with the highest storage costs) and the second is spending units of commodity money. For reference purposes, call these the pivotal trades.

In the second step, assume that we have a particular equilibrium, for instance, the fundamental equilibrium. Given the trade strategies that are consistent with the equilibrium, there emerges a distribution of inventories. Subsequently, households continue to expect the same distribution because deviation strategies by an individual household leave the aggregate inventory distribution unchanged, owing to the assumption that each household is infinitesimal. That is, the same expectations engender the same trade strategies as those consistent with the equilibrium.

\textsuperscript{4}I do not consider collective deviations. That is, deviation strategies of individual households are examined while taking the evaluated commodity money distribution as given (as consistent with the steady state under review). This greatly reduces the computational burden during value function iteration.
because, for the current inventory distribution (for the current matching probabilities), the discounted value of returns to each household from deviation strategies is lower than the payoff to the original strategy.\footnote{In the model, the inventory distribution is directly mirrored by the matching probabilities, similar to KW. I use both phrases interchangeably.}

In other words, given the inventory distribution (or matching probabilities) in the fundamental equilibrium, the pivotal household makes no pivotal trades as described above. This way, the matching probabilities are self-fulfilling and engender the continuation of the same trade strategies as those consistent with the fundamental equilibrium. Similarly, given the inventory distribution in the speculative equilibrium, the pivotal household always engages in the pivotal trades, whenever such trade opportunities arise. Again, the matching probabilities in this equilibrium are self-fulfilling and engender the continuation of trade strategies that are consistent with the speculative equilibrium. In short, since expectations of the matching probabilities are self-fulfilling, the same parameters can support both steady states.

An obvious question that arises is that, why does such an outcome not emerge in KW? To aid the explanation, I first shed light on how coexistence is attained computationally in the model, and then show why the same mechanism is not possible in KW. Starting off with a set of parameters that satisfy the fundamental equilibrium, I evaluate a hypothetical distribution of inventories assuming agents played the speculative strategy instead, given the same parameters. I undertake value function iteration to check whether the pivotal household finds it optimal to engage in pivotal trades when they observe this hypothetical distribution. If not, I conclude that pivotal households are not receiving sufficient trade surplus during bargaining in such pivotal trades in order to sustain a speculative equilibrium.
Next, I change parameters such as the marginal utility of trade partners so as to shift more of the trade surplus to pivotal households during bargaining in pivotal trades. I continue this until the set of parameters can confirm both equilibria.\textsuperscript{6} This process is not possible in KW because trade is one-for-one exchange. Specifically, it is not possible to shift trade surpluses between types to achieve coexistence.

Another significant result is that when goods with high storage costs start to circulate, they drive out the circulation rate of goods with low storage costs. This conclusion depends on the storage capacity assumption. By this assumption, agents have limited assets and the decision to accept certain objects as money constrain the acceptance of others at the same time. This assumption resonates to the modern economy because of the fundamental problem of scarcity and the fact that households possess limited resources.

This paper adds to the literature on commodity money, with related papers including Star (1972), Ostroy (1973), Jones (1976), Iwai (1988), Aiyagari and Wallace (1991), Marimon, McGrattan and Sargent (1990), Burdett, Trejos and Wright (2001) and Molico (1999). The next section presents the model studied. I impose sufficient structure to permit stationary equilibria, characterize these equilibria and present simulated results in section 1.3. Section 1.4 addresses issues on coexistence, welfare and velocity. I conclude in section 1.5.

1.2 Economic Environment

Time is discrete. Next period variables are denoted with primes. The economy is inhabited by a large number of decision-making households who are divided equally into three types, $h = 1, 2, 3$. Similar to Shi (1997, 1999) each household is

\textsuperscript{6}Changing parameters can have a second order feedback effect on the inventory distributions.
assumed to have a unit measure of member agents and each agent has a unit storage capacity. There are three types of perfectly divisible goods, 1, 2, 3. Households of type $h$ consume commodity $h$ and produce commodity $h + 1$, modulo 3. Household $h$ neither produces nor consumes good $h + 2$ and has no reason to hold good $h + 2$ except for use as intermediate units for future trade, thus, as commodity money. Good $h + 2$ therefore emerges as commodity money only if $p_{h,h+2} > 0$, where $p_{h,k}$ is the proportion of storage space used to store good $k$ between periods at household $h$. Alternatively, $p_{h,k}$ represents the proportion of household $h$’s agents holding good $k$. The first subscript ($h$) refers to the household type, while the second ($k$) refers to the commodity type held. I assume that households do not delay consumption, hence, $p_{h,h} = 0$. That is, a household does not carry its consumption good between periods. Production in each period ensures that each household always holds positive stocks of its production commodity: $p_{h,h+1} \in (0, 1]$, $\forall h$. Unit storage capacity means that $p_{h,h+1} + p_{h,h+2} = 1$ at every date.

Let $m_h \in [0, 1]$ be household $h$’s stock of commodity money and $1 - m_h$ the stock of produced goods. The state variable $m_h$, thus, completely describes the household’s portfolio at a given time.\textsuperscript{7} The cost of storing such a portfolio between periods is denoted $c_h (m_h)$. I choose the functional form:

$$c_h (m_h) = \varpi_{h,h+1} [2 - m_h] + \varpi_{h,h+2} [I + m_h]$$

(1.1)

where $I$ is an indicator function that takes the value one if $p_{h,h+2} > 0$ and zero otherwise. To be consistent with the ranking of storage costs in KW, I assume that $\varpi_{h,1} < \varpi_{h,2} < \varpi_{h,3}, h = 1, 2, 3$. Household members carry these goods individually.

\textsuperscript{7}With divisible goods, we have $m_h \in [0, 1)$ and not $m_h \in \{0, 1\}$ as in KW. Notice that $m_h = 0$ does not mean that the household has no stocks, but instead, has $1 - m_h = 1$ in produced goods. An implicit assumption is that total stocks are kept to unit capacity at the end of each period. This assumption becomes redundant if marginal utility is sufficiently large.
along to the market, yet storage costs are borne collectively at the household level.

When entering the market, an agent cannot carry two types of goods at the same time. Therefore, $p_{h,h+2}$ also represents the number of agents to hold the $m_h$ units of commodity money, each of whom enter the market with $\frac{m_h}{p_{h,h+2}} = 1$ unit. Thus, each member holding the produced units enters the market with the quantity $\frac{1-m_h}{p_{h,h+1}} = 1$.

A household has one choice variable, being an accept or reject decision for units of commodity money in the current trade period. This decision is communicated to member agents holding produced units on whether or not to accept commodity money in exchange for those goods. This decision is denoted $D_h(m_h)$, or simply $D_h$ for household $h$, where the dependence on the aggregate portfolio distribution has been suppressed. $D_h$ takes the value 1 if agents are instructed to accept commodity money and zero otherwise.

**Figure 1.1**

Timing of Events

\[
\begin{array}{cccc}
\text{Choose} & \text{Markets Open} & \text{Markets Close} & \text{Storage Costs} \\
\rightarrow & \rightarrow & \rightarrow & \\
\text{D}_h(m_h) & \frac{1-m_h}{p_{h,h+1}} & \text{Poolig} & c(m_h') \\
\frac{m_h}{p_{h,h+2}} & \text{Consumption} & \\
\text{Pair-wise matching} & \text{Production} & \\
\text{Trade qyts, Trade} & \\
\end{array}
\]
Next, markets open. Once in the market, types are fully observable. Agents match randomly and individually with other agents. Matched agents may trade depending on the household’s accept or reject instructions.

Agents make decisions, but only in bargaining during successful matches. A successful match is one in which both agents hold goods that the other wants. A successfully matched agent decides on \( s \), being the quantity to sell and \( b \), the quantity to request in return. The variables \( s \) and \( b \) are subject to the stock availability constraints and the bargaining paradigm. Household \( h \) agents do not buy back type \( h + 1 \) goods from the market, since these are their production specialty.

### 1.2.1 Matching

Pair-wise matching and bargaining yield trade quantities which eventually appear in the household’s optimization. Hence, I outline the matching and bargaining environment as well as the terms of trade. An agent from household \( h \) matches with other agents of types 1, 2, 3, each at the probability \( \frac{1}{3} \). Agents may engage in two main types of trades, *direct* and *indirect*. A direct trade occurs whenever an agent spends units of his household produced goods in direct exchange for consumption units. An indirect trade on the other hand occurs when an agent spends or acquires intermediate units, that is, commodity money. This second type of transactions are indirect in the sense that (i) when commodity money is spent to buy consumption units, such a trade was preceded by the acquisition of commodity money in previous periods and (ii) when commodity money is acquired in a transaction, such a trade is succeeded by an ultimate trade for consumption units. The subsequent discussion sheds further light on the nature of direct and indirect trades.

- When matched with fellow type \( h \) agents, there cannot be beneficial trade
since preferences and household instructions are alike.

- Now, suppose an agent from household $h$ goes to the market with produced goods $h + 1$. Denote this agent as $p_{h,h+1}$.

  - Meetings between $p_{h,h+1}$ and $p_{h+1,h+2}$ can result in trade only if $D_h = 1$. The probability of such a match occurring is $\frac{1}{3}p_{h,h+1}p_{h,h+2}$. From the viewpoint of household $h$, this trade is an indirect trade to acquire money (since household $h$ neither produces nor consumes good $h + 2$).

  - If $p_{h,h+1}$ meets $p_{h+1,h}$, there can be trade leading to consumption in both households. From the viewpoint of household $h$, this is a direct trade with a consumer. Firstly, this trade is direct as described earlier. Secondly, the trade is with a consumer since agent $p_{h+1,h}$ is acquiring good $h + 1$, his consumption units.

  - If $p_{h,h+1}$ meets $p_{h+2,h}$, there can only be trade if $D_{h+2} = 1$. This leads to consumption only at household $h$. From the viewpoint of household $h$, this is a direct trade with a money seeker. Again, this trade is direct as described earlier. Secondly, the trade is with agent $p_{h+2,h}$, who is making money (good $h + 1$) for his household.

  - And if $p_{h,h+1}$ meets $p_{h+2,h+1}$, there will not be trade since both agents hold the same commodity type ($h + 1$).

- Next, suppose an agent from household $h$ enters the market with money units $h + 2$. Denote this agent as $p_{h,h+2}$.

  - If matched with $p_{h+1,h+2}$, both agents hold the same commodity type ($h + 2$) and there is no trade.
– If $p_{h,h+2}$ meets $p_{h+1,h}$, there will not be trade because agent $h + 1$ does not buy back production units $h + 2$ from the market.

– If $p_{h,h+2}$ meets $p_{h+2,h}$, there will be trade and consumption in both households. From the viewpoint of household $h$, this trade is an *indirect trade to spend money* (since household $h$ neither produces nor consumes good $h + 2$).

– Finally, if $p_{h,h+2}$ meets $p_{h+2,h+1}$, there will not be trade since $h$ does not buy back production units $h + 1$ from the market.

To summarize, there are four possibilities for trade. They are:

1. *Indirect trade to acquire money*
2. *Direct trade with a consumer*
3. *Direct trade with a money seeker* and
4. *Indirect trade to spend money*

The probabilities of these other matches follow similarly (just as for the first case - meeting between $p_{h,h+1}$ and $p_{h+1,h+2}$). There are four possibilities for trade, three of which can lead to consumption at household $h$. Household $h$ acquires commodity money during meetings between $p_{h,h+1}$ and $p_{h+1,h+2}$, and spends it during matches between $p_{h,h+2}$ and $p_{h+2,h}$. 
1.2.2 Bargaining

Next, I consider bargaining and terms of trade. Bargaining involves sharing the surplus from trade. There are established bargaining methods in the literature where buyers and sellers are clearly differentiable - a buyer being a bearer of money and a seller being a bearer of goods for sale. In the case of barter however, a buyer of goods simultaneously acts as a seller of some other goods. One way to get around the problem is to assign bargaining power $\theta = \frac{1}{2}$ to each trader. However, the extent to which stock availability constraints bind is not the same across all successful matches. Since symmetry cannot be imposed across household types, the problem expands to solving several bargaining problems, each subject to its own stock availability constraints depending on the type of agents involved in the match.

To simplify therefore, I assume the bargaining power $\theta \in \{0, 1\}$, taking each value with probability $\frac{1}{2}$. In other words, one of the two matched agents is chosen at random and assigned all the bargaining power. That agent makes a take-it-or-leave-it offer. In making or accepting offers, agents are assumed to consider only
the immediate implication of the exchange. These immediate effects are captured in stages 2 and 3 of the timing sequence. Agents are also assumed to be naı̈ve about the impact of their trade decisions on storage costs and the continuation value.

Let $\delta y$ be the disutility from producing $y$ units of output, $\delta \geq 0$. Also, let $u_h(c) = u'_h c$ be the utility that household $h$ gains from consuming $c > 0$ units of good $h$ and $u_h(0) = -\infty$, with $u'_h > \delta$. $u'_h$ in this case is the constant, time-invariant marginal utility when $c > 0$.

1. **Direct trade with a consumer and indirect trade to spend money**: For matches between $p_{h,k}$ and $p_{k,h}$, $k \neq h$, suppose agent $p_{h,k}$ offers to sell the quantity $s$ and in the process, buy the quantity $b$. Household $h$ receives net utility $u'_h b - \delta s$, while household $k$ gets $u'_k s - \delta b$. On occasions when agent $p_{h,k}$ makes the take-it-or-leave-it offer, the optimal quantities are such that:

$$u'_k s - \delta b = 0 \quad \text{and} \quad b = 1.$$  

That is, since $u'_h > \delta$, agent $p_{h,k}$ exhausts the stocks of agent $p_{k,h}$, while leaving the latter with zero net surplus. The quantities exchanged are thus $s = \frac{\delta}{u'_k}$ and $b = 1$. The other half of the time when the other agent, $p_{k,h}$, makes the offer, she chooses $S = \frac{\delta}{u'_h}$ and $B = 1$. For household $h$, the total net utility from both offers is $\frac{1}{2} \left[ u'_h - \delta \frac{\delta}{u'_k} \right] + \frac{1}{2} \left[ u'_k \frac{\delta}{u'_h} - \delta \right]$, or simply $\frac{1}{2} \left[ u'_h - \delta^2 \frac{1}{u'_k} \right]$. This analysis pertains to when $p_{h,h+1}$ matches with $p_{k,h}$, $k = h + 1, h + 2$, which are two of the four possible trade matches.

2. **Direct trade with a money seeker**: Consider meetings between $p_{h,h+1}$ and $p_{h+2,h}$, with $D_{h+2} = 1$. If the former makes the offer $(s, b)$, the latter household
receives zero trade surplus. Hence, \( b = s = 1 \), since again, \( p_{h,h+1} \) exhausts stocks of the trade partner. Household \( h \) therefore receives \( \frac{1}{2} [u'_h - \delta] \) in trade surplus. When agent \( h + 2 \) makes the offers, then again \( S = \frac{\delta}{u_h} \) and \( B = 1 \) and household \( h \) gets zero surplus.

3. **Indirect trade to acquire money**: Finally, consider matches between \( p_{h,h+1} \) and \( p_{h+1,h+2} \), with \( D_h = 1 \). If the former makes the offer, then I set \( u'_{h+1} s - \delta b = 0 \) and \( b = 1 \). Thus, \( s = \frac{\delta}{u_{h+1}} \). However, if the latter agent makes the offer, she sets \( -\delta B + \delta S = 0 \), hence, \( B = S = 1 \). Household \( h \) gets total net utility \( \frac{1}{2} \delta \left[ 1 - \frac{\delta}{u_{h+1}} \right] \) from both cases. We employ the above bargaining solution because, as explained above, using others, such as the generalized Nash bargaining solution impose considerable challenges.\(^8\)

The bargaining results above enable us to ascertain the net utility gained from trade transactions in the market as follows:

\[
U_h = p_{h,h+1}p_{h+1,h+2} \frac{D_h}{6} \delta \left[ 1 - \frac{\delta}{u'_{h+1}} \right] + p_{h,h+1}p_{h+1,h+2} \frac{1}{6} \left[ u'_h - \frac{\delta^2}{u_{h+1}} \right] + p_{h,h+1}p_{h+2,h} \frac{D_{h+2}}{6} \left[ u'_h - \delta \right] + p_{h,h+2}p_{h+2,h} \frac{1}{6} \left[ u'_h - \frac{\delta^2}{u'_{h+2}} \right], \tag{1.2}
\]

\( h = 1, 2, 3 \). The terms in this equation follow directly from the preceding discussion. The first, second, third and fourth terms on the right hand side are trade surpluses from the bargaining problems in “indirect trades to acquire money”, “direct trades with consumers”, “direct trades with money seekers” and “indirect trades to spend

\(^8\)For example, applying to trade match between \( p_{12} \) and \( p_{2,1} \), we have: \( \max_{s,b} [u_1 (b) - \delta s]^{\theta} \times [u_2 (s) - \delta b]^{1-\theta} + \lambda_1 \left[ \frac{1-m_1}{p_{1,2}} - s \right] + \lambda_2 \left[ \frac{m_2}{p_{2,1}} - b \right] \). This is only one trade match, yet with two Lagrange multipliers. Since we cannot impose symmetry among household types, we need to keep track of all multipliers separately and once we consider all possible trade encounters, the problem can become rather large.
money” respectively.

The commodity money accumulation equation is given as:

\[
m'_h = m_h + p_{h,h+1}p_{h+1,h+2} \frac{D_h}{6} \left\{ 2 - p_{h,h+2}p_{h+2,h} \frac{1}{6} \left\{ \frac{\delta}{u'_{h+2}} + 1 \right\} \right\}, \ h = 1, 2, 3 \quad (1.3)
\]

Household \( h \) acquires commodity money units in “indirect trades to acquire money”, when \( p_{h,h+1} \) meets \( p_{h+1,h+2} \). For both values of \( \theta, \theta \in \{0, 1\} \), agent \( p_{h,h+1} \) acquires a unit quantity. Commodity money is spent in “indirect trades to spend money”, when \( p_{h,h+2} \) meets \( p_{h+2,h} \). Half of the time, agent \( p_{h,h+2} \) spends \( \frac{\delta}{u_j} \) of a unit and the other half, spends an entire unit. Choosing \( D_h = 0 \) in the current period does not preclude trading away commodity money, since stocks of previously acquired units may exist. If no stocks exist, \( p_{h,h+2} = 0 \), since the household cannot allocate members to hold units that do not exist. This eliminates the last terms in equations (1.2) and (1.3).

### 1.2.3 Household’s Problem

Agents are altruistic towards fellow household members. Let \( v(m_h, \mu(m)) \) be the value function of a representative type \( h \) household, where \( \mu(m) \) is the aggregate distribution of money across all types. Household \( h \) chooses \( D_h (m_h, \mu(m)) \in \{0, 1\} \) so as to solve:

\[
v(m_h, \mu(m)) = \max_{D_h \in \{0, 1\}} R_h + \beta Ev(m'_h, \mu(m')) \quad (1.4)
\]

subject to (1.2) and (1.3), where \( \beta \) is the time discount factor and \( R_h \) is the return function given as:

\[
R_h = U_h - c_h(m'_h), \ h = 1, 2, 3. \quad (1.5)
\]
1.3 Equilibrium and Simulation Results

Suppose the household’s problem yields the policy function $D_h(m_h, \mu(m))$, $h = 1, 2, 3$. An $e$ equilibrium - $e$ being “fundamental” or “speculative” - is defined as the value functions $v(m^e_h, \mu(m^e))$ and a stationary distribution of commodity money $\mu(m^e)$ such that, given the random matching function, all household types solve their respective maximization problems and yield the time-consistent policy functions $D^e_h(m^e_h, \mu(m^e))$, $h = 1, 2, 3$.

