

Publicity and Clustering of IPO Underpricing*

Melanie Cao
York University, Canada
and
Queen's University, Canada

Shouyong Shi
Indiana University, USA
and
Queen's University, Canada

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Abstract

In this paper we construct a signalling model to explain why underpriced IPOs cluster in time and industry. Our model emphasizes the industry-wide rather than firm-specific uncertainty in earnings. The central assumption is that underpriced IPOs generate publicity for the industry which in turn increases the market's expectations on earnings of each individual firm in the industry and increases the market price of shares. We show that there are two co-existing, self-fulfilling market equilibria where high-quality firms successfully separate themselves from low-quality firms. In one, firms do not underprice their IPOs and the industry's publicity is low. In the other, the industry's publicity is high and all high-quality firms underprice their IPOs. The industry's expected publicity can magnify small differences between firms' intrinsic qualities into large differences in the market price of shares, leading to large clustered underpricing which fulfills the expectations of a high publicity for the industry. The model predicts that the clustering of underpriced IPOs (i) occurs in particular industries at particular times, (ii) is fragile with respect to market expectations of the industry's future, and (iii) is more likely to occur when the marginal cost of non-equity funds is low or when the average return to firms is high.

Keywords: Initial public offerings; Signalling; Publicity; Clustering; Multiple equilibria.
JEL classifications: G30, D82.

* Corresponding author: Melanie Cao, Department of Economics, Queen's University, Kingston, Ontario, Canada, K7L 3N6 (fax: 613-533-6668, e-mail: caom@qed.econ.queensu.ca). This paper has been presented at the Western Finance Association meeting (Idaho, 2000), ABN AMRO International Conference on IPO (Amsterdam, 2000), Pacific-Basin Capital Market/Financial Management Association Meeting (Melbourne, 2000), Queen's University and University of Toronto. We thank Jay Ritter, Kent Womack, Deborah Lucas, Ralph Winter, Kathryn Wong, Douglas Cumming, and the conference/workshop participants for comments and discussions. Both authors gratefully acknowledge the financial support from the Social Sciences and Humanities Research Council of Canada. All errors are ours alone.

1. Introduction

Internet firms in 1999 created a phenomenal “hot-issues” market in initial public offerings (IPOs). Many such IPOs had large price gains, which implied large underpricing in the offer price. For example, the share price of MarketWatch.com rose from an offer price of \$17 to a market price \$97.50 on the first trading day. Like previous hot issues documented by Ritter (1984), the hot issues of 1999 were specific to a particular industry, accompanied by the lackluster performance of concurrent IPOs in other industries.¹ In this paper we construct a signalling model to explain why underpriced IPOs cluster in time and industry.

Our model emphasizes the industry-wide rather than firm-specific uncertainty in earnings. The central assumption is that underpriced IPOs generate publicity for the industry which in turn increases the market’s expectations on earnings of each firm in the industry and increases the market price of shares. We show that there are two co-existing, self-fulfilling market equilibria where high-quality firms successfully separate themselves from low-quality firms.² In one, firms do not underprice their IPOs and the industry’s publicity is low. In the other, the industry’s publicity is high and all high-quality firms underprice their IPOs. The industry’s expected publicity can magnify small differences between firms’ intrinsic qualities into large differences in the market price of shares, leading to large clustered underpricing which fulfills the expectations of a high publicity for the industry.

The industry in our model is comprised of many firms that try to raise funds through IPO. Some are of high quality and the others low quality. The quality is private information. Firms try to signal quality by choosing the offer price and the number of shares issued in IPO. Each firm’s earnings contain an uncertain, industry-wide component. Market expectations on this

¹In fact, there was an increase in cancellations and withdraws from the IPO market by non-Internet firms. As the chief executive of a large dry pet food company complained, “If you look at the IPO market, there’s large-capitalization activity and dot.com activity, but little else. I feel sorry for small-cap companies that are nondot.com, and which need to complete their deals.” (Prial 1999)

²All equilibria we focus on in this paper are separating equilibria that are refined by the intuitive criterion of Cho and Kreps (1987). For each value of expected earnings, there is a unique separating equilibrium in the signalling game, but there are multiple values of expected earnings that are consistent with rational expectations.

component increase with the industry's publicity