To find an equilibrium, our approach is to conjecture the equilibrium, characterize it, and then show that it is in fact an equilibrium. Given a consistent choice, $D_h$, by all households $h = 1, 2, 3$, the state variables settle to a constant distribution. The commodity money stocks are given by a solution to a system of simultaneous equations. In a conjectured equilibrium $e$ therefore, a household that accepts commodity money will hold some constant quantity $m^e_h \in (0, 1)$, while a household that rejects will hold $m^e_h = 0$.

1.3.1 The Fundamental and Speculative Equilibria

If good 1 emerges as the sole medium of exchange, this is characterized by $m_2 = p_{2,1} \in (0, 1)$, $m_1 = p_{1,3} = m_3 = p_{3,2} = 0$ and $p_{1,2} = p_{3,1} = 1$. Finally, $D_1 = D_3 = 0$, while $D_2 = 1$. Imposing stationarity on (1.3), we have:

$$
\mu(m^F) = \{m_1^F, m_2^F, m_3^F\} = \left\{0, \frac{2u_1'}{3u_1' + \delta}, 0\right\} \equiv \{p_{1,3}^F, p_{2,1}^F, p_{3,2}^F\}
$$

Equation (1.6) is the distribution of commodity money stocks among households in the fundamental equilibrium. This contrasts with $\{0, 0.5, 0\}$ in KW, dependent only on the matching rate. The difference is accounted for by bargaining, which
reduces to one-for-one exchange in environments with indivisible goods. Commodity money holding by a type 2 household is increasing in the marginal utility of household 1 types and decreasing in the production disutility of the same household types. In a match between \( p_{2,1} \) and \( p_{1,2} \) in which the former makes the offer, the higher the value \( u'_1 \), the higher the surplus agent \( p_{2,1} \) derives from the match. Agent \( p_{2,1} \) exploits this surplus by giving fewer units of commodity money to agent \( p_{1,2} \). Thus, agent \( p_{2,1} \) holds on to more units, raising \( m_2 \) at the household level. The reverse is the case for \( \delta \).

From (1.5), the instantaneous return for each household type is:

\[
R^F_1 = p^F_{2,1} \frac{1}{6} \left[ u'_1 - \frac{\delta^2}{u'_2} \right] - 2\tau_{1,2} \tag{1.7}
\]

\[
R^F_2 = p^F_{2,1} \frac{1}{6} \left[ u'_2 - \frac{\delta^2}{u'_1} \right] + p^F_{2,3} \frac{1}{6} \delta \left[ 1 - \frac{\delta}{u'_3} \right] - \bar{c}_{2,3} \left[ 2 - m^F_2 \right] - \bar{c}_{2,1} \left[ 1 + m^F_2 \right] \tag{1.8}
\]

\[
R^F_3 = p^F_{2,3} \frac{1}{6} \left[ u'_3 - \delta \right] - 2\tau_{3,1} \tag{1.9}
\]

where \( p^F_{2,3}, p^F_{2,1} \) and \( m^F_2 \) are as in (1.6).

For goods 1 and 3 to emerge as media of exchange in the Speculative equilibrium, we require \( D_1 = D_2 = 1 \), while \( D_3 = 0 \). Again, imposing stationarity on (1.3) for \( h = 1, 2 \), we have a system of two equations for the values \( m_1 (= p_{13}) \) and \( m_2 (= p_{21}) \). This I can solve simultaneously or iteratively to deliver \( \mu (m^S) = \{ m^S_1, m^S_2, m^S_3 \} \), where \( m^S_3 = 0 \). The resulting return functions, \( R^S_h, h = 1, 2, 3 \), can also be derived likewise.
1.3.2 Simulation and Results

I employ value function iteration to confirm these equilibria. To verify the fundamental equilibrium for instance, I start by quantifying the related equilibrium distribution $\mu(m^F)$, as in (1.6). Next, I form a grid for each type as follows:

Type 1: $\{m_1^F, \ldots, m_{1,Dev}^F\}$

Type 2: $\{m_2^F, \ldots, m_2^F\}$

Type 3: $\{m_3^F, \ldots, m_3^F,Dev\}$

where $m_{h,Dev}^F$ is the steady state money stock of a type $h$ household that consistently deviates from the fundamental strategy, given that all other households (including other type $h$ households) continually play fundamental. For example, if household 2 consistently deviates to reject good 1, stocks of $m_2$ eventually run out. Thus, we know from inspection that $m_{2,Dev}^F = 0$. Call these grids $m_h$. Next, I use (1.3) and $p_{hj}$ values from (1.6) to get a new grid, $m'_h$. The new grid is approximated back onto type $h$'s original grid, $m_h$, for both choices: $D_h = \{0, 1\}$. This gives the next period’s state variable, depending on current state and decision. Now, one can iterate the value function for the representative type $h$ household until stationary. The fundamental equilibrium is confirmed if $D_1(0) = 0$, $D_2(m_2^F) = 1$, and $D_3(0) = 0$. A similar approach is used for the Speculative equilibrium. The resulting distribution of money in each equilibrium is:

---

9 In the value function iteration for the representative type $h$ household, we hold all other household’s (including other type $h$ household’s) state and decision variables constant and consistent with the fundamental equilibrium. Thus, we do not consider collective deviations.

10 See the appendix for the choice of parameters.
Similar to KW, the speculative equilibrium arises here because the marketability benefits to household 1 from accepting good 3 outweighs the storage cost disadvantage, given the distribution in each equilibrium. This outcome indicates that the results in KW can hold more generally and does not depend on their restrictive environment with indivisible goods.

### 1.4 Velocity, Coexistence and Welfare

#### 1.4.1 Velocity

Let the velocity of good $h$ in equilibrium, $e$, be defined as $V_h^e = \frac{q_h}{m_{h,e}^2}$, modulo 3. $q_h$ refers to the quantity of commodity $h$ that exchanges hands each period. Using (1.3), the velocity of money becomes:

$$V_h^e = \left[ \frac{p_{2,3}^h p_{3,1}^h}{3} + \frac{p_{2,1}^h p_{1,2}^h}{6} \left( 1 + \frac{\delta_1}{u_1^e} \right) \right] \frac{1}{2m_2^e}, \ e = F, S \quad (1.10)$$

$$V_3^S = \left[ \frac{p_{1,2}^S p_{2,3}^S}{3} + \frac{p_{1,3}^S p_{3,1}^S}{6} \left( \frac{\delta_3}{u_3^S} + 1 \right) \right] \frac{1}{2m_3^S}. \quad (1.11)$$

In the above equations, commodity money 1 circulates in both equilibria. Hence, $V_1^e$ is defined for both $e = F$ and $e = S$. Using the money distribution above in each equilibrium, we have:

<table>
<thead>
<tr>
<th>Table 1.1</th>
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<tbody>
<tr>
<td>Money Distribution</td>
</tr>
<tr>
<td>-----------------------------------</td>
</tr>
<tr>
<td>$h$</td>
</tr>
<tr>
<td>-----</td>
</tr>
<tr>
<td>1</td>
</tr>
<tr>
<td>2</td>
</tr>
<tr>
<td>3</td>
</tr>
</tbody>
</table>
TABLE 1.2

<table>
<thead>
<tr>
<th></th>
<th>$V_{1}^{e}$</th>
<th>$V_{3}^{e}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$e = F$</td>
<td>0.1683</td>
<td>n.a.</td>
</tr>
<tr>
<td>$e = S$</td>
<td>0.1123</td>
<td>0.1683</td>
</tr>
</tbody>
</table>

This implies that in the fundamental equilibrium, a unit of commodity money (good 1) takes on average 5.94 periods ($= \frac{1}{0.1683}$) to exchange hands. When commodity 3 begins to act as money however, the turnover period for good 1 jumps to 8.91. When bad money with higher storage costs begin to circulate, it crowds out the circulation rate of good money, leading to a version of the Gresham’s Law. In fact, the sharp increase in the stocks of good 1 held as money in the speculative steady state (see Table 1.1) is a direct result of the decline in its circulation rate. In the fundamental equilibrium, commodity money 1 bearers ($p_{2,1}$) have more agents to trade with (specifically, agents of measure $p_{1,2}^{F} = 1$). In the speculative equilibrium however, the number of these trade partners fall to $p_{1,2}^{S} < 1$. Thus, household 2 disposes off money far less often, resulting in a rise in $m_{2}$ as well as a fall in the circulation rate. This result is different from Renero (1999). In his constructed equilibrium with coexistence, Renero showed that when agents employ mixed strategies and are indifferent between holding their respective non-consumption goods, the circulation rate of good 1 increases. With mixed strategies, agents adopt an always-trade policy, leading to higher frequency of trade and higher welfare.

11 Although the results on Gresham’s Law in the model relies on inventory restrictions imposed on agents, a similar result is possible if the marginal cost of storage is sufficiently increasing.
1.4.2 Coexistence

The decision of household 1, $D_1$, is pivotal in outlining which equilibrium emerges. Suppose that a type 1 household is confronted with the money distribution $\{0, m^F_2, 0\}$ and chooses $D_1 = 0$. Further, suppose economic fundamentals remain unchanged and the same household, this time observing $\{m^S_1, m^S_2, 0\}$, chooses $D_1 = 1$. We conclude that economic fundamentals are right for coexistence of equilibria, since both are steady states for this economy. For the parameters chosen in the above simulation, I find that this indeed is the case. The no-coexistence conclusion in KW is a consequence of indivisibility, which limits the possible distributions and bargaining outcomes facing households.\(^{12}\) With divisible goods however, economic fundamentals allow more flexibility for the steady state distribution space and bargaining outcomes. I find regions of the parameter space for which the discounted value of deviation strategies is less than those consistent with both equilibria.

Since it is not practical to provide intuition using the discounted value of an infinite horizon of deviation strategies, I resort to examining the instant utility functions. From (1.2), the instant utility of a type 1 household (the pivotal type) can be derived as:

$$U^F_1 = p^F_{2,1} \frac{1}{6} \left[ u'_1 - \frac{\delta^2}{u'_2} \right]$$ and

$$U^S_1 = p^S_{1,2} p^S_{2,3} \frac{1}{6} \left[ 1 - \frac{\delta}{u'_2} \right] + p^S_{1,2} p^S_{2,1} \frac{1}{6} \left[ u'_1 - \frac{\delta^2}{u'_2} \right]$$

$$+ p^S_{1,3} p^S_{3,1} \frac{1}{6} \left[ u'_1 - \frac{\delta^2}{u'_3} \right],$$

(1.13)

\(^{12}\)In their model with indivisible goods, coexistence requires that: $\mathfrak{r}_{1,3} - \mathfrak{r}_{1,2} = (p^F_{3,1} - p^F_{2,1}) \frac{1}{3} u'_1 = (p^S_{3,1} - p^S_{2,1}) \frac{1}{3} u'_1$. This is not feasible in their environment since $p^F_{3,1} = p^S_{3,1} = 1$ and $p^F_{2,1} \neq p^S_{2,1}$. 

21
since $D_3 = 0$ in both the fundamental and speculative equilibria and $D_1 = 0$ in the fundamental equilibrium. To check for coexistence, I start off from a choice of variables for which the fundamental equilibrium is attained. Next, I assume $D_1 = 1$ and generate $p_{2,1}^S$ (hence, $p_{2,3}^S$) using (1.3). I then simulate the model again to ensure that $D_1 = 1$ is indeed the equilibrium choice of household 1. If instead $D_1 = 0$ is the equilibrium choice, one can conclude that type 1 households are not receiving sufficient instant returns in order to choose $D_1 = 1$ and, in so doing, sustain the speculative equilibrium. In the final step, I change parameters to alter instant utility in order to attain coexistence. Examining (1.12) and (1.13), an ideal parameter to change to increase $U_1^S$ without affecting $U_1^F$ is $u_3$. An increase in $u_3$ results in an increase in $U_1^S$. The simulation is run again for $e = F, S$, for each household type, to confirm both equilibria until coexistence is attained.

An intuitive interpretation of the adjustment in the parameter is that the higher the value of $u_3$, the higher the trade surplus household 1 can retain when selling commodity money (good 3) to household 3. That is, if household 1 is not receiving sufficient instant utility in the speculative equilibrium, increasing the trade surplus they derive from holding good 3 can deliver coexistence. With divisible goods and bargaining, parameters can be chosen to affect trade surpluses and inventory distributions so as to attain coexistence.

1.4.3 Welfare

The table below shows the effect on welfare between the two equilibria. With the bargaining framework used, each household receives positive return on half of the trades and loses nothing on the other half. That is, for household $h$, higher

\[ u_{3} \]
turnover for production good \( h + 1 \) translates monotonically into higher household consumption in equilibrium. Given the results on velocity, the welfare implications are therefore below:

**TABLE 1.3**

| Welfare |  
|---------|---------|---------|---------|
|         | \( h = 1 \) | \( h = 2 \) | \( h = 3 \) |
| \( R_h^F \) | -0.4081 | -3.9563 | 8.9635 |
| \( R_h^S \) | 2.2254 | 3.4907 | 7.7055 |

The speculative equilibrium sees an improved turnover for good 3, whilst that of good 1 declines. Therefore, due to the decline in the circulation of its produced goods, household 3 (the producer of good 1) suffers, while household 2 benefits.

### 1.4.4 Discussion

Popular modeling alternatives for dealing with tractability include the numerical simulation approach, as in Molico (1999), and the rebalancing of portfolios in centralized markets, as in Lagos and Wright (2005). Both approaches partially defeat the purpose of this paper. For the first approach, one is unable to provide close form solution to the distribution of commodity money in equilibrium. The Lagos and Wright approach short-circuits the effect of the decentralized market matching function on the distribution of commodity money at the end of each period, owing to the rebalancing of portfolios in the centralized market. Although not necessarily an undesirable outcome on its own, it reduces the comparability of the results with those in KW. This is against the background that in KW, only two factors affect the distribution of commodity money in equilibrium, being (i) the accept or reject decision of agents and (ii) the matching function.
Perhaps the single most significant limitation to our results is that we impose uniformity in the allocation of stocks among household agents. Once stocks, $m_h$, $1 - m_h$ are determined, agents are allocated accordingly, with each agent holding an equal volume $[m_h/p_{h,h+2} = (1 - m_h)/p_{h,h+1} = 1]$. This is closely tied to the assumption that each agent has a unit storage capacity. It may be useful to modify this assumption. Firstly, this will require a more general functional form for utility (and/or production disutility) and a more elaborate bargaining specification. As explained earlier, one cannot impose symmetry among household types. Hence, one must keep track of several Lagrange multipliers. The second challenge in the broader environment is that of finding a steady state in which a specific good or set of goods consistently emerges as medium(s) of exchange. Households will potentially vary their commodity stocks in order to affect the bargaining outcomes. It is also possible that periods of over-accumulation of commodity money may lead households to reject goods that they may other times decide to accept. That is, goods may cyclically gain or lose acceptability.

I implicitly assumed that prior to matching, each agent of type $h$ can hold stocks of either good $h + 1$ or $h + 2$, but never both. However, after the match, they can carry quantities of both units at the same time. This essentially is a simplifying assumption, without which an agent can be in a match in which she has the option to sell units of either commodity $h + 1$ or $h + 2$, if she is allowed to carry both prior to the match. This choice is not a trivial one and rather than delving into that debate, I used the above assumption [see Rupert, Schindler, Schevchenko and Wright (2000)]. If agents are allowed to hold both units into matches, our results on velocity are likely to fail. Trade, consumption and production can rise leading possibly to a Pareto improvement.

In the bargaining framework used above, I did not include storage costs when
stating a household’s surplus from each trade encounter. As a result, trade partners do not have to compensate for storage cost implications of trade with household $h$. This is unlikely to affect the results in this paper. If a household does not hold commodity money in a given equilibrium, it will be impossible during a match to bribe a single member agent to accept commodity money for two reasons. Firstly, the bribe needs to be high enough to at least compensate for the constant cost part of storing positive units of the good $(I_{c,h,h+2})$. Secondly, no such compensation can be adequate for the second reason, being the fact that each trade encounter is infinitesimal. The implication is that no commodity can emerge as money for the sole reason that there was positive surplus in a trade encounter which facilitated a bribe.

1.5 Conclusion

In this paper, I tested the robustness of results found by Kiyotaki and Wright (1989) that attempts to identify commodity money in a strategic sequential random matching framework. The environment discussed is similar to theirs to allow for comparison. The equilibria described by these authors are readily identifiable and for the same reasons.

The contributions of this paper are as follows. I find that the existence of the fundamental and speculative steady states is robust to the indivisibility assumption. I am unable to test the second result of KW, which states that no other equilibria exist. Although I find no new equilibria, this is not conclusive evidence against the possible existence of other equilibria, since only a limited section of the parameter space is testable by any computational exercise.

Unlike in Renero (1999) and KW, I demonstrate how the equilibrium distrib-
ution of commodity money depends not only on the matching function, but also on model parameters. Through bargaining, households can influence their mix of goods, without relying solely on the matching function for this outcome. This distribution is important, since commodity money is the only state variable for households in this class of models. The distribution of stocks also has important implications for velocity as well as welfare. We arrive at a non-trivial distribution of commodity money compared to earlier papers.

The third contribution of this paper is that the no-coexistence result in KW is a direct result of indivisibility. With divisible goods, the two equilibria can coexist. This is because bargaining outcomes and the distribution of inventories are not tied to the indivisibility assumption. In the case of coexistence with mixed strategies as in Renero (1999), velocity of the lower storage cost good increases since agents are less unwilling to part with it. Under coexistence with divisible goods however, the velocity of the lower storage cost good declines as its circulation is crowded out by the higher storage cost good. Velocity is closely tied with production and consumption. Since velocity of the lower storage cost good declines in the speculative equilibrium, its producer engages in less production, consumption and has lower welfare.
Appendix

<table>
<thead>
<tr>
<th>Parameters</th>
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<tbody>
<tr>
<td>$u'_h$</td>
</tr>
<tr>
<td>$c_{h,1}$</td>
</tr>
<tr>
<td>$c_{h,2}$</td>
</tr>
<tr>
<td>$c_{h,3}$</td>
</tr>
<tr>
<td>$\delta$</td>
</tr>
<tr>
<td>$\beta$</td>
</tr>
</tbody>
</table>

TABLE 1.4

$h = 1, 2, 3$
Chapter 2

Inflation and the Underground Economy

In this paper, I develop a model with congestible markets and a tax-evading sector. I show that inflation causes money and buyers to exit congested markets if there exists an alternative less-congested market where money can be spent faster. In other words, if the tax on using cash is high enough, then agents will move to the market where they can spend it quickly, even if it means that they have to pay taxes. Specifically, if the tax-evading sector is more congested with buyers, inflation moves money and buyers to the formal sector market and this reduces tax evasion. Therefore, inflation finance becomes attractive. The optimal inflation rate can be high and comparable to the actual inflation rates observed in some poor countries. If the formal market is more congested for buyers however, inflation increases tax evasion as buyers move to the underground market. The usefulness of inflation finance is reduced and the optimal inflation rate is low, even in an economy with prevalent tax evasion.
2.1 Introduction

One of the traditional arguments advanced for positive inflation is that, in the prevalence of a tax-evading underground economy, governments should rely heavily on seigniorage financing. Regular taxes tend to shift economic activity underground and are therefore distortionary. To reduce these distortions, the tax burden must be spread over all goods and services, including the liquidity services that money provides. Inflation tax is particularly convenient since it needs not be legislated or collected. In the presence of a tax-evading sector, the typical result is that the Friedman rule is not optimal. By modeling markets as decentralized exchange, this paper sheds new light on how inflation affects the underground economy. I show that if the tax on using cash is high enough, then agents will move to the market where they can spend it quickly, even if it means that they have to pay taxes.

Throughout this paper, the underground economy is defined as a collection of activities where taxes are evaded and where goods are of inferior quality. This definition is convenient for finding the data to discipline the model.\(^1\) In terms of magnitudes, the underground-to-formal sector \textit{output ratio} is estimated to be about 8.8\% in the US, 44\% in Peru and 76\% in Nigeria [Schneider and Enste (2000)]. The question asked in this paper is as follows: What is the optimal rate of inflation in an economy characterized by bilateral trade and a tax-evading underground sector? To the best of my knowledge, there are three papers in the literature that address the question of optimal inflation with underground production.\(^2\) All three papers consider tax evasion in environments with centralized market clearing

\(^1\)There is no universally accepted definition of the underground economy. For the purpose of this paper, I focus on this narrow definition. Inferior quality can be interpreted to mean that there are no legal guarantees protecting consumers of underground goods. Note that while similar activities and tasks may be performed in the production of goods in both sectors, the resulting output is taxed in one sector but not in the other.

using the Walrasian auctioneer. To the contrary, bilateral exchange (one-on-one anonymous meetings between buyers and sellers) seems to be the more plausible trade arrangement that facilitates tax evasion. Apart from being the natural way to model tax evasion, this paper shows that the mechanism of bilateral trade can have pivotal implications for the optimal rate of inflation. I find that under one set of bilateral trade market conditions, which I explain shortly, the optimal rate of inflation is high and comparable to some of the rates suggested in the literature. Under a different set of bilateral trade market conditions however, the optimal rate of inflation is extremely low, even for an economy with the same output ratio.

In the environment examined, households have buyers. Some buyers are sent to the formal market, while others are sent to the underground market. If underground goods are of poor quality, a household sends relatively more buyers to the formal market. Each household acts similarly and private interest overwhelms the social optimum. There is a tendency for overcrowding of buyers in the formal sector and trade opportunities become few for each buyer in this sector.\(^3\) If the inflation rate increases, households try to spend money faster at current prices rather than at future higher prices. They divert buyers to the underground market, where the overcrowding of buyers is less. The turnover of goods in the underground market increases and underground output increases relative to the formal sector. Since inflation increases tax evasion, seigniorage financing becomes less attractive and the optimal rate of inflation is low compared to the literature. This result defies conventional wisdom, which claims that in the presence of a significant tax-evading sector, governments should resort to inflation tax.

On the other hand, if underground goods are of considerably good quality,

\(^3\)This of course depends on the allocation of sellers as well. For a full description of how I treat sellers, see section 2.2. Also, one can think of “fewer trade opportunities” as equivalent to a lower probability of finding a match with a seller.
the underground market tends to be more crowded for underground buyers. In response to higher inflation, buyers move to the “less-crowded” formal market to spend money faster. Thus, inflation reduces tax evasion and seigniorage financing becomes more attractive. The optimal inflation rate is high and close to the actual inflation rates observed in some poor countries.

For a given size of the underground economy, optimal inflation depends crucially on market conditions. An environment with market crowding is essential for generating this outcome. In particular, notice that the results are not driven by the extrinsic quality of underground goods, but rather by differences in market crowding. Compare the above analysis to an equivalent economy with Walrasian market clearing, while retaining the assumption that underground goods are of lower quality. In such an economy, higher inflation still brings higher urgency to spend money. However, the distribution of goods from sellers to buyers is fully and equally efficient in both sectors due to the Walrasian auctioneer in both sector markets. Money can thus be spent equally fast in both sectors and households need not adjust buyer allocations in order to spend money faster. Inflation on its own does not affect the sectoral distribution of the economy even though underground goods are inferior. Since inflation does not increase or decrease tax evasion, the optimal rate of inflation is unaffected. The crowding effect is unique to search models and is sometimes termed the “extensive margin” or the “market congestion effect”.

The advantage of modeling both sectors as decentralized markets is that it enables me to demonstrate the realistic outcome that buyer congestion can be worse in either market. Following this formulation, an interesting result is that depending on the relative congestion of the two sector markets, inflation can either increase

---

4For instance, modeling the formal sector as Walrasian would imply that buyer congestion is always worse in the underground market, unless there are an infinite number of sellers underground.
or decrease the underground economy. How does this result compare with the literature? Koreshkova (2006) introduced an environment in which credit services are produced solely in the formal sector. Inflation causes agents to trade more with credit, thereby increasing the formal sector at the expense of the underground sector. This approach supports a negative relationship between changes in inflation and changes in underground output. Although intuitively coherent, the data on the other hand is far less conclusive. In Figure 2.4, I compare changes in inflation to changes in underground output for several countries. There is very little if any such negative correlation. Although there may exist an endogeneity problem, this only cements the need for comprehensive modeling of the underground economy to investigate the evidence.

In relation to Figure 2.4, the results in this paper can be interpreted as follows. At a given point in time, two countries can take opposite positions on the relative congestion of their formal and underground markets for buyers. Inflation therefore impacts their underground sectors in opposite directions. Secondly, over time, a single country can switch states in the relative congestion of the two sector markets for buyers. Thus, inflation moves underground output in reverse directions over time. Putting these together, one can get data points that wrongly suggest no relationship between changes in inflation and changes in the output ratio, similar to Figure 2.4.

Wright (2005) identifies four major areas where the existing literature on micro foundations of money needs further extension. Two of these are (i) extensions to include fiscal policy variables to examine their interaction with monetary policy and (ii) quantitative analysis to enable numerical policy proposals. This paper makes

\footnote{Regressing changes in inflation on changes in the output ratio generates coefficients that are statistically not different from zero.}
a significant contribution towards the integration of elaborate schemes of public finance into the monetary search literature, following recent progress by Aruoba, Waller and Wright (2006). I show that these models are indeed computable to generate numerical results that are relevant for policy. Using decentralized trade provides a coherent account of the underground economy by integrating anonymity, which directly motivates tax evasion. Finally, I show that the relative congestion of the two sector markets is important for optimal inflation.

This paper adds to the monetary literature on the informal sector, alongside Koreshkova (2006), Cavalcanti and Villamil (2003) and Nicolini (1998). Optimal policy in the presence of externalities follows fundamentals by Sandmo (1975).6 The next section presents a two-sector monetary search framework that replicates properties of the underground-formal dichotomy. In section 2.3, I characterize the model and describe the equilibrium. Section 2.4 derives the price and output ratios and examines how households adjust decisions when inflation changes. In section 2.5, I calibrate the model to data from Peru and present quantitative estimates of the optimal inflation rate. Section 2.6 considers robustness and extensions. I conclude in section 2.7.

2.2 Economic Environment

I extend the framework introduced by Shi (1999) to allow for two sectors, formal and underground (informal). These are denoted by the subscripts $f$ and $i$ respectively and are assumed to be on separate islands. Goods are perishable between periods, irrespective of the sector in which they are produced. By this, I preclude the emergence of commodity money. Self-produced goods yield no utility. There-

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6 Also see Ng (1980), Goulder (1995) and Bovenberg and van der Ploeg (1998).
fore, trade is essential for worthwhile consumption. Some of these restrictions are standard in monetary search models because they permit trade and an endogenous role for fiat money.

Time is discrete, denoted $t$. Aggregate money stock and household money holdings are the only state variables. The economy is inhabited by a large number of anonymous and infinitely-lived agents who are either buyers or sellers/producers. For tractability, I collect agents into decision-making families or households. A household is constituted by the measure $s$ of sellers and $b$ of buyers, $s \in (0, \infty)$, $b \in (0, s]$. For simplicity, sellers are allocated exogenously between sectors, with $s_f + s_i = s$ and $s_j \in (0, s)$, $j = f, i$ [section (2.6) contains a discussion on this assumption]. There are a large number of households, and each household is infinitesimal compared to the aggregate. The focus is on the representative household, who’s state and choice variables are in lower-case letters. Capital-case variables represent those of other households and the aggregate economy, which the representative household takes as given. Economy-wide money supply is $M_t$, of which the representative household has $m_t$. There is no population growth, the number of households, sellers and buyers being exogenous constants.

2.2.1 Market Congestion

The key mechanism driving the results in this paper is the potential for differences in market congestion in the two sectors. Hence, I present this mechanism first.

Each household sends a fraction of its buyers to each sector market. Let $B_{jt}$ and $S_j$ be the aggregate number of buyers and sellers entering market $j$, $j = f, i$. These

---

agents match one-on-one and may trade if the match is successful. A successful match occurs when any buyer meets a seller from a household other than his own. The total number of successful matches, \( X_{jt} \), is derived from the matching function:

\[
X_{jt} = B_{jt}^\alpha S_{jt}^{1-\alpha}, \quad \alpha \in (0,1), \quad j = f, i.
\]  

(2.1)

Also, define \( B_{jt} \) and \( S_{jt} \) as:

\[
B_{jt} = \frac{X_{jt}}{B_{jt}} = \left( \frac{S_{jt}}{B_{jt}} \right)^{1-\alpha}
\]  and

\[
S_{jt} = \frac{X_{jt}}{S_{jt}} = \left( \frac{B_{jt}}{S_{jt}} \right)^{\alpha}, \quad j = f, i.
\]  

(2.2)

(2.3)

Then, \( B_{jt} \) and \( S_{jt} \) are the average matching rates per buyer and per seller respectively. These can also be interpreted as the market congestion rates for buyers and sellers respectively. Since each household is infinitesimal, they take congestion rates as given. The larger the number of buyers entering market \( j \), the higher is the market congestion for buyers in that sector and the fewer the trade opportunities for each buyer in that sector.

Suppose there are more trade opportunities for each underground buyer than for each formal buyer: \( B_{it} > B_{jt} \). In other words, the formal market is more congested for buyers than the underground sector. Then, an increase in inflation moves buyers to the less-congested underground market, given higher urgency to spend money stocks. Buyers are moved underground to take advantage of better trade opportunities there. On the aggregate level, the turnover of goods increase underground relative to the formal sector. Since inflation can increase tax evasion,

---

\(^8\)Note that \( B_{jt}B_{jt} = S_{jt}S_{jt}, \quad j = f, i \). Since it takes two to trade, one successfully matched seller implies a successfully matched buyer. See Petrongolo and Pissarides (2001) for a survey of related matching functions.
seigniorage financing is unattractive and the optimal rate of inflation is low. The opposite is the case when $B_{it} < B_{jt}$. I focus on the market congestion rate for buyers only since the allocation of sellers is exogenous.

2.2.2 Household’s Problem

Household agents are altruistic towards fellow members. Let $U_t$ be instantaneous utility from consumption, net of the disutility of production. $\Phi(Q_{jt}) = Q_{jt}^\phi$, $\phi > 1$ is the disutility of producing $Q_{jt}$ units inside a match. Also, let the pair $\{q_{jt}, x_{jt}\}$ be the terms of trade whenever the representative household’s buyers engage in purchases and $\{Q_{jt}, X_{jt}\}$ when the sellers engage in sales. Here, $q_{jt}$ (or $Q_{jt}$) is the quantity to be traded and $x_{jt}$ (or $X_{jt}$) is the monetary payment in currency. The terms of trade will be discussed later but for now, it suffice to take these values as given. The household’s problem is:

$$v(m_t) = \max_{b_{jt}, m_{jt}, m_{t+1}, j = f, i} U_t + \beta Ev(m_{t+1}) \ , \ \beta \in (0, 1) \ , \ (2.4)$$

subject to the terms of trade as well as:

$$U_t = c_{ft} + \eta c_{it} - s_f S_{ft} \Phi(Q_{ft}) - s_i S_{it} \Phi(Q_{it}) \ , \ (2.5)$$

$$c_{ft} = (1 - \tau) b_{ft} B_{ft} q_{ft} - Q_t^g \ , \ (2.6)$$

$$c_{it} = b_{it} B_{it} q_{it} \ , \ (2.7)$$

$$b_{ft} + b_{it} \leq b \ , \ (2.8)$$

$$m_{ft} + m_{it} \leq m_t \ , \ (2.9)$$

$$m_{t+1} - m_t \leq s_f S_{ft} X_{ft} + s_i S_{it} X_{it} + P_t Q_t^g - b_{ft} B_{ft} x_{ft} - b_{it} B_{it} x_{it} \ , \ (2.10)$$
\[ m_{jt}, x_{jt}, c_{jt}, b_{jt} \geq 0, \quad j = f, i \quad \text{and} \quad m_t \geq 0 \quad \forall t. \]

Given the market congestion rates, total successful matches for household agents sent to market \( j \) are \( b_{jt}B_{jt} \) for buyers and \( s_jS_{jt} \) for sellers. Total purchases are thus \( b_{jt}B_{jt}q_{jt} \), while total disutility is \( s_jS_{jt}\Phi(Q_{jt}) \), \( j = f, i \). In (2.5), formal and underground goods are perfect substitutes in consumption but underground goods may be of inferior quality: \( \eta \leq 1 \).\(^9\) I define composite consumption as \( c_t = c_{ft} + \eta c_{it} \), where \( c_{jt} \) is consumption of sector \( j \) goods. A fraction, \( \tau \), of formal sector purchases is paid as a commodity tax. Also, the government buys off the quantity \( Q_t^g \) from formal buyers and pays for these units by printing money. Due to perishability, the household consumes all goods instantly. In (2.10), incoming funds from sales, \( X_{jt} \), arrive simultaneously as outgoing funds, \( x_{jt} \), during purchases. Hence, the former cannot be used to finance the latter within the same period. Nominal income from sales to the government is \( P_tQ_t^g \), where \( P_t \) is the per-unit price paid by the government.

**Figure 2.1**

**Timing of Events**

<table>
<thead>
<tr>
<th>( t )</th>
<th>( t + 1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Decisions</td>
<td>Markets Open</td>
</tr>
<tr>
<td>( b_{jt}, m_{jt} )</td>
<td>Buyers ( \rightarrow \frac{m_t}{\eta_{jt}} )</td>
</tr>
<tr>
<td>( m_{t+1} )</td>
<td>Taxes Paid</td>
</tr>
<tr>
<td>Terms of trade</td>
<td>Match, Bargain</td>
</tr>
<tr>
<td></td>
<td>Govt. Purchases: ( Q_t^g )</td>
</tr>
<tr>
<td></td>
<td>Produce, Trade</td>
</tr>
<tr>
<td></td>
<td>Consumption</td>
</tr>
</tbody>
</table>

\(^9\)An alternative formulation is to consider \( \eta \) as representing a less-efficient production technology in the underground sector.
I specify the timing of events next. Starting a period with money holdings $m_t$, the representative household makes decisions on the allocation of buyers and money. The household also instructs its buyers and sellers on the terms of trade, which include the offers to make and those to accept in all successful matches. Next, the markets open, with formal agents visiting only the formal market, while informal agents go to the underground market. Once in the market, agents match one-on-one according to the matching function. Anonymity forbids credit transactions and trade is *quid pro quo*. After a bargain is reached, a successfully matched seller produces the desired output and trade is then finalized. As markets close, goods exiting the formal market gates are all taxed. Each formal buyer compulsorily sells some quantity $Q_g^t$ to the government and receives money. Agents return to their respective households where purchased goods and sales receipts are gathered. There is consumption, and the period ends.

### 2.2.3 Terms of Trade

Notice that the terms of trade, $\{q_{jt}, x_{jt}\}$, essentially establishes the per-unit price, $p_{jt}$, which is implied by $p_{jt} = \frac{x_{jt}}{q_{jt}}, j = f, i$. After the money and buyer allocations, a representative buyer enters his assigned market $j$ with $\frac{m_{jt}}{q_{jt}}$ units of money, $j = f, i$. In each successful match, trade can occur if the offer is acceptable to both sides. For each implementable offer, monetary payments cannot exceed the buyer’s money holding upon entering the match: $x_{jt} \leq \frac{m_{jt}}{q_{jt}}, j = f, i$. This feasibility constraint is intrinsic to the environment, given that trade is *quid pro quo*.\textsuperscript{10}

Let $\omega_t$ (or $\Omega_t$) be the value of money. Then, for an offer to be accepted, it must

\textsuperscript{10}Market clearing models of the underground economy are useful due to the ease of incorporating credit. For ways to include credit in models with anonymous agents, see Berentsen, Camera and Waller (2005).
satisfy the seller’s individual rationality constraint. This is simply 
\( x_{jt} \Omega_t \geq \Phi(q_{jt}) \), 
\( j = f, i \). In both sectors, I allow buyers to hold all the bargaining power and to make take-it-or-leave-it offers. Optimal offers ensure that the individual rationality constraint holds with equality. Combined with the feasibility constraint, we have:

\[
\frac{m_{jt}}{b_{jt}} \geq \frac{\Phi(q_{jt})}{\Omega_t}, \ j = f, i .
\] (2.11)

Inequality (2.11) is named the cash-and-carry constraint and is the final constraint on the household’s problem.

Sellers act as “offer takers” and take the quantity requested as given. Temporarily assume that money is valued, which allows the cash-and-carry constraint to bind in both sectors. Then, one can rewrite the level of output-per-trade in each sector as:

\[
q_{jt} = \frac{m_{jt}}{b_{jt}} \Omega_t \right)^{\frac{1}{\phi}} , \ j = f, i .
\] (2.12)

With quantities determined, the quantity-per-trade ratio, \( \frac{q_{jt}}{X_{jt}} \), can be readily derived. I will return to this later.

To summarize, the terms of trade is simply 
\( x_{jt} = X_{jt} = \frac{m_{jt}}{b_{jt}} \) and \( q_{jt} (= Q_{jt}) \) given by (2.12). Having established this, I now address \( P_t \), which is the price that the government pays for goods. I allow formal sector agents to charge a premium on all sales to the government to take account of matching costs. Specifically, I assume \( P_t = \frac{p_{ft}}{X_{jt}} \).

### 2.2.4 Government

The definition of a sector as “underground” suggests the existence of an authority that makes this distinction. There is a centralized government that implements
both monetary and fiscal policies. Money supply, $M_t$ per capita household, grows at the rate $\gamma$ per period. There is no government debt. Instead, newly printed money, $(\gamma - 1) M_t$, is used by the government in the market as payment for $Q^g_t$. That is, $Q^g_t$ is real seigniorage income. The real government budget constraint is:

$$G = \tau b_f t B_f q_{ft} + Q^g_t, \quad \text{(2.13)}$$

where $G$ is an exogenous expenditure each period. Since part of government revenues are nominal while expenditure is real, the government faces a liquidity constraint much like private households. Following Cooley and Hansen (1991),

$$(\gamma - 1) M_t = P_t Q^g_t. \quad \text{(2.14)}$$

Note that the money growth rate and tax rate are endogenous. Consider a reduction in $\tau$. The government’s liquidity constraint goes into deficits as consistent with the optimal region of the Laffer curve. This requires an adjustment in transfers to supply the funds necessary to alleviate the fiscal position, which, in turn, changes $\gamma$. Thus, (2.13) and (2.14) emphasize the inherent interaction between the fiscal and monetary policy variables $\tau$ and $\gamma$.

### 2.3 Characterizing the Equilibrium

This section examines the Euler conditions that characterizes the equilibrium. Let $\lambda_{jt}, j = f, i$, be the Lagrange multiplier on the cash-and-carry constraint in each successful match. $m_{jt}$ is chosen such that the cash-and-carry constraint binds to an equal extent in expectation in each sector: $B_f \lambda_{ft} = B_i \lambda_{it}$. The implied Euler
condition for money is:

\[ \frac{\omega_t}{\beta} = \omega_{t+1} + B_{jt+1}\lambda_{jt+1}, \ j = f, i. \]  

(2.15)

Money kept between periods delivers its discounted value in the next period as well as helps alleviate the cash-and-carry constraint in future trade matches. From (2.15), it can be shown that both cash-and-carry constraints bind in all successful matches in equilibrium if the return on money is sufficiently low: \( \gamma > \beta \). From this point on, I assume this to be the case.

Next, I turn to the optimal quantity of output that is demanded in each trade match. The associated first order conditions are derived as:

\[ 1 - \tau = \lambda_{ft} \frac{\Phi(q_{ft})}{\Omega_t} \frac{\phi}{q_{ft}} + \omega_t \frac{dx_{ft}}{dq_{ft}} \]  and

(2.16)

\[ \eta = \lambda_{it} \frac{\Phi(q_{it})}{\Omega_t} \frac{\phi}{q_{it}} + \omega_t \frac{dx_{it}}{dq_{it}}. \]  

(2.17)

Demanding a higher quantity yields marginal utility from the additional units. The marginal cost is incurred at two levels. At the buyer level, demanding a larger quantity requires of the buyer to pay more money, thus making the corresponding cash-and-carry constraint more binding. The rate at which this constraint becomes more binding depends on how much is required to motivate the seller to deliver the additional quantity, which, in turn, depends on the seller’s production disutility costs on the margin. Secondly, as buyers purchase higher quantities from the market and need more money to do so, the household is pressured to deliver more money to its buyers. This causes the liquidity constraint (2.10) to become more binding.
The first order condition for $b_{ft}$ is given as:

$$
B_{ft} \left[ (1 - \tau) q_{ft} - \lambda_{ft} \frac{\Phi(q_{ft})}{\Omega_t} - \omega_{ft} x_{ft} \right] = B_{it} \left[ \eta q_{it} - \lambda_{it} \frac{\Phi(q_{it})}{\Omega_t} - \omega_{it} x_{it} \right].
$$

(2.18)

Allocating more buyers to the formal sector generates more formal sector purchases and yields the associated marginal benefits in consumption utility. All things being equal, as more buyers visit the formal sector, $\frac{m_{ft}}{\sigma_{ft}}$ declines and the cash-and-carry constraint binds further in this sector. The household is pressured to deliver more money to formal sector buyers, causing the liquidity constraint to become more binding as well. A similar effect pertains to the underground sector. For the marginal buyer, the net benefits must be equal between sectors in expectation.\footnote{The matching rates $B_{ft}$ and $B_{it}$ can be interpreted in terms of probabilities.}

All households are alike and so I apply symmetry as usual. The only state variables are money holdings per household and aggregate money supply. To proceed to describe an equilibrium therefore, it is essential to ensure that this variable evolves at a constant rate. Assuming a fixed inflation rate $\gamma$, the Euler condition for money holding in steady state reduces to:

$$
\lambda_{jt} = \frac{\gamma - \beta}{\beta B_j} \Omega_t , \ j = f, i.
$$

(2.19)

Substituting this into (2.16) to (2.18) gives (2.20) to (2.22) below.

### 2.3.1 The Equilibrium

**Definition 1** A symmetric monetary search equilibrium is defined as the inflation rate $\gamma$, the set of household choices $(b_{f}, m_{ft})_{t=0}^{\infty}$ and the implied value of money $(\omega_t)_{t=0}^{\infty}$ such that given $\tau$, the following requirements are met: (i) each household
solves its optimization problem; (ii) the representative household’s variables replicate the aggregate equivalents; (iii) prices are positive, though bounded (the value of money is positive and bounded); and (iv) the government budget balances.

In particular, an equilibrium involves a solution to a system of four equations for \( b_f, m_{ft}, \omega_t \) and \( \gamma \):

\[
1 - \tau = \left[ 1 + \frac{\gamma - \beta}{\beta B_f} \right] \Omega_t \frac{m_{ft}}{b_f} \phi f,
\]

\[
\eta = \left[ 1 + \frac{\gamma - \beta}{\beta B_i} \right] \Omega_t \frac{m_{it}}{b_i} \phi i,
\]

\[
\frac{m_{it}}{m_{ft}} = \frac{\gamma - \beta + \beta B_f}{\gamma - \beta + \beta B_i},
\]

\[
G = \tau b_f B_f q_f + (\gamma - 1) \frac{M_t}{P_i}.
\]

Variables without the time subscript represent equilibrium real values. Those with time subscripts are nominal values that depend on the money stock at date \( t \).

Given \( \tau \), there exists an equilibrium. The equations (2.20), (2.21) and (2.22) deliver values for the household variables \( b_f, m_{ft} \) and \( \omega_t \), all in terms of \( \gamma \). The required inflation rate that balances the budget, given \( \tau \), is then derived from (2.23). All other variables - such as \( q_j, c_j, B_j, \lambda_{jt}, x_{jt}, p_{jt} \) and \( P_t \) - can be derived as functions of the four in the definition.

Equation (2.22) plays a central role in understanding the implications of the model. Firstly, the sector with the higher buyer congestion rate always has the higher money holding per buyer. If market congestion is worse for formal buyers, each is compensated with higher sums of money. In other words, if \( B_f < B_i \), households take advantage of the intensive margin when buying from the formal sector.
and the extensive margin when buying underground goods. Secondly, suppose there is an increase in $\gamma$, with $B_f < B_i$. All things being equal, more money is diverted to underground buyers per capita and $q_{lt}$ increases relative to $q_{ft}$. That is, the erosive effect of inflation on household money stock increases tax evasion and seigniorage financing becomes less attractive. The reverse is the case when market congestion is worse for buyers in the underground market. A discussion of the effect of inflation follows in the next section.

### 2.4 Size, Prices and Inflation

The quantity-per-trade ratio describes trade within an underground match relative to a formal sector match and is denoted $R_I = \frac{q_i}{q_f}$. Summing over all such trade encounters in each sector gives the aggregate output ratio in trades involving all household buyers. This is denoted $R = \frac{b_i B_i q_{it}}{b_f B_f q_{ft}}$. The subscript $I$ is used to denote the intensive margin.

#### 2.4.1 Relative Quantities and Relative Price

Since the cash-and-carry constraint binds in both sectors, (2.12) gives the quantity-per-trade in each sector. Using this outcome together with (2.22), the equilibrium quantity-per-trade ratio becomes:

$$R_I = \left[ \frac{\gamma - \beta (1 - B_f)}{\gamma - \beta (1 - B_i)} \right]^{\frac{1}{\phi}},$$

which completely describes the intensive margin. The intensive margin concerns the quantity traded within each successful match, which depends on the amount
of money each buyer takes into a match. If the formal market is congested for buyers, households take advantage of each successful formal match to acquire large quantities, which implies the expense of higher sums of money in formal matches compared to underground matches. In other words, high market congestion for formal buyers reduces the intensive ratio.$^{12}$

Next, the aggregate trades equivalent is:

$$R = \frac{b_i B_i}{b_f B_f} R_I \equiv \frac{s_i}{s_f} \left[ \frac{\eta}{1 - \tau} \right]^\alpha R_I^{1-\alpha},$$

(2.25)

which is the underground-to-formal sector output ratio. Comparing with the intensive ratio, $R$ stresses the effect of the matching rate on aggregate market outcomes. Suppose $R_I$ is given. Then, for the representative buyer sent to each island, the congestion of the underground market relative to the formal market, $\frac{B_i}{B_f}$, determines the quantity of expected purchases by an underground buyer relative to a formal buyer: $\frac{B_i}{B_f} R_I$. Preference and policy parameters $\eta$ and $\tau$ are reflected in $R$ because households are mindful of the effect of their buyer allocation decisions on the eventual mix of goods that they consume. Given the bargaining outcome and market congestion conditions, households employ their buyer allocation decision to edge closer to their preferred mix of goods. The allocation of buyers and its effect on market congestion and aggregate trade outcomes is termed the extensive margin. This margin is conclusively captured by $R$ and a search model is essential for separating $R$ from $R_I$.

Price in each transaction as determined from the terms of trade is $p_{jt} = \frac{m_{jf}}{b_j} \frac{1}{q_j}$, $j = f, i$ in equilibrium. Using (2.22), the relative price ratio in private trades

---

$^{12}$One can consider the effect of technology as another dimension of the intensive margin. Superior technology in the formal sector means that even with equal financial compensation, formal sector sellers can deliver higher quantities within each trade meeting.
reduces to \( \frac{p_{it}}{p_{ft}} = \frac{1}{R_I} \frac{m_{it}}{b_i} / \frac{m_{ft}}{b_f} \), or:

\[
\frac{p_{it}}{p_{ft}} = \left[ \frac{\gamma - \beta (1 - B_f)}{\gamma - \beta (1 - B_i)} \right]^{1-\frac{1}{\phi}}.
\] (2.26)

Similar to (2.25), the relative price is not only a function of preferences and taxes but also an endogenous outcome of monetary policy, unlike in the earlier papers. With relatively high market congestion for formal buyers, each brings more money into a match and this increases the formal sector price relative to that underground. If \( B_i < B_f \), it is possible to generate higher prices in the underground sector. It is worth noting however that \( p_{ft} \) is price before taxes. The effective price ratio after tax is \( \frac{p_{it}}{p_{ft}} (1 - \tau) \), which I report in section 2.5.

The ratio \( R \) has been the subject of virtually all of what is known in the literature on underground economy. The environment presented above enables us to use published empirical estimates of \( R \) and back out the micro level ratio \( R_I \) as well as the price ratio \( \frac{p_{it}}{p_{ft}} (1 - \tau) \) as demonstrated. Some of these results may be particularly useful since empirically, micro level data is unattainable in studies on the underground economy.

### 2.4.2 Effect of Inflation

In this subsection, I assume that monetary injections are via lump sum transfers to households and also that \( \frac{d\pi}{d\gamma} = 0 \).\textsuperscript{13} In the equivalent case in Cavalcanti and Villamil (2003) as well as Nicolini (1998), firms and households do not adjust portfolios when inflation increases. In particular, inflation has no effect on sectoral allocations. \( \frac{dR}{d\gamma} \bigg|_{\tau} \) is strictly negative in Koreshkova (2006) since inflation causes agents to use more

\textsuperscript{13}Specifically, government simply hands money to each buyer, instead of requesting \( Q_f^g \) units of output. For now, ignore the effect on the government budget.
credit, which is exclusively produced in the official sector. In the model proposed however:

\[
\frac{dR_i}{d\gamma} \bigg|_\tau = [B_i - B_f + \varphi] \frac{R_f}{A} \quad (2.27)
\]

by quotient rule, where

\[
A = \frac{\phi}{\beta} [\gamma - \beta + \beta B_f] [\gamma - \beta + \beta B_i] \quad \text{and} \quad (2.28)
\]

\[
\varphi = [\gamma - \beta + \beta B_i] \frac{dB_f}{d\gamma} \bigg|_\tau - [\gamma - \beta + \beta B_f] \frac{dB_i}{d\gamma} \bigg|_\tau . \quad (2.29)
\]

Notice that \(B_j > 0 \forall b_j \in [0, b]\), hence, \(A > 0 \forall \gamma \geq \beta\). Secondly, \(\frac{dB_i}{d\gamma} \bigg|_\tau > 0\) and \(\frac{dB_f}{d\gamma} \bigg|_\tau < 0\) whenever \(B_i - B_f > 0\) and vice versa. Thus, \(\varphi\) is a function of the same sign as \(B_i - B_f\). Assume that underground buyers have better matching success: \(B_i - B_f > 0\). When \(\gamma\) increases, households seek to spend nominal balances faster and they divert some buyers from the formal market to the less congested underground market, as consistent with (2.22). Since \(b_i\) increases, \(\frac{dB_i}{d\gamma} \bigg|_\tau < 0\) and the household compensates each underground buyer with more money per capita, which increases \(R_i\). Due to the increase in \(b_i\), aggregate matches \((b_iB_i)\) increase underground relative to the formal sector. The effect on the extensive ratio \(R\) is therefore in the same direction as \(R_f\).

Even with \(\frac{d\tau}{d\gamma} = 0\), monetary policy affects the relative price. If \(B_i - B_f > 0\), the underground price level rises relative to the formal sector price as the rate of inflation increases. Again, by quotient rule:

\[
\frac{d\frac{p_u}{p_{ft}}}{d\gamma} \bigg|_\tau = [B_i - B_f + \varphi] (\phi - 1) \frac{p_u}{p_{ft}} A . \quad (2.30)
\]

Intuitively, increased inflation implies that each underground buyer starts to
hold more money compared to previously (if $B_i - B_f > 0$). Thus, underground buyers begin to demand higher quantities in each trade. They need to pay higher prices to motivate the additional units, owing to the convex cost of production ($\phi > 1$). This change in the relative price implies a marginal decline in $R_I$. However, this effect is of second order and does not reverse the initial rise in $R_I$ and $R$. When $B_i - B_f < 0$, the relative size and relative price ratios respond in the opposite direction of the corresponding effect above as inflation increases.

2.4.3 The Ramsey Problem (Optimal Inflation)

Bailey (1956) and Phelps (1973) brought the subject of optimal inflation into the fold of public finance. In this seminal contribution, Phelps advocates for a positive tax on the liquidity services that money provides if taxes on other goods and services are distortionary. This argument favours a positive nominal interest rate, or simply, positive inflation. Tax distortions are socially costly, while inflation presents the usual welfare consequences. The task facing a benevolent government is to find the best trade-off between the deadweight loss from tax financing and that from seigniorage financing.

I focus on the Ramsey problem that seeks to find the optimal mix of consumption and inflation taxes when government can commit to the announced policy. Without money, the cash-and-carry constraint (2.11) cannot be satisfied and the economy described degenerates into autarky. Following Kiyotaki and Wright (1989), money acts as an intermediate commodity that facilitates trade. Diamond and Mirrlees (1971) established a general result emphasizing the undesirability of taxing the intermediate goods sector when all final goods and services fall under the tax radar. In application to monetary economics, their conclusion implies that inflationary
tax should not be used despite the distortions caused by taxes on the final goods sector.\textsuperscript{14} However, where there is a third sector - the underground economy - that evades regular taxes, the optimal policy set may include positive seigniorage.

The formalized Ramsey problem is to solve the social planner’s problem subject to the government’s budget constraints and the first order conditions in section (2.3). Let the variables with tildes represent the Ramsey allocations: \(\{\tilde{b}_f, \tilde{m}_{ft}, \tilde{q}_f, \tilde{q}_i\}\).

The required inflation and tax rates are respectively:

\[
\tilde{\gamma} = \beta + \beta \left[ \frac{\tilde{m}_{it}}{b_i} - \frac{\tilde{m}_{ft}}{b_f} \right]^{-1} \left[ \tilde{B}_f \frac{\tilde{m}_{ft}}{b_f} - \tilde{B}_i \frac{\tilde{m}_{it}}{b_i} \right] \quad \text{and} \quad (2.31)
\]

\[
\tilde{\tau} = \frac{G - (\tilde{\gamma} - 1) M_t/\tilde{P}_t}{b_f \tilde{B}_f \tilde{q}_f}. \quad (2.32)
\]

Market clearing models of the underground economy have commonly used credit to establish reasons why the Friedman rule is suboptimal. Even in the absence of credit, search frictions rule out any possibility that the Friedman rule may be optimal, except for the special case where market congestion rates are equal between the two sectors. From (2.22), one can show that second term on the right hand side of (2.31) is non-negative. Firstly, \(\frac{\tilde{m}_{it}}{b_i} = \frac{\tilde{m}_{ft}}{b_f}\) whenever \(\tilde{B}_f = \tilde{B}_i\). Secondly, when \(\tilde{B}_f \neq \tilde{B}_i\), \(\frac{\tilde{m}_{it}}{b_i} - \frac{\tilde{m}_{ft}}{b_f}\) and \(\tilde{B}_f \frac{\tilde{m}_{ft}}{b_f} - \tilde{B}_i \frac{\tilde{m}_{it}}{b_i}\) are of the same sign, in which case the second term is strictly positive.

The trade-off between taxes and inflation in the presence of congestion externalities warrants further explanation. In the typical environment with market clearing and evadable taxes, optimal policy considers (i) real distortions created by formal sector taxes and (ii) the welfare cost of inflation. Assume that the implied optimal

\textsuperscript{14}Also, see Kimbrough (1986), Faig (1988), Guidotti and Vegh (1992), Chari, Christiano and Kehoe (1996).
policy set in this case is the pair $\{	ilde{\tau}_a, \tilde{\gamma}_a\}$. The additional dimension provided in the framework with bilateral exchange is that there is a role for inflation in correcting any imbalances in market congestion rates in the two sectors.\footnote{For more on second best taxation in environments with externalities, see Sandmo (1975), Ng (1980), Goulder (1995) and Bovenberg and van der Ploeg (1998).}

**The Trade-off with market congestion - Case 1**

To illustrate, suppose sellers are distributed evenly between the two sectors. Then, to minimize search frictions and maximize aggregate matches, buyers must also be allocated equally between sectors. Now suppose the allocation of buyers is skewed towards the underground sector, causing high market congestion for buyers in that sector. Then, the optimal policy set includes low taxes ($\tilde{\tau}_1 < \tilde{\tau}_a$) and high seigniorage ($\tilde{\gamma}_1 > \tilde{\gamma}_a$). Low taxes edge buyers back into the formal market and improves the coordination problem. That is, if the tax on using cash is high enough, then agents will move to the market where they can spend it quickly, even if it means that they have to pay taxes.

In this illustration, two factors account for the negative relationship between $\tau$ and $\gamma$. The first is the traditional argument that as $\gamma$ increases, seigniorage income rises, which finances the government and helps reduce $\tau$. The second is that as $\gamma$ increases, buyers move to the formal sector via the extensive margin in order to spend money faster. Thus, more goods become taxable, which also means the tax rate can adjust downwards even further. For both of these factors, $\frac{d\tau}{d\gamma} < 0$ when the underground market has the higher market congestion for buyers. Apart from the trade-off between distortionary taxes and the welfare cost of inflation, the Ramsey problem also seeks to even out market congestion rates in the two sectors and improve the coordination problem.
The Trade-off with market congestion - Case 2

When the market is more congested for formal buyers, the trade-off between these two taxes is less clear. Consider a marginal reduction in inflation. Seigniorage incomes decline, but tax revenues increase even with no change to the tax rate. This is because buyers return to the congested formal market, given a lower urgency to spend money. Depending on the influx of buyers into the formal market, the rise in tax revenues can outweigh losses in seigniorage income. That is, marginal reductions in $\tau$ also become affordable. In summary, lower inflation and a lower tax rate are jointly feasibly: $\frac{d\tau}{d\gamma} > 0$.16

If government can lower the welfare cost of inflation (by lowering $\gamma$) and at the same time lower tax distortions (by lowering $\tau$), then is the Friedman rule optimal? Not necessarily, because of market congestion. Since $B_f < B_i$, lower taxes and lower inflation both have the same effect of moving buyers to the congested formal market. Thus, for sufficiently low levels of $\gamma$ and $\tau$, too many buyers enter the already-crowded formal market and the coordination problem worsens. This hinders aggregate trade and reduces welfare. Optimal policy includes $\{\tilde{\tau}_2, \tilde{\gamma}_2\}$ such that $\tilde{\tau}_1 < \tilde{\tau}_2 < \tilde{\tau}_a$ and $\tilde{\gamma}_2 < \tilde{\gamma}_a$, but this does not guarantee that the Friedman rule becomes optimal. In this case as well, the Ramsey problem finds optimal policy after considering not only tax distortions and the welfare cost of inflation, but also market congestion.

---

16The analysis here is aimed at explaining our simulation results as in section 2.5, for the case where $B_f < B_i$. See the upper right panel of Figure 2.3.
2.5 Calibration and Results

This section calibrates the model to match data from Peru and identifies the optimal rate of inflation. I normalize the number of sellers, s, to unity. Using time diary data, Juster and Stafford (1991) estimate that US residents spend, on the average, 23.9 hours on paid work and 6.8 hours shopping per week. b is set to $\frac{6.8}{23.9}$. This value is adopted for Peru, but considered a lower bound for time spent shopping in that country.\(^\text{17}\)

| TABLE 2.1

<table>
<thead>
<tr>
<th>PERU: Model Parameters and Economic Indicators</th>
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<tbody>
<tr>
<td><strong>Parameters</strong></td>
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<tr>
<td>Period</td>
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Data on tax revenue as a percentage of GDP is retrieved from the World Development Indicators (WDI) database of the World Bank. The average for 2000 to 2004 is used to represent $\tau$. Also collected from the same database is average annual CPI inflation for 2000 to 2005, which is used to represent $\gamma^{12} - 1$. Finally, an estimate of the underground-to-formal sector output ratio is taken from Schneider and Enste (2000) and used to represent $R$.

Specifically, the equations I calibrate are (2.20) to (2.25). Temporarily assume that we know $s_f$ (hence, $s_i$). Then, given the above values for $\tau$, $G$, $\gamma$, $b$, $\beta$ and $\alpha$, equations (2.20) to (2.23) are used to get $b_f$, $m_{ft}$, $\omega_t$ and $Q^g$. The remaining\(^\text{17}\)The appendix includes sensitivity analysis on $b$, $\phi$ and $\alpha$.
requirement is to verify $s_f$. The model is simulated for the value $s_f$ such that the relative size of the underground economy, $R$, equals 0.44, as consistent with (2.25). This completes the calibration.\footnote{For the sake of comparison, I also calibrate the US economy for which data is collected similarly and from the same sources, with $R = .088$, $\tau = .1073$ and $\gamma^{12} = 1.028262$. Note that the percentages included in this calibration exclude home production.}

For the first case, I assume that underground goods are just as good as formal sector goods: $\eta = 1$. In Table 2.2, households send relatively more buyers to the underground sector, causing high market congestion for underground buyers ($B_i < B_f$). Each underground buyer is handed a relatively high sum of money: $\frac{m_u}{m_t} > \frac{b_i}{b}$ (which also means that $\frac{m_u}{b_i} > \frac{m_f}{b_f}$). Since each underground buyer holds more money per capita, they can buy more units and the intensive margin ensures that $R_I > 1$. In order to match the output ratio of $R = .44$, I assign sufficiently few sellers to the underground market. The value of $s_f$ derived is retained for all other simulations for this first case. For each policy set fed into the model, (2.25) is then used to evaluate the new level of $R$, given $s_f$ constant.
<table>
<thead>
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<th>( \eta )</th>
<th>( \frac{b_i}{b} )</th>
<th>( \frac{s_i}{s} )</th>
<th>( \frac{m_i}{m_t} )</th>
<th>( B_i )</th>
<th>( B_f )</th>
<th>( q_i )</th>
<th>( q_f )</th>
<th>( R_I )</th>
<th>( R )</th>
<th>( \frac{p_{ft}}{p_{fft}} (1 - \tau) )</th>
<th>( Q^g )</th>
<th>( G )</th>
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In percentages

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<tr>
<th>( \frac{Q^g}{G} )</th>
<th>( \frac{G}{b_f B_f q_f} )</th>
<th>( \frac{G}{b_f B_f q_f + b_i B_i q_i} )</th>
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<td>.60</td>
<td>10.79</td>
<td>9.92</td>
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</table>
For the second case, I assume that $\eta = .85$. Market congestion is reversed, with the formal sector being more congested for buyers. Market congestion is lower for each underground buyer, requiring lower money allocation to these buyers: $\frac{m_{it}}{m_t} < \frac{b_i}{b}$. Since each underground buyer bears lower money stocks, they buy fewer units per capita compared to formal buyers and $R_I < 1$. Here again, the model is simulated to deliver $s_f$ such that $R = .44$. The value of $s_f$ derived is retained for all other simulations for this second case. The quantity $Q^g$ is real government revenue from seigniorage spending. The values are however small compared to the total government budget, $G$.

### 2.5.1 Optimal Inflation Tax

In solving the for the optimal rate of inflation, I focus on maximizing the steady state value of the return function $U$ of the representative household, for alternative combinations of $\tau$ and $\gamma$. Notice that in the model studied in this paper, there is no transitory response to changes in policy. Instead, there is a single-period adjustment to a new steady state, given a change in policy. Hence, maximizing the steady state value of $U$ is equivalent to solving the Ramsey problem.

The optimal policy set is in Table 2.3. It is important to note that the higher optimal inflation recommended for the economy with $\eta = 1$ is not because that economy has higher tax evasion. In fact, in both economies, I start off with $R = 0.44$ as shown in Table 2.2. Instead, the economy with $\eta = 1$ has higher optimal inflation because of higher market congestion for buyers in the underground market. Inflation does not only bring seigniorage income, it also reduces tax evasion as buyers start to take advantage of lower market congestion in the formal market. That is, if the tax on using cash is high enough, then agents will move to the market where
they can spend it quickly, even if it means that they have to pay taxes. This acts as an additional incentive for seigniorage financing and explains the optimal rate of 42.69% in Peru. This result is robust for marginally inferior underground goods: \( \eta = 1 - \varepsilon \), \( \varepsilon \) being an arbitrarily small positive number. That is, inflation increase the consumption of higher-quality formal sector goods. This result is new, and opposite to that found in Peterson and Shi (2004), where inflation causes households to compromise on the quality of goods they consume. In this case seigniorage contributes significantly to the government budget.\(^{19}\)

When the congestion of buyers is higher in the formal sector, inflation increases \( R \), which acts as a disincentive to seigniorage financing. The optimal inflation rate here is 1.48%, despite the large tax-evading sector. In Figure 2.3, seigniorage income \( (Q^g/G) \) rises with \( \gamma \), as consistent with models with centralized markets. Given \( G \), seigniorage helps alleviate tax financing. However, as \( \gamma \) increases, buyers exit the formal market in search for better matching rates underground \( (B_i > B_f) \). The turnover of taxable goods decline, along with tax revenues, at the going tax rate. Tax revenues decline at a rate faster than the gains from seigniorage, requiring \( \tau \) to rise.

The large variation in optimal inflation is not driven by differences in the quality of goods but rather by differences in market congestion. In the equivalent economy with market clearing, inflation on its own does not alter the extent of tax evasion, irrespective of the relative quality of underground goods. The optimal rate of inflation is therefore unaffected.

\(^{19}\)For different configurations of relative credit-use, Nicolini (1998) finds optimal annual interest rates between 7.34% and 19.17%. In Table 2.3, I convert these estimates into inflation rates using the Fisher equation as in section 2.6. The tax rate in that paper is calibrated differently and not compared. For an economy with 40% output ratio, Koreshkova (2006) estimates the optimal rate of inflation to be approximately 60% per annum. Her base economy is calibrated to US data. Hence, the tax rates are also not comparable.
To better understand the welfare implications of the simulations, I define the index:

\[
\% \Delta c^\gamma = \frac{U^\beta - U^\gamma}{c^\beta} \times 100\% .
\]  

\((2.33)\)

\(U^\gamma\) is the instantaneous return to the household [see equation (2.5)] in an equilibrium with inflation rate \(\gamma\), using a corresponding tax rate that balances the government budget. Similarly, \(c^\beta\) is the composite consumption level at \(\gamma = \beta\).

<table>
<thead>
<tr>
<th>TABLE 2.3</th>
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<tr>
<td>Peru: Optimal Inflation Policy</td>
</tr>
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<table>
<thead>
<tr>
<th>(\eta)</th>
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<tr>
<td>(\tilde{\gamma}^{12} - 1)</td>
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<td>1.48</td>
</tr>
<tr>
<td>(\tilde{\tau}(\tilde{\gamma}))</td>
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<td>12.47</td>
</tr>
<tr>
<td>(% \Delta c^\gamma)</td>
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<td>-.049</td>
</tr>
<tr>
<td>(\frac{Q^\eta}{G})</td>
<td>9.18</td>
<td>.266</td>
</tr>
<tr>
<td>(\frac{Q^\eta}{b_f B_f q_f + b_i B_i q_i})</td>
<td>.78</td>
<td>.023</td>
</tr>
<tr>
<td>(R)</td>
<td>28.53</td>
<td>42.72</td>
</tr>
</tbody>
</table>

Matching Rates (not in %)

| \(B_f\) | 2.0501 | 1.7701 |
| \(B_i\) | 1.1592 | 2.1057 |

Inflation Data %

| 1976-1995 (average) | 525 |
| 2005 | 1.6 |

Optimal Inflation by: %

| Nicolini (1998) | 14.95 to 3.54 |
| Koreshkova (2006) | 60 |
Figure 2.2: Effect of Inflation in Peru (the case of high optimal inflation)
Figure 2.3: Effect of Inflation in Peru (the case of low optimal inflation)
Starting from the Friedman rule, the value $\% \Delta c^\gamma$ denotes the proportional increase in consumption required to compensate the representative household for the transition to a new equilibrium with $\gamma > \beta$. In Table 2.3, the difference in the welfare effects of inflation in the two cases is accounted for by (i) the size of the optimal inflation rate in each case, which affects the extent to which the coordination problem is corrected and (ii) the change in tax distortions that is achieved via the adjustment to the new tax rate.

In the first case ($\eta = 1$), I compare the optimal policy $\{\tilde{\gamma} = 42.69\%, \tilde{\tau} = 9.98\%\}$ and the results in the first column of Table 2.3 to the actual policy $\{\gamma = 2.24\%, \tau = 12.71\%\}$ and its associated results in the first column of Table 2.2. High optimal inflation goes a long way to (i) improve the coordination problem (reduce $|B_f - B_i|$) as well as (ii) reduce tax distortions due to lower taxes. The combined effect is such that a reduction in inflation from this optimal value down to the Friedman rule requires $\% \Delta c^\gamma = -14.09\%$.

In the second case ($\eta = .85$), optimal policy $\{\tilde{\gamma} = 1.48\%, \tilde{\tau} = 12.47\%\}$ is compared to the actual $\{\gamma = 2.24\%, \tau = 12.71\%\}$ in the same fashion. Since the optimal inflation rate is lower than the actual, this (i) worsens the coordination problem by increasing $|B_f - B_i|$. However, buyers return to the congested formal market given less urgency to spend money, thereby increasing the volume of goods under the tax radar. As this happens, a tax rate marginally lower than the actual becomes feasible, thus (ii) reducing tax distortions. The net effect of these two accounts for $\% \Delta c^\gamma = -.049\%$.

The results are worth comparing with those in Nicolini (1998), also calibrated to Peru. Nicolini considers an economy in which an exogenous segment of the formal

\textsuperscript{20}Benabou (1991) reconciles a related index of the welfare effects of inflation to the area under the real money demand function.
sector commodity space has trade conducted only with credit, while another segment has trade strictly with cash. A mutually exclusive set of goods are produced in a tax-evading underground sector and traded only with cash. Over-reliance on tax financing widens the tax burden between the formal-cash goods and the underground-cash goods sectors. On the other hand, over-reliance on seigniorage financing widens the tax burden between the formal-credit and formal-cash goods sectors. Optimal policy employs a mix of both sources of financing. For different relative sizes of the formal-credit sector, he documents variations in optimal inflation in the range shown in Table 2.3.

The results in this paper show that even without any assumptions regarding credit-use, there can be large variations in the optimal rate of inflation if trade is decentralized. This is against evidence provided by Besley and Levenson (1996), which calls to question the role played by credit. They document a high prevalence of Rotating Saving and Credit Associations in Taiwan, allowing informal sector agents access to financial intermediation. Participation rates were found to be as high as 45% at the highest income percentile, which is significant compared to the relative size of the underground economy. The micro foundations of money alone can explain why the Friedman rule fails to be optimal. Augmenting the current framework with credit can lead to higher optimal rates of inflation, perhaps close to estimates found in Koreshkova (2006). Her paper finds that in a country with 40% underground economy, the optimal rate of inflation is about 60% per annum. In contrast, decentralized market conditions can instead support high or low inflation rates even for closely similar rates of tax evasion.
2.6 Discussion

The economic environment examined is directly equivalent to one in which households interact with a centralized market for government bonds. Augmenting the household’s liquidity constraint with bonds, the Euler condition for bonds is \( \frac{\omega_t}{\beta} = \omega_{t+1} (1 + r_{t+1}) \), where \( r_t \) is the net nominal interest rate. Comparing this Euler condition with (2.15), the interest rate is derived as:

\[
    r_t = B_f \frac{\lambda_f}{\omega_t} = B_i \frac{\lambda_i}{\omega_t} \equiv \frac{\gamma - \beta}{\beta}.
\]  

(2.34)

The Friedman rule involves setting \( \gamma \) to \( \beta \), or alternatively, \( r_t \) to zero.

In the environment studied, the allocation of sellers between sectors is exogenous. The configuration \( \{s_f, s_i\} \) is nevertheless consistent with the equilibrium. Due to take-it-or-leave-it offers by buyers, sellers exit each trade match with zero net surplus in both sectors. Households are therefore indifferent in the allocation of sellers between sectors when I endogenize the seller allocation decision. Using (2.16) and (2.17), it is easy to show that the first order condition for \( s_f \) holds true for all values of \( s_f \in [0, s] \). The result is an infinite set of equilibria, including the point \( \{s_f, s_i\} \) used in section 2.2. Employing Nash bargaining may narrow the set of equilibria. Such an extension is likely to strengthen the results discussed in this paper. I conjecture that in response to changes in the inflation rate, sellers are likely to move in the same direction as buyers, further strengthening the results on the extensive margin.

In the extended environment with both seller and buyer allocations being endogenous, appropriate restrictions can be imposed on the model to prevent corner solutions. An example of such restrictions can be found in Liu and Shi (2006),
where the authors present a related environment with two countries. They employ matching functions with local congestion of buyers in the sense that, by entering a goods market, a buyer increases congestion more for other buyers from his own country than for the buyers from the other country. The goods market is also not be fully integrated for sellers. In particular, transaction costs make more sellers sell in their domestic currency area than in the foreign currency area. These restrictions act as restraints on their model’s behaviour and prevents corner solutions in which goods market trade occurs exclusively in one country.

This paper generates endogenous micro level trade ratios including the quantity-per-trade ratio and the relative price. A somewhat related paper in the literature is McLaren (1998). He considers a non-monetary economy with markets for imported goods. There are several markets, each for a specific class of imported goods. Depending on the tax rate and the concentration of tax inspectors in a given market, traders decide either to import legally and pay the associated taxes or to smuggle at a risk of detection. Quantity per importer is fixed and only the choice of sector is endogenously influenced by policy. In equilibrium, traders in the market for a particular class of good are all simultaneously legitimate importers or all smugglers. This is an outcome of market clearing. Although separate prices can be derived for the two sectors, only one is operational for each commodity class. He then studied the optimal tax and audit rates in a Ramsey-type equilibrium. The current paper on the other hand endogenizes production quantities, prices and sector choice, and these depend on fundamentals as well as economic policy, including money.

A possible extension is to introduce capital into the environment examined in this paper. Notice that the model presented above can be interpreted as one with constant returns to scale production technology involving labour: $q_{jt} = l_{jt}$, $j = f, i$, where $l_{jt}$ is labour input. In this case, the disutility of production reverts
to disutility of labour: $\Phi(l_{jt})$. The introduction of capital simply involves employing a more general production function and an appropriate capital accumulation equation. This extension will facilitate interesting dynamic and business cycle applications. One is however compelled to take a stand on which good(s), formal or underground, can be accumulated into capital, if not both. How exactly are they combined in the constitution of a uniform capital stock?

2.7 Conclusion

There are two main conclusions to draw from this paper. Firstly, the data fails to support the conventional wisdom that higher inflation strictly reduces the size of the underground economy. There are data points for which decreases (increases) in the underground-to-formal sector output ratio were indeed accompanied by higher (lower) inflation. However, there are just as many data points that rather suggest the reverse. I develop a theoretical framework that explains the evidence. The solution I propose is that the relative congestion of the formal and underground markets for buyers can be different across countries. Where the formal market is more congested for buyers, inflation causes households to compromise on the quality of goods they consume and commit more money and more buyers to the underground sector. In this case, underground output increases both on the intensive and extensive margins relative to the formal sector. When the underground sector is more congested for buyers, inflation achieves the opposite result. In short, inflation can move underground output in both directions, as consistent with the data.

The second conclusion is as follows. In the presence of an underground sector, tax distortions are socially costly, while inflation presents the usual welfare conse-
quences. If both sector markets are characterized by Walrasian market clearing, the task facing a benevolent government is to find the best trade-off between the deadweight loss from tax financing and that from seigniorage financing. With bilateral trade however, optimal policy also seeks to correct the coordination problem that exists when market congestion is unbalanced between sectors. When the underground market is more congested for buyers, the benevolent government reduces the formal sector tax rate to encourage buyers back into the formal sector. Optimal policy thus involves high seigniorage financing and low taxes. I find optimal inflation rates as high as 42.69% per annum for Peru. Although this rate is lower than the rates observed in that country from the mid 1970s to the mid 1990s, it does offer a general explanation for the high rates of inflation in some poor countries within the context of optimal public finance policy. One way to summarize this result is that, if the tax on using cash is high enough, agents will move to the market where they can spend it quickly, even if it means that they have to pay taxes.

When the formal sector is more congested for buyers, optimal policy seeks to reduce the overcrowding of buyers in the formal sector to improve the coordination problem. This requires high taxes combined with low seigniorage spending. Nevertheless, the Friedman rule fails to be optimal because when the rate of inflation is too low, there is no urgency to spend money. Buyers languish in the already congested formal market, thereby worsening trade frictions. For the relevant configuration of the model, I generate an optimal annual inflation rate of 1.48% for Peru, which is close to the rate observed in that country in 2005. In Peru, the size of underground output relative to the formal sector is estimated at 44%. With such high rates of tax evasion, a familiar assertion in the literature calls for high reliance on seigniorage financing. Further, Cooley and Hansen (1991) showed quantitatively that when inflation tax revenue is replaced by revenue from other distortionary
taxes, the welfare effect is negative. The results in this paper show that the optimal inflation rate can be far lower than suggested in the literature, even though formal sector taxes are distortionary.

The results of this paper must not be taken to imply that within the range of low to high inflation, only the extreme policies are optimal. The environment examined ignores other important considerations for inflation tax, including the cost of administering alternative forms of taxation, the availability of other stores of value apart from money and the redistributive implications of inflation. On the theory front, I make significant inroads in integrating fiscal policy instruments into the literature on the micro foundations of money. I showed that the model is adaptable for the inclusion of capital, thus allowing the familiar dynamic and business cycle analysis. The environment proposed is flexible and permits applications to other sectoral divisions of the economy such as manufacturing versus services. Instead of matters concerning two-sector economies, further extensions may consider two-country applications.
Appendix

The household solves:

$$v(m_t) = \max_{b_{jt}, m_{jt}, q_{jt}, m_{t+1}, j=f,i} c_{ft} + \eta c_{it} - s_f S_{ft} \Phi(Q_{ft}) - s_i S_{it} \Phi(Q_{it})$$

$$+ \beta Ev(m_{t+1}) + b_f \mathcal{B}_f \lambda_{ft} \left[ \frac{m_{ft}}{b_{ft}} - \frac{\Phi(q_{ft})}{\Omega_t} \right] + b_i \mathcal{B}_i \lambda_{it} \left[ \frac{m_{it}}{b_{it}} - \frac{\Phi(q_{it})}{\Omega_t} \right]$$

$$+ \omega_t [m_t + s_f S_{ft} X_{ft} + P_t Q_t^g + s_i S_{it} X_{it} - b_f \mathcal{B}_f x_{ft} - b_i \mathcal{B}_i x_{it} - m_{t+1}].$$

The Euler conditions (2.15) to (2.18) follow directly from the above setup. Equations (2.20), (2.21) and (2.22) are arrived at as follows. If money is valued, then $$\lambda_{jt} \geq 0, j = f, i$$, hence, $$x_{jt} = \frac{\Phi(q_{jt})}{\Omega_t}$$, with $$\frac{dx_{jt}}{dq_{jt}} = \frac{\Phi(q_{jt})}{\Omega_t} \omega_{jt}, j = f, i$$. This substituted into (2.16), (2.17) and (2.18) yields:

$$1 - \tau = \left[ \lambda_{ft} + \omega_t \right] \frac{\Phi(q_{ft})}{\Omega_t} \phi_{q_{ft}}.$$

$$\eta = \left[ \lambda_{it} + \omega_t \right] \frac{\Phi(q_{it})}{\Omega_t} \phi_{q_{it}}$$ and

$$\mathcal{B}_f \left[ (1 - \tau) q_{ft} - (\lambda_{ft} + \omega_t) \frac{\Phi(q_{ft})}{\Omega_t} \right] = \mathcal{B}_i \left[ \eta q_{it} - (\lambda_{it} + \omega_t) \frac{\Phi(q_{it})}{\Omega_t} \right].$$

With a constant money growth rate $$m_{t+1} = \gamma m_t$$, the value of money declines at the growth rate of money: $$m_{t+1} \omega_{t+1} = m_t \omega_t$$. Thus the Euler for money gives
\[ \gamma \omega_t m_t = \beta \omega_{t+1} m_{t+1} + \beta B_f \lambda_{ft+1} m_{t+1} \]. Rearranging,

\[ B_f \lambda_{ft} = B_i \lambda_{it} = \frac{\gamma - \beta}{\beta} \omega_t \]. (2.35)

Due to the restriction that \( \gamma \geq \beta, \lambda_{jt} \geq 0, j = f, i \), the cash-and-carry constraints bind in all transactions:

\[ \frac{m_{jt}}{b_{jt}} \omega_t = q^\phi \omega_t , \ j = f, i \]. (2.36)

Substituting (2.35) and (2.36) in the three conditions and imposing symmetry (\( \omega_t = \Omega_t \) and \( Q_{jt} = q_{jt} , j = f, i \) etc), we have:

\[ 1 - \tau = \frac{\gamma - \beta}{\beta B_f} \omega_t \frac{m_{ft} \phi}{b_{ft} q_{ft}} \] (2.20)

\[ \eta = \frac{\gamma - \beta}{\beta B_i} \omega_t \frac{m_{it} \phi}{b_{it} q_{it}} \] (2.21)

\[ B_f \left[ (1 - \tau) q_{ft} - \frac{\gamma - \beta}{\beta B_f} \omega_t \frac{m_{ft}}{b_{ft}} \right] = B_i \left[ \eta q_{it} - \frac{\gamma - \beta}{\beta B_i} \omega_t \frac{m_{it}}{b_{it}} \right] . \]

Simplifying this last condition using (2.20) and (2.21) gives:

\[ \frac{m_{it}}{b_{it}} \frac{m_{ft}}{b_{ft}} = \frac{\gamma - \beta + \beta B_f}{\gamma - \beta} . \] (2.22)

At the government side of the model, (2.23) follows easily from (2.13) and (2.14).
The derivation of the ratios are explained in the paper. The ratio of (2.20) and (2.21) gives:

$$\frac{B_i}{B_f} = \frac{1 - \tau q_{ft}}{\eta q_{it}}. \quad (2.37)$$

Simplifying further gives

$$\frac{b_i}{b_f} = \frac{s_i}{s_f} \left[ \frac{q_{it}}{1 - \tau q_{ft}} \right]^{1/\alpha},$$

or:

$$\frac{b_i}{b_f} = \frac{s_i}{s_f} \frac{q_{it}}{\eta q_{it}} \left[ \frac{\eta q_{it}}{1 - \tau q_{ft}} \right]^{1/\alpha}. \quad (2.38)$$

Notice that

$$R = \frac{b_i}{b_f} \frac{B_i}{B_f} \frac{q_{it}}{q_{ft}},$$

which involves the product of (2.37), (2.38) and \(R_I\). The outcome is (2.25). Finally, (2.31) follows from (2.22) and (2.32) from (2.23).

In the equivalent model with centralized market clearing and cash-in-advance, the household’s problem is

$$v(m_t) = \max_{c_{jt},l_{jt},m_{jt},m_{t+1},j=f,i} \left( 1 - \tau \right) c_{ft} + \eta c_{it} - \Phi (l_{ft}) - \Phi (l_{it}) + \beta Ev(m_{t+1})$$

$$+ \lambda_{ft} [m_{ft} - p_{ft} c_{ft}] + \lambda_{it} [m_{it} - p_{it} c_{it}]$$

$$+ \omega_t [m_t + W_{ft} l_{ft} + W_{it} l_{it} - p_{ft} c_{ft} - p_{it} c_{it} + \Pi_{ft} + \Pi_{it} - m_{t+1}] ,$$

where \(l_{jt}\) is labour, \(W_{jt}\) the wage rate and \(\Pi_{jt}\) is firm profit in sector \(j\). Firms solve

$$\Pi_{jt} = \max_{l_{jt}} p_{jt} q(l_{jt}) - W_{jt} l_{jt}, \quad j = f, i,$$

where \(q(l_{jt}) = l_{jt}\). The government budget constraint is

$$G = \tau c_{ft} + Q_{ft}^g,$$

where \(Q_{ft}^g = (\gamma - 1) \frac{M}{p_{ft}}\). Market clearing requires that

$$q_{ft} = c_{ft} + Q_{ft}^g$$

and \(q_{it} = c_{it}\). Given the linear nature of the firm’s problem, the auctioneer sets

$$p_{jt} = W_{jt},$$

while the first order conditions of the household’s problem are

$$\lambda_{ft} = \lambda_{it}, \quad 1 - \tau = (\lambda_{ft} + \omega_t) p_{ft}, \quad \eta = (\lambda_{it} + \omega_t) p_{it}$$

and \(\omega_t W_{jt} = \Phi' (l_{jt})\).
In equilibrium:

\[
\begin{align*}
\frac{p_{it}}{p_{ft}} &= \frac{\eta}{1 - \tau}, \\
\frac{q_{it}}{q_{ft}} &= \left[ \frac{\eta}{1 - \tau} \right]^{\frac{1}{\phi - 1}}. 
\end{align*}
\]

A summary comparison of the models is in Table 2.4.

| TABLE 2.4 |
|---|---|---|
| **Comparison of Models** | **Centralized** | **Decentralized** |
| **Price Ratio** | $\frac{p_{it}}{p_{ft}}$ | $\eta \frac{1}{1 - \tau}$ | $\left[ \frac{\gamma - \beta (1 - B_f)}{\gamma - \beta (1 - B_i)} \right]^{1 - \frac{1}{\phi}}$ |
| **Quantity Ratios** | $R_I$ | n.a | $\left[ \frac{\gamma - \beta (1 - B_f)}{\gamma - \beta (1 - B_i)} \right]^{\frac{1}{\phi}}$ |
| | $R$ | $\left[ \frac{\eta}{1 - \tau} \right]^{\frac{1}{\phi - 1}}$ | $\frac{\eta_i}{\eta_f} \left[ \frac{\eta}{1 - \tau} \right]^{\frac{\alpha}{\alpha - 1}} R_f^{\frac{1}{\alpha - 1}}$ |
Data

1. Data on the relative size of the underground economy \((R)\) was retrieved from Schneider and Enste (2000). The other ratios, \(R_t\) and \(\frac{p_t}{p_f}\), are derived using these values of \(R\) and formulas outlined in section 2.4.

2. Figure 2.4 does not show country names against the data points due to overcrowding. The data is available upon request. The regression \(\Delta UE = \beta_0 + \beta_1 \Delta Inf.\) gives \(\beta_0 = 1.2486, \beta_1 = 0.0872, R^2 = 0.0007\) and \(p\)-values of 0.7913 and 24.9586 respectively.

3. In Table 2.5, simulations with \(p_{it} > p_{ft}\) must be read with caution. As explained in section 2.4, the relevant price ratio is \(\frac{p_{it}}{p_{ft}} (1 - \tau)\), which is less than unity in all cases.
TABLE 2.5

Sensitivity Analysis

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Figure 2.4: Changes in Inflation and Changes in Underground Economy

See point 2 on page 71 for comments on countries associated with these data points.
Figure 2.5: Inflation and the Underground Economy
Figure 2.6: Government Spending and Underground Economy

[Graph showing a scatter plot with countries represented by different symbols and lines indicating the relationship between government spending and underground economy as a percentage of reported GDP.]
Figure 2.7: Taxation and the Underground Economy

![Graph showing the relationship between Tax Revenue as a percentage of reported GDP and Undergr. Econ. (as a percentage of reported GDP)]
Chapter 3

A Flat Tax Reform and Informal Production

One of the highly debated economic policy questions in recent history concerns the proposed restructuring of the personal tax system in the United States (US) from the current progressive design to a flat tax regime. Proponents argue that flat tax is fair, simple, eliminates the current double taxation of capital income and promotes capital accumulation. Opponents reiterate that the likely effect of this policy is that it will worsen existing inequalities in earnings and wealth distribution. Several authors have proposed elaborate economic models that mimic properties of the US economy aimed at providing answers to some of these hypotheses. However, a common assumption in these papers is that working agents are automatically presumed to be completely covered under the tax radar. This assumption prevents the models from capturing the realistic feature that individuals with different income levels or facing different marginal tax rates may have different incentives to evade taxes. In this paper, I study quantitatively the effect of a revenue-neutral flat tax reform on
the extent of tax evasion and the distribution of wealth and earnings in the United States.

3.1 Introduction

Economists have long identified the tax burden as a statistically significant determinant of tax evasion and informal production in several countries.\(^1\) Under the existing progressive personal tax regime in the United States, marginal tax rates differ widely for taxpayers at different levels of the earnings ladder. As a result, marginal incentives for tax evasion vary considerably. It is therefore not immediately clear whether environments with heterogeneous agents but no tax evasion completely describe the potential effects of tax reforms. The aim of this paper is to study the effect of a revenue-neutral flat tax reform on the relative size of the informal sector in the U.S.

The working definition of the informal sector is that production in this sector is labour intensive and producers evade taxes. Further, there are no competitive markets for employing informal sector inputs. While agents may perform similar tasks and activities in the production of goods in both sectors, the resulting output is taxed in one sector but not in the other.

As described by Hall and Rabushka (1995), the suggested reforms involve phasing out (i) the tax on corporate profits and (ii) the progressive tax on personal income from all sources (such as labour wages, interest, dividends etc.). These are replaced by a fixed marginal tax rate imposed on the portion of an agent’s labour earnings that are in excess of a specified exemption amount. For an agent earning labour income beyond the exemption level, his marginal tax rate exceeds his

\(^1\)See Loayza (1996) and Enst and Schneider (2000).
average tax rate due to the exemption amount. In other words, the proposed flat
tax system actually contains an element of progressive taxation. Further, the new
system taxes all capital income at a flat rate equal to the fixed marginal tax rate
on wages. Since there is no exemption amount for capital income, the average tax
rate indeed equals the marginal tax rate, eliminating all elements of progressive-
ness. Capital income in this case is defined to include all corporation income paid
to shareholders after allowing for deductions such as wages, other input costs and
even capital accumulation.

Addressing the implications of tax reform in a heterogenous agents general equi-
librium model with informal production is not an easy task. This is partly because
such an analysis requires the construction of models capable of matching (i) fea-
tures of pre-reform empirical earnings or wealth distributions, while, in our case,
also matching (ii) the pre-reform size of the informal sector relative to the formal
sector. The model economy considered is populated by a unit measure of infinitely-
lived agents. Although there is no aggregate or individual uncertainty, agents have
permanent differences in their labour productivity levels. As a result, the dynam-
ics confronting each individual agent do not coincide with those of the aggregate
economy in equilibrium, thereby enabling a look at earnings and wealth distribu-
tions. There is a uniform durable consumption commodity that is produced in two
sectors, formal and informal. A solitary formal sector firm hires labour and capital
at competitive rates for production. In the informal sector however, there are a
unit measure of firms, each of which produces goods using a diminishing marginal
product function involving labour only. More specifically, each informal sector firm
is individually owned and operated by one of the unit measure of agents. Each
agent commits part of his own labour supply to his privately operated informal
I find that a flat tax reform leaves the informal-to-formal sector output ratio unchanged. The mechanism driving this result is derived from the counteractive response of agents at the two ends of the earnings distribution in relation to the reforms. Firstly, the implication of the above model characteristics is that agents take the formal sector wage rate as given, while having control over the marginal return to their informal labour input. Agents in the lower spectrum of the productivity ladder each supply minimal effective labour units and as a result, their marginal return to effective labour is high in the informal sector compared to the formal sector wage rate. Such agents supply most of their labour to the informal sector. A flat tax reform increases the marginal tax rate on labour income for these low productivity agents. This further reduces the effective return to formal sector employment for these agents, urging them to substitute further towards informal production. Thus, from the lower end of the earnings distribution, a flat tax causes an increase in tax evasion. The second part of the mechanism concerns the upper end of the earnings ladder. High earners reduce their participation in informal production under a flat tax regime since a flat tax rate is lower than the pre-reform marginal tax rate for such agents. Quantitatively, I find that these two effects cancel out on the aggregate and the informal-to-formal sector output ratio remains relatively unaffected by the flat tax plans examined in the paper.

A second result is that I uncover a theoretical distribution for informal sector earnings characterized by a much lower concentration compared to the formal sector earnings distribution. The formal sector earnings distribution was replicated by calibrating productivity levels of individual agents. Yet, the implied theoretical

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2 A closely related assumption can be found in Aiyagari (1995) in an idiosyncratic risk model involving market versus household production.
distribution of informal sector earnings for the same population is completely different because of the additional assumption that there is no competitive market for informal sector inputs. As consistent with the literature, a flat tax reform worsens the distribution of formal sector earnings. In contrast, a flat tax reform reduces further earnings inequality in the informal sector. This is because, as stated earlier, high earners reduce their evasive activities, while low earnings increase theirs under flat tax.

This paper contributes to the literature on tax reform and the elimination of the double taxation of capital income, along side Domeij and Heathcote (2004), Díaz-Giménez and Ríos-Rull (2003). It also adds to the literature on informal sector, underground economy and home production, with notable contributions including Aiyagari (1995), Parente, Rogerson and Wright (2000) and Caballé and Panadés (2002). The next section presents the model economy. Section 3.3 presents the calibration and section 3.4 the results of a flat tax reform. I conclude in section 3.5.

3.2 Economic Environment

There is a uniform durable consumption commodity that can be produced in at least one of two sectors, formal and informal. These sectors are denoted by the subscripts \( f \) and \( i \) respectively. Heterogeneity originates from permanent labour productivity differences between agents. In this regard, let the subscript \( e \) denote an agent’s type.
### 3.2.1 Agents

The economy is inhabited by a large number of infinitely lived agents, their total measure being unity. Each period, each agent is endowed with one unit of time, which he allocates between work and leisure. An agent of type $e$ maximizes:

$$\sum_{t=0}^{\infty} \beta^t U(c_{et}, l_{et}) \mid s_e, k_{e0},$$

where $0 < \beta < 1$ is his subjective time discount factor, $c_{et}$ is his consumption level and $l_{et}$ his labour supply in period $t$, while $k_{e0} \in (0, \infty)$ is his stock of capital at date zero.\(^3\) All agents have identical utility functions, $U$, defined over consumption and leisure $(1 - l_{et})$. Similar to Conesa and Krueger (1999) and Ventura (1999),

$$U(c_{et}, l_{et}) = \frac{c_{et}^\eta (1 - l_{et})^{1-\eta}}{1 - \sigma} \cdot \sigma > 0, \eta \in (0, 1),$$

where $\sigma$ is the intertemporal elasticity of substitution and $\eta$ is the share parameter for consumption in the composite consumption function. To cut down on notation, I drop the time subscript for most of what follows and instead use primes to indicate next-period variables.

Over time, agents remain identical except for their stock of capital and their labour productivity levels. Possible values of labour productivity are within the set $S \in \{s_1, s_2, ..., s_5\}$, $0 < s_e < \infty \forall e$ and $s_{e+1} > s_e$, $e = 1, 2, 3, 4$. This productivity level impacts labour supply multiplicatively and effective labour supply is $s_e l_e$. There is no individual or aggregate uncertainty.

\(^3\)Note that the labour supply is elastic in the current model. Part of leisure can be interpreted alternatively as home production, which is not taxed under most tax codes. Thus, this model distinguishes home production from the activities in the informal sector.
The constraints facing an agent of type $e$ are:

$$s_el_e \geq l_{ef} + l_{ei}, \quad (3.2)$$

$$k'_e + c_e \leq k_e + y_{ef} + y_{ei} \quad \text{and} \quad (3.3)$$

$$c_e \geq 0, \quad k_e \geq k, \quad l_{ej} \geq 0, \quad j = f, i.$$ 

An agent divides his effective labour supply into two for allocation to the two sectors. $l_{ef}$ and $l_{ei}$ are individual labour supplied to the formal and informal sectors respectively. In (3.3), $y_{ej}$ is income from sector $j$, $j = f, i$, both of which are perfectly substitutable in consumption.\(^4\) Specifically,

$$y_{ef} = [1 - \tau_b (\cdot)] w_{lef} + [1 - \bar{\tau}_b (\cdot)] r_{ke} + \epsilon_b (\cdot) \quad \text{and} \quad (3.4)$$

$$y_{ei} = A_i l_{ei}^{1-\alpha}, \quad A_i \in [0, \infty), \quad \alpha \in (0, 1), \quad (3.5)$$

where

$$\bar{\tau}_b (\cdot) = \begin{cases} \tau_b (\cdot), & \text{under progressive taxation} \\ 0, & \text{under a flat tax system}. \end{cases}$$

In the above, $w$ is the wage rate in the formal sector and $r$ the interest rate. An agent’s marginal tax rate is $\tau_b$ and $\epsilon_b$ is his tax rebate. The subscript $b$ indicates the agent’s tax bracket, which depends on his reported incomes: $w_{lef}$ and $r_{ke}$. Clearly, $\tau_b$ and $\epsilon_b$ are both fixed for each given tax bracket.\(^5\) Under progressive taxation, capital income $r_{ke}$ is treated in the exact same way as labour income. Hence, the

\(^4\)Income $y_{ei}$ should not be confused with home production, as the latter is unlikely to be perfect substitutes for market goods. Instead, the leisure component of the model $(1 - l_e)$ can be interpreted to represent home production.

\(^5\)The purpose of $\epsilon_b$ is to take care of the exemption level as well as progressive taxation. An agent’s total formal income $(w_{lef} + r_{ke})$ is taxed at the marginal rate $\tau_b$. After this, a rebate $\epsilon_b$ is given to take account of the fact that the first few units of income may be tax exempt or taxed at a lower marginal rate.
marginal tax rate is a function of the agent’s total reported income: \( w_{ef} + r k_{ef} \).
For now, assume that the tax on capital income under a flat tax system is zero.\(^6\) By implication, the tax bracket of an agent, \( b \), depends only on his labour income under a flat tax regime. \( y_{ei} \) is the production function in each privately operated informal firm, while \( A_i \) is a constant that is the same for all agents. It may be realistic to assume that informal production is more labour intensive than the output produced by the official economy [see Tanzi (1999)]. I adopt the extreme position by assuming that informal production requires labour input only.

Equations (3.2) and (3.5) imply that there is zero mobility of informal sector labour across informal firms. Similar to Aiyagari (1995), each agent can use only part of his own labour for allocation to his informal production and he cannot hire labour services from other agents.\(^7\) If \( A_i > 0 \), then \( \frac{dy_{ei}}{dl_{ei}} \bigg|_{l_{ei} \to 0} = \infty \). Hence, \( l_{ei} > 0 \) for every agent in every period and in every state.

As is the tradition in models with heterogeneous agents, it is useful to rewrite the agent’s problem in recursive form. The associated Bellman equation is:

\[
V(s_e, k_e) = \max_{k_{e}'l_{e}, l_{ef}, l_{ei}} U(c_e, l_e) + \beta V(s_e, k_{e}') ,
\]

subject to (3.1) to (3.5) and the non-negativity constraints. One-period ahead variables are indicated with primes and the dependence of the value function on aggregate variables has been suppressed.

\(^6\)I include the flat tax on capital income on the firm’s side of the model.

\(^7\)Aiyagari (1995) does not explicitly assume the non-existence of a competitive market for labour supplied to household production. Instead, he imposes a constraint limiting labour commitment to the household sector between zero and the endowment amount. Heterogeneity in his model is derived from shocks to the productivity of the household production technology of individual agents. However, agents with transitory high productivity of their household production know-how are unable to hire additional labour inputs from the market to maximize output. This restriction is equivalent to the one imposed in this paper.
3.2.2 The Formal Firm

There is a single official sector firm that hires labour and capital competitively and pays wages and interest equal to the marginal product of each input. The formal production technology is:

\[ Y_f = K^\alpha L_f^{1-\alpha} - \delta K, \]

where \( L_f \) and \( K \) are total effective units of labour and total capital demanded by the formal firm. \( \delta \) is the rate of depreciation of capital. The constant return formal technology and diminishing marginal product informal technology are similar to those used in the market and home production model of Aiyagari (1995).8,9

Since effective labour services are homogeneous, there is a single wage rate per efficient unit of labour, given by:

\[ w = (1 - \alpha) K^\alpha L_f^{-\alpha}. \]

Interest paid to agents per unit of capital is given by:

\[ r = (1 - \tau_j) \left[ \alpha K^{\alpha-1} L_f^{1-\alpha} - \delta \right], \]

where \( \tau_j = \tau_k \) under progressive taxation and \( \tau_j = \tau_{Flat} \) under a flat tax system. Notice that \( \tau_k \) and \( \tau_{Flat} \) are aggregate variables in the sense that they are independent of the agent’s type. The above formulation simulates the controversial double taxation of capital income in the existing progressive tax system since interest is also taxable under the agent’s problem. This is one of the problems that a flat tax

---

8 The diminishing marginal product informal technology is similar to the representative agent model in Koreshkova (2006).

9 Further, in comparison to the formal sector production function, the informal technology \( A_i r_{ei}^{1-\alpha} \) implies that there exists a fixed factor that distinguishes between the formal and informal sectors.
reform is aimed at resolving. Under a flat tax regime, capital income net of depreciation allowance is taxed at a flat rate, which equals the flat marginal tax rate on labour income and no exemption amount is considered for individual agents. This is equivalent to applying this tax directly to the marginal product of capital at the firm level rather than at the household level.

Each agent views $r$ and $w$ as the pre-tax marginal return to capital and effective labour supplied to the formal firm. Agents take these rates as given. However, they have control over the marginal return to their informal labour input. A low productivity agent supplies few effective labour units. His marginal return to effective labour is high in the informal sector compared to the formal sector wage rate. Such an agent places more emphasis on informal production. In fact, for sufficiently low productivity levels, an agent opts out of formal employment by self-selection, resulting in an endogenous limited participation in formal sector employment. Some of these model properties are consistent with empirical observations in Tanzi (1999) that informal sector workers are often less educated and less experienced than their official sector counterparts, or are mostly past their productive prime (such as retirees). This is further supported by a related model on the quality of managerial talent by Amaral and Quintin (2006). They show theoretically that under certain conditions, the quality of managerial talent in the formal sector is at least as high as that in the informal sector.
I describe the timing of events next. Early in the period, an agent takes stock of his capital. $K$ is the aggregate stock of capital and $\Gamma (k_e)$ is the distribution of capital among agents. Both aggregate variables are public knowledge. Taking into account his own productivity level, the agent decides on $l_e$, being the number of hours to work in the period. For an agent with productivity level $s_e$, these hours are automatically transformed into $s_el_e$ effective labour units. Given that the wage rate is publicly known, he chooses how to divide his labour units between the sectors. Next, there is production, and incomes and taxes are paid. The agent makes a consumption decision and the period ends.\textsuperscript{10} Tax revenues received by the government, net of rebates, are spent on $G$, a public service such as national defence. For simplicity, this expenditure does not enter the agent’s utility function or the production functions.\textsuperscript{11}

\textsuperscript{10}The fact that consumption occurs after incomes are paid ensures that the initial distribution of wealth is irrelevant for the long run equilibrium. The essential assumption is that $K_0 > 0$. That is, at least one agent has positive capital stock at date zero.

\textsuperscript{11}See Loayza (1996) and Dessy and Pallage (2003) for alternative formulations that integrate productive public services.
3.2.3 The Market

Although there are no shocks, agents are heterogeneous with respect to their labour productivity level and their capital holdings. To define an equilibrium requires that I establish a probability measure $\Gamma(k_e, e)$ over each element of the state space $(k_e, e)$. $\Gamma(k_e, e)$, then, is the percentage of agents in that state. However, one of these indexes can be eliminated since agents of a given type, $e$, all hold equal stocks of capital in equilibrium. A stationary equilibrium consists of a sequence of prices $\{r, w\}$, a set of decision rules $k'_e(k_e)$, $l_{ef}(k_e)$, $l_{ei}(k_e)$ and $c_e(k_e)$, and the time-invariant distribution $\Gamma^*(k_e)$ such that:

1. Given the sequence of prices, the individual decision rules solve (3.6).

2. The formal sector input markets clear:

\[
L_f = \sum_{k_e} \Gamma^*(k_e) l_{ef} \quad \text{and} \quad K = \sum_{k_e} \Gamma^*(k_e) k_e.
\]  

(3.7)  

(3.8)

3. The goods market clears:

\[
G + \sum_{k_e} \Gamma^*(k_e) c_e = Y_f + \sum_{k_e} \Gamma^*(k_e) y_{ei}.
\]  

(3.9)

4. $\Gamma^*$ is given by:

\[
\Gamma^*(k'_e) = \sum_{k_e \in \psi(k'_e)} \Gamma^*(k_e),
\]  

(3.10)

where $\psi(k'_e) = \{k_e : k'_e \in K'_e(k_e)\}$. 

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5. The government budget constraint is satisfied:

\[ G = \sum_{k_e} \Gamma^* (k_e) \left[ \tau_b w l_{ef} + \tau_b r k_e - \epsilon_b \right]. \tag{3.11} \]

Given that there is no individual uncertainty, the law of motion for the distribution of capital coincides with the exogenous distribution of types in equilibrium.

### 3.2.4 Characterizing the Equilibrium

For a given tax bracket \( b \), the first order conditions are:

\[ c_e = A_i \eta s_e \frac{(1 - \alpha)}{1 - \eta} l_{e_i}^{-\alpha} (1 - l_e) \quad \text{and} \tag{3.12} \]

\[ (1 - \tau_b) w \leq (1 - \alpha) A_i l_{e_i}^{-\alpha}. \tag{3.13} \]

Equation (3.12) is the trade-off between consumption and informal labour supply. This condition holds with equality since in each period, \( c_e, l_{e_i} > 0 \). (3.13) is the condition that describes the optimal allocation of labour between the two sectors. For a given marginal tax rate \( \tau_b \), if (3.13) holds with equality, then \( l_{ef} > 0 \). Agents for whom (3.13) is slack choose \( l_{ei} = s_e l_e \) and \( l_{ef} = 0 \). An agent is never too wealthy or too unproductive to work at least in the informal sector. The above conditions are important because they provide the solution to the intratemporal optimization problem of each agent, being the choice between \( l_e, l_{ef} \) and \( l_{ei} \).

\[ ^{12} \text{The appendix includes computational details.} \]
3.3 Calibration

The calibration exercise involves choosing 7 parameters to match 7 targets. The first aim is to edge as close as possible to the observed earnings distribution in the US. Specifically, the empirical earnings distribution quintiles imply 5 targets, which I calibrate by choosing the 5 productivity levels \( \{ s_1, s_2, s_3, s_4, s_5 \} \). Since agents of a given productivity type each hold the same quantity of capital in equilibrium, they also each receive equal amounts in interest income. Secondly, agents of a given productivity type each supply the same amount of labour to the formal sector in each period, hence, receive equal amounts in wages. By implication, formal earnings \( y_{ef} \) is equal among all agents of the same type.\(^{13}\) Thus, the economy-wide reported earnings distribution contains only 5 possible earnings levels given by \( \{ y_1f, y_2f, y_3f, y_4f, y_5f \} \). The only feasible way to distribute these over a range of quintiles is to set \( \Gamma(e) \equiv \Gamma(k_e) = \{ \frac{1}{5}, \frac{1}{5}, \frac{1}{5}, \frac{1}{5}, \frac{1}{5} \} \). Next, I normalize \( s_5 \) to unity, and then choose \( s_1, s_2, s_3 \) and \( s_4 \) such that the resulting earnings distribution \( \{ \frac{y_1f}{5Y_f}, \frac{y_2f}{5Y_f}, \frac{y_3f}{5Y_f}, \frac{y_4f}{5Y_f}, \frac{y_5f}{5Y_f} \} \) approximates the empirical earnings quintiles in the US, where \( Y_f \) is average official earnings:\(^{14}\)

\[
\widetilde{Y}_f = \sum_{k_e} \Gamma^*(k_e) [w_{ef} + rk_e] . \tag{3.14}
\]

The second aim of the calibration is to approximate the relative size of informal production in the US. Schneider and Enst (2000) estimate the informal-to-formal sector output ratio in the US to be about 8.8%\(^{15}\). To obtain this sixth target, I

\(^{13}\)Informal earnings \( y_{ei} \) are also equal among all agents of the same type.

\(^{14}\)Normalizing \( s_5 \) to unity rather than \( s_1, s_2, s_3 \) or \( s_4 \) is convenient. It limits the equilibrium aggregate capital stock to a low value, allowing me to use a low capital grid size during value function iteration without compromising accuracy.

\(^{15}\)As explained in section 3.2, part of leisure \((1 - l_e)\) can be interpreted alternatively as home production. In relation to the calibration, the estimate of the size of informal output in the US
define the ratio:

\[ R = \frac{1}{Y_f} \sum_{k_e} \Gamma^*(k_e) y_{ei} . \]

I choose \( A_i \) appropriately in order to match \( R \).\(^{16}\)

The final aim is to attain average working hours - \( l_e \) for each agent unadjusted for productivity - as a percentage of the unit time endowment to be:

\[ \sum_{k_e} \Gamma^*(k_e) l_{ef} \approx .33 . \]

This fraction is approximately the value supported by the business cycle literature for infinitely-lived agents [see Prescott (1986)]. I choose the consumption coefficient \( \eta \) to attain this seventh target. Labour is elastic in the model and part of leisure can be interpreted as home production. Since leisure is not taxed in the US, the calibration distinguishes between informal and home production activities.

The model period is set to one year. Agents can only hold non-negative stocks of capital. That is, the lower bound of the capital grid \( k \) is set to zero.

| TABLE 3.1 |
| Parameters |

<table>
<thead>
<tr>
<th>Period</th>
<th>( \beta )</th>
<th>( \delta )</th>
<th>( \eta )</th>
<th>( A_i )</th>
<th>( \alpha )</th>
<th>( \sigma )</th>
<th>( \tau_k )</th>
<th>( s_1 )</th>
<th>( s_2 )</th>
<th>( s_3 )</th>
<th>( s_4 )</th>
<th>( s_5 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Year</td>
<td>.95</td>
<td>.05</td>
<td>.56</td>
<td>.081</td>
<td>1/3</td>
<td>1.5</td>
<td>.102</td>
<td>.0106</td>
<td>.0451</td>
<td>.0795</td>
<td>.2510</td>
<td>1.00</td>
</tr>
</tbody>
</table>

I set the share of capital in formal sector production (\( \alpha \)) to \( 1/3 \), the typical value used in neoclassical growth models. Thus, the elasticity of informal output with (8.8% relative to formal sector output) is reasonable since this estimate excludes home production.\(^{16}\)

\(^{16}\)Note that \( Y_f = \bar{Y}_f \) only if \( \tau_k = \tau_{Flat} = 0 \).
respect to labour input is $\frac{2}{3}$. This is marginally lower than the value used for
the informal sector in the representative agent model of Koreshkova (2006). The
elasticity of intertemporal substitution is set to 1.5 and the appendix provides
robustness simulations for $\sigma = 4$.

In calibrating taxes, I follow closely the rates used in the single sector model
of Ventura (1999). The tax rate on interest, $\tau_k$, is taken from his paper. The
case of income taxes is more complicated. I follow closely Ventura’s procedure by
first guessing a value for $\bar{Y}_f$, being the average income reported by agents. Using
this guess, I establish tax bracket bendpoints as in Table 3.2. The marginal tax
rates corresponding to these brackets are similar to Ventura’s paper. Using these
brackets and tax rates, I solve each agent’s problem, which delivers the reported
earnings of each type: $wl_{ef} + rk_e$. In the final step, I verify and update the guess
of $\bar{Y}_f$ until stationarity using the formula for finding the average reported earnings,
given by (3.14).

<table>
<thead>
<tr>
<th>TABLE 3.2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Personal Income Tax Profile</td>
</tr>
<tr>
<td>----------------------------------</td>
</tr>
<tr>
<td>$b$</td>
</tr>
<tr>
<td>-----</td>
</tr>
<tr>
<td>1.00</td>
</tr>
<tr>
<td>2.00</td>
</tr>
<tr>
<td>3.00</td>
</tr>
<tr>
<td>4.00</td>
</tr>
<tr>
<td>5.00</td>
</tr>
<tr>
<td>6.00</td>
</tr>
</tbody>
</table>

Given the above parameterization, the model is simulated and the value of gov-
ernment consumption, $G$, is evaluated using (3.11). This value is retained as the
revenue requirement in the experimented tax reform.

The computation of the agent’s problem is done in two stages. In the first, I assume knowledge of the intertemporal choice of each agent: \( \Delta = k_e' - k_e \). With this guess, I solve the intratemporal optimization to determine the choices of \( l_{ef} \) and \( l_{ei} \) for each agent using the Euler conditions (3.12) and (3.13). Next, I verify the guess of the intertemporal decision via value function iteration. Since the value function is concave, I use a logarithmic transformation to attain an uneven asset grid that is finer at the lower wealth levels. The appendix contains further details.

3.4 Quantitative Results

3.4.1 Calibration Results

\textit{Formal Sector: Earnings Distribution}

The empirical U.S. earnings distribution in Table 3.3 is taken from Castañeda, Díaz-Giménez and Ríos-Rull (2003). The reported (formal sector) quintiles distribution attained by the model simulation matches favourably with the empirical distribution with marginally less success on the third quintile. Regardless, the Gini coefficient obtained by the calibration is considerably lower than that from the data. The reason for this discrepancy is that in the model examined in this paper, there is perfect earnings equality for all agents within a given quintile. In the data however, there is earnings inequality even within each quintile, which makes the empirical Gini higher than that attained by the model.\(^{17}\)

\textit{Informal Sector: Size and Earnings Distribution}

\(^{17}\)For instance, Castañeda, Díaz-Giménez and Ríos-Rull (2003) reports that although the 5th quintile accounts for 61.39% of total earnings, 14.76% goes to the top 1% of agents.
For $A_i = .081$, the aggregate informal-to-formal sector output ratio is 8.85%, which is approximately the desired calibration target. Since each informal sector firm is individually owned and operated, the model has the advantage of generating an implied theoretical distribution for informal sector earnings. This is included in Table 3.3. Informal sector earnings show much less concentration compared to formal sector earnings. The reason is that agents in the low productivity spectrum earn low informal incomes due to their inefficiency, while highly productive agents on the other hand supply most of their labour to the formal sector, thereby earning equally dismal incomes in the informal sector. This finding can be supported by several other reasons for which informal sector earnings may indeed be less concentrated than formal sector earnings. In the enforcement literature for instance, a wealthy agent’s ability to conceal his informal sector earnings from tax authorities is unlikely to be overwhelmingly better than a poor agent’s ability to conceal his.\footnote{For instance, Cremer and Gahvari (1993) use a convex concealment cost function.}

Although the results are insightful, there is no comparable empirical evidence on informal sector earnings distribution since by their very nature, such earnings go unreported.

**Wealth and Consumption Distribution**

The empirical wealth distribution in Table 3.4 is taken from Castañeda, Díaz-Giménez and Rios-Rull (2003). Also taken from the same source is the distribution of US consumption of nondurables. The same table also reports the wealth and consumption distribution from model simulations. These outcomes fall short in matching the empirical quintiles because the calibration is focused on matching earnings rather than the above pair. Integrating social mobility schemes such as idiosyncratic shocks and life cycles and programs for social inclusion in the form of social security and unemployment insurance may provide improvements in approx-
imating the empirical wealth and consumption distributions.

3.4.2 Results of Flat Tax Reforms

I undertake two flat tax policy experiments similar to those examined in Ventura (1999). Flat Tax Plan 1 shows results for the revenue-neutral tax reform in which the labour income exemption amount is set to 40% of pre-reform mean earnings: $0.4\bar{Y}_f$. That is, $\bar{Y}_f$ is taken from the benchmark calibration. Flat Tax Plan 2 performs the same experiment as in plan 1, except that the labour income exemption amount set to 20% of pre-reform mean earnings. The analyses are restricted to comparative static of equilibria.19

Similar to results in Ventura (1999), Altig et al. (2001) and Díaz-Giménez and Pijoan-Mas (2005), a flat tax reform worsens the observed earnings distribution. In the model examined however, the unobserved informal earnings distribution moves in the opposite direction, becoming less concentrated under a flat tax regime. In terms of earnings therefore, the informal sector acts as a safety net that cushions the effect of reforms on the lower end of the earnings distribution. Further, a flat tax reform worsens wealth distribution, another result consistent with the literature. Interest on capital investment is earned by a select few, thereby accounting in part for the changes in reported earnings distribution. In both policy experiments, aggregate capital increases owing to the elimination of the double taxation of capital income. Although there is an increase in the wage rate due to a flat tax reform, weighted average utility declines.

19See Erosa and Ventura (2002) for the dynamic properties of a related environment also with agents having permanent non-idiosyncratic differences in endowments.
<table>
<thead>
<tr>
<th></th>
<th>Formal Earnings</th>
<th>Informat Earnings</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Gini, Quintiles</td>
<td>Gini, Quintiles</td>
</tr>
<tr>
<td></td>
<td>1st 2nd 3rd 4th 5th</td>
<td>1st 2nd 3rd 4th 5th</td>
</tr>
<tr>
<td>US Data</td>
<td>.6300 –0.40 3.19 12.49 23.33 61.39 n.a n.a n.a n.a n.a</td>
<td>n.a n.a n.a n.a n.a</td>
</tr>
<tr>
<td>Calibration Results</td>
<td>.5912 .00 .0305 .0999 .2305 .6390 1759 .1397 .1569 .1569 .2171 .3295</td>
<td></td>
</tr>
<tr>
<td>Castañeda et al (2003)</td>
<td>.6300 .00 .0374 .1459 .1599 .6568</td>
<td>n.a n.a n.a n.a n.a</td>
</tr>
<tr>
<td>Flat Tax Plan 1</td>
<td>.6551 .00 .0307 .0543 .1616 .7534 1331 .1527 .1582 .1582 .2655 .2655</td>
<td></td>
</tr>
<tr>
<td>Flat Tax Plan 2</td>
<td>.6581 .00 .0303 .0493 .1652 .7552 1035 .1474 .1496 .2344 .2344 .2344</td>
<td></td>
</tr>
<tr>
<td>Source</td>
<td>Gini</td>
<td>Wealth Quintiles</td>
</tr>
<tr>
<td>-------------------------------</td>
<td>------</td>
<td>------------------</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1st 2nd 3rd 4th 5th</td>
</tr>
<tr>
<td>US Data</td>
<td>.7949</td>
<td>-.39 1.74 5.72 13.43 79.49</td>
</tr>
<tr>
<td>Calibration Results</td>
<td>.6343</td>
<td>.00 .00 .00 .4142 .5858</td>
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<tr>
<td>Castañeda et al (2003)</td>
<td>.79</td>
<td>.0021 .0121 .0193 .1468 .8197</td>
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<tr>
<td>Flat Tax Plan 1</td>
<td>.80</td>
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<tr>
<td>Flat Tax Plan 2</td>
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<td>.00 .00 .00 .00 1.00</td>
</tr>
<tr>
<td></td>
<td>Calibrated Results</td>
<td>Flat Tax 1</td>
</tr>
<tr>
<td>--------------------------------</td>
<td>--------------------</td>
<td>------------</td>
</tr>
<tr>
<td><strong>R</strong></td>
<td>.0885</td>
<td>.0911</td>
</tr>
<tr>
<td>$\tau_{Flat}$</td>
<td>n.a</td>
<td>.2281</td>
</tr>
<tr>
<td>Interest Rate ($r$)</td>
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<td>.0552</td>
</tr>
<tr>
<td>Wage Rate</td>
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<td>1.1042</td>
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<tr>
<td>Av. Labour Hours (out of unity)</td>
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<td>.4246</td>
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<tr>
<td>Aggregate Capital</td>
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<td>.6059</td>
</tr>
<tr>
<td>Weighted Average Utility</td>
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<td>-7.7268</td>
</tr>
<tr>
<td><strong>Gini Coefficients</strong></td>
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</tr>
<tr>
<td>Wealth (Capital)</td>
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<td>.800</td>
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<tr>
<td>Total Earnings (Formal and Informal)</td>
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<td>.6544</td>
</tr>
<tr>
<td>Consumption</td>
<td>.5623</td>
<td>.6411</td>
</tr>
</tbody>
</table>
Next, I turn to the central focus of the paper, which is to address the effect of such reforms on the relative size of the informal sector. Agents in the higher earnings brackets have a higher incentive to evade taxes and to under-report earnings due to their higher marginal tax rates. A flat tax reform reduces the marginal tax rate of these agents, causing them to decrease their evasion activities. Agents in the lower tax brackets may, in turn, encounter higher marginal taxes compared to previously. Hence, they evade a larger percentage of their total earnings. On the aggregate, it is not immediately clear the direction of change of the relative size of the tax-evading sector, thus motivating this study. This is of particular concern since a flat tax reform is known to cause fundamental changes in the earnings and wealth distributions. In both flat tax plans experimented in this paper, I find that the reforms hardly cause changes in the aggregate informal-to-formal output ratio. In other words, increased evasion by low earners is just about compensated for by reduced evasion on the part of high earners.

3.5 Conclusion

Several authors have proposed elaborate models that mimic properties of the US economy aimed at investigating the likely effect of flat tax reforms in the US.\(^{20}\) A familiar assumption in these papers is that working agents are automatically presumed to be completely covered under the tax radar. This paper relaxes this assumption.

The main conclusion is that a flat tax reform leaves informal sector output rela-

\(^{20}\) Ventura (1999) and Díaz-Giménez and Pijoan-Mas (2004) consider environments with uninsured idiosyncratic risk in dynamic life-cycle models that also include social security schemes. Altig et al (2001) consider the sensitivity of output to several tax reform systems including flat tax.
tive to that of the formal sector unchanged. Although individual agents alter their participation in tax evading activities, these actions cancel out on the aggregate in equilibrium. The paper also reveals a distribution for informal sector earnings that is far less concentrated than the official sector earnings distribution. Although a flat tax reform worsens the inequality of earnings in the latter distribution, the reverse is documented for the former. This suggests that the informal sector acts as a safety net for poor agents when a flat tax reform is implemented.

The results on the distribution of informal earnings are compelling. However, it must be emphasized that these results are at best theoretical conclusions, as there are no empirical comparables for such unobserved distributions. The results presented indicate that as a percentage of their respective total earnings (formal and informal), low earners evade more taxes than high earning agents. On one hand, this conclusion resonates as reasonable, since it may be easier for agents at the lower end of the earnings distribution to evade large percentages of their earnings, since in absolute terms, the evaded sums are small. On the other hand, an equally credible argument can be made that agents at the upper end of the earnings distribution have the ways and means to evade larger percentages of their earnings. A possible extension to the current model is to include heterogeneity in the ability to evade taxes. In particular, if the ability to evade taxes is correlated with labour productivity levels, the model can generate the outcome that agents who are at the two extreme ends of the distribution evade larger percentages of their earnings than those in the center of the distribution.

Further extensions to the model presented in this paper may include schemes of social mobility, such as idiosyncratic shocks and life cycles, as well as programs for social inclusion, such as social security and unemployment insurance. These can provide further insight to the empirical plausibility of the findings on the informal
sector earnings distribution, and its response to different tax reforms.
Appendix

Computation

\( \tilde{Y}_f \) is required in order to establish the tax bendpoints. I make guesses for \( K \) and \( L_f \). Notice that the wage and interest rates are independent of individual agent decisions. Hence, \( \tilde{Y}_f = \sum_k \Gamma^*(k) \) \text{[}w_{ef} + r_k\text{]} \equiv w \sum_k \Gamma^*(k) l_{ef} + r \sum_k \Gamma^*(k) k_e = wL_f + rK \). Thus, the guesses \( K \) and \( L_f \) deliver \( \tilde{Y} \) and one can find the tax bendpoints using Table 3.2.

**Intratemporal Optimization**

1. Assume that the intertemporal decision has already been made. Obviously, I use a large grid size but for the purpose of illustration, suppose I used the \( 1 \times g \) asset grid \{0, 0.33, 0.79, 1.43, 3.5\}, where \( g = 5 \) in this example. Define the \( g \times g \) matrix of possible changes in individual asset holdings as \( \Delta \). For instance, for an agent moving from \( k_e = 0.33 \) to \( k_e' = 1.43 \), this implies \( k_e' - k_e = \Delta_{24} = 1.1 \) and so forth. Thus, \( \Delta \) is given as the numbers in the rectangle below.

<table>
<thead>
<tr>
<th>( k_e' )</th>
<th>.00</th>
<th>0.33</th>
<th>0.79</th>
<th>1.43</th>
<th>3.5</th>
</tr>
</thead>
<tbody>
<tr>
<td>.00</td>
<td>( \Delta_{11} = 0 )</td>
<td>( \Delta_{12} = .33 )</td>
<td>( \Delta_{13} = .79 )</td>
<td>( \Delta_{14} = 1.43 )</td>
<td>( \Delta_{15} = 3.5 )</td>
</tr>
<tr>
<td>0.33</td>
<td>( \Delta_{21} = -.33 )</td>
<td>( \Delta_{22} = 0 )</td>
<td>( \Delta_{23} = .46 )</td>
<td>( \Delta_{24} = 1.1 )</td>
<td>( \Delta_{25} = 3.17 )</td>
</tr>
<tr>
<td>0.79</td>
<td>( \Delta_{31} = -.79 )</td>
<td>( \Delta_{32} = -.46 )</td>
<td>( \Delta_{33} = 0 )</td>
<td>( \Delta_{34} = .63 )</td>
<td>( \Delta_{35} = 2.70 )</td>
</tr>
<tr>
<td>1.43</td>
<td>( \Delta_{41} = -1.43 )</td>
<td>( \Delta_{42} = -1.1 )</td>
<td>( \Delta_{43} = -.63 )</td>
<td>( \Delta_{44} = 0 )</td>
<td>( \Delta_{45} = 2.07 )</td>
</tr>
<tr>
<td>3.5</td>
<td>( \Delta_{51} = -3.5 )</td>
<td>( \Delta_{52} = -3.17 )</td>
<td>( \Delta_{53} = -2.70 )</td>
<td>( \Delta_{54} = -2.07 )</td>
<td>( \Delta_{55} = 0 )</td>
</tr>
</tbody>
</table>

2. Choose a productivity level \( s \). For each of these \( g \times g \) transition paths in \( \Delta \), find whether the agent will participate in formal production.
(a) Take for instance the case of $\Delta_{24} = 1.1$. Firstly, assume that he does not participate in the formal labour market ($l_{ef} = 0$). Then,

$$
c_e = [1 - \bar{\tau}_b(\cdot)] r k_e + \epsilon_b(\cdot) + A_{i} l_{ei}^{1-\alpha} - 1.1 \quad \text{and} \quad \ l_{ei} = s_e l_e ,
$$

where $k_e$ here equals .33 as assumed above and $\bar{\tau}_b$ and $\epsilon_b$ are the tax and rebate levels associated with the reported income level $rk_e$. Comparing the above two equations with (3.12), we have:

$$
[1 - \bar{\tau}_b(\cdot)] r k_e + \epsilon_b(\cdot) + A_{i} l_{ei}^{1-\alpha} - 1.1 = \frac{A_{i} \eta s_e(1 - \alpha)}{1 - \eta} l_{ei}^{1-\alpha} \left( 1 - \frac{l_{ei}}{s} \right) .
$$

Since we know $s$, this is one equation with one unknown and I can evaluate $l_{ei}$.$^{21}$ Repeat this exercise for each cell in the $g \times g$ matrix $\Delta$. The result is a $g \times g$ matrix, $L_{ei}$.

(b) Secondly, using each element of $L_{ei}$, check that (3.13) is satisfied for each cell. If yes, then indeed $l_{ef} = 0$ for that cell. This gives a $g \times g$ matrix, $L_{ef}$, with some cells being zero and others with $l_{ef} > 0$, the exact values yet unknown. Temporarily assign a value $l_{ef} = a$ for these unknown cells.

3. For cells of $L_{ei}$ for which (3.13) is not satisfied, assign $l_{ei} = a$. For all “a” cells in $L_{ei}$ and $L_{ef}$, $l_{ef} > 0$. Thus, for these cells, (3.13) holds with equality. For each of these cells, use the two equalities (3.13) and

$$
A_{i} l_{ei}^{1-\alpha} + [1 - \tau_b(\cdot)] w l_{ef} + [1 - \bar{\tau}_b(\cdot)] r k_e + \epsilon_b(\cdot) - 1.1 = \frac{A_{i} \eta (1 - \alpha)}{1 - \eta} s_e l_{ei}^{1-\alpha} (1 - l_e)
$$

$^{21}$Alternatively, $[1 - \bar{\tau}_b(\cdot)] r k_e + \epsilon_b(\cdot) + A_{i} l_{ei}^{1-\alpha} - 1.1 - \frac{A_{i} \eta s_e(1 - \alpha)}{1 - \eta} l_{ei}^{1-\alpha} (1 - \frac{l_{ei}}{s}) = 0$ For a given tax bracket $b$, the left hand side of this is monotonically increasing, allowing for a unique $l_{ei}$. 

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to find the true cell values \( l_{ef} \) and \( l_{ei} \). Note that in this last equation, I use \( \Delta_{24} = 1.1 \) only for illustration.

4. Using \( L_{ef} \) and \( L_{ei} \), find the corresponding \( g \times g \) matrix \( C_e \) using (3.3). These three - \( L_{ef}, L_{ei} \) and \( C_e \) - are the policy choices that describe the intratemporal decision of each agent for each intertemporal transition path in \( \Delta \). Also note that given an agent’s stock \( k_e \), the choice of \( l_{ef} \) automatically implies the choice of a tax bracket \( b \).

5. Notice that using the three \( g \times g \) matrices \( L_{ef}, L_{ei} \) and \( C_e \), I get another \( g \times g \) matrix \( U \), which is the instantaneous utility in accordance with (3.1). Find all cells in \( C_e \) with negative values. Replace the values in the corresponding cells in \( U \) with \(-\infty\).

6. Repeat the steps 2 to 5 for all five realizations of \( s \).

**Intertemporal Optimization**

Here, I use standard value function iteration. This is preferred to policy function iteration in this simulation because the value function has kinks, owing to the different tax brackets and tax rebate amounts. The grid size chosen is \( g = 111 \) within the interval \([\underline{k}, \overline{k}] = [0, 3.5]\).

The last stage of the computation involves updating the guesses of \( L_{ef} \) and \( K_e \) until stationarity. This is equivalent to checking the initial guess of \( \bar{Y}_{ef} \).

The uneven asset grid is chosen as follows. First set \( \kappa = \log (1 + k) \) and \( \pi = \log (1 + \overline{k}) \). Next, I form a preliminary equally-spaced grid of size \( g \) between \( \kappa \) and \( \pi \). Finally, each component \( \kappa_z \) of this preliminary grid is transformed back into the final using \( k_z = e^{\kappa_z} - 1 \). This final grid is finer at lower asset levels.
TABLE 3.6

Earnings Distributions with Informal Production ($\sigma = 4$)

<table>
<thead>
<tr>
<th></th>
<th>Formal Earnings</th>
<th></th>
<th>Informal Earnings</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Gini</td>
<td>Quintiles</td>
<td></td>
<td>Gini</td>
</tr>
<tr>
<td></td>
<td>1st</td>
<td>2nd</td>
<td>3rd</td>
<td>4th</td>
</tr>
<tr>
<td>US Data</td>
<td>.6300</td>
<td>-0.40</td>
<td>3.19</td>
<td>12.49</td>
</tr>
<tr>
<td>Benchmark Results</td>
<td>.5912</td>
<td>.00</td>
<td>.0305</td>
<td>.0999</td>
</tr>
<tr>
<td>Model with $\sigma = 4$</td>
<td>.5944</td>
<td>.00</td>
<td>.0302</td>
<td>.0861</td>
</tr>
<tr>
<td>Castañeda et al (2003)</td>
<td>.6300</td>
<td>.00</td>
<td>.0374</td>
<td>.1459</td>
</tr>
<tr>
<td>Flat Tax Plan 1 ($\sigma = 4$)</td>
<td>.6560</td>
<td>.00</td>
<td>.0304</td>
<td>.0538</td>
</tr>
<tr>
<td>Flat Tax Plan 2 ($\sigma = 4$)</td>
<td>.6584</td>
<td>.00</td>
<td>.0302</td>
<td>.0493</td>
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</tbody>
</table>

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<table>
<thead>
<tr>
<th></th>
<th>Gini</th>
<th>Wealth Quintiles</th>
<th>Gini</th>
<th>Consumption Quintiles</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>1st</td>
<td>2nd</td>
<td>3rd</td>
</tr>
<tr>
<td>US Data</td>
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<td>1.74</td>
<td>5.72</td>
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<td>Benchmark Results</td>
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<tr>
<td>Model with $\sigma = 4$</td>
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<td>.00</td>
<td>.00</td>
<td>.0115</td>
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<tr>
<td>Castañeda et al (2003)</td>
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<td>.0021</td>
<td>.0121</td>
<td>.0193</td>
</tr>
<tr>
<td>Flat Tax Plan 1 ($\sigma = 4$)</td>
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<td>.00</td>
<td>.00</td>
<td>.00</td>
</tr>
<tr>
<td>Flat Tax Plan 2 ($\sigma = 4$)</td>
<td>.80</td>
<td>.00</td>
<td>.00</td>
<td>.00</td>
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</tbody>
</table>
**TABLE 3.8**
Flat Tax reforms and Summary Economic Indicators with Informal Production ($\sigma = 4$)

<table>
<thead>
<tr>
<th></th>
<th>Benchmark Results ($\sigma = 1.5$)</th>
<th>Model with $\sigma = 4$</th>
<th>Flat Tax 1 ($\sigma = 4$)</th>
<th>Flat Tax 2 ($\sigma = 4$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R$</td>
<td>.0885</td>
<td>.0912</td>
<td>.0916</td>
<td>.0894</td>
</tr>
<tr>
<td>$\tau_{Flat}$</td>
<td>n.a.</td>
<td>n.a.</td>
<td>.2094</td>
<td>.1846</td>
</tr>
<tr>
<td>Interest Rate ($r$)</td>
<td>.0683</td>
<td>.0793</td>
<td>.0588</td>
<td>.0549</td>
</tr>
<tr>
<td>Wage Rate</td>
<td>1.0606</td>
<td>1.0350</td>
<td>1.0911</td>
<td>1.1236</td>
</tr>
<tr>
<td>Av. Labour Hours (out of unity)</td>
<td>.3216</td>
<td>.3366</td>
<td>.4250</td>
<td>.4216</td>
</tr>
<tr>
<td>Aggregate Capital</td>
<td>.5250</td>
<td>0.4836</td>
<td>.5859</td>
<td>.6461</td>
</tr>
<tr>
<td>Weighted Average Utility</td>
<td>-7.5326</td>
<td>-122238.83</td>
<td>-122243.16</td>
<td>-122245.01</td>
</tr>
<tr>
<td>Gini Coefficients</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Wealth (Capital)</td>
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<td>.6854</td>
<td>.80</td>
<td>.80</td>
</tr>
<tr>
<td>Total Earnings (Formal and Informal)</td>
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<td>.5937</td>
<td>.6552</td>
<td>.6576</td>
</tr>
<tr>
<td>Consumption</td>
<td>.5623</td>
<td>.5672</td>
<td>.6436</td>
<td>.6501</td>
</tr>
</tbody>
</table>
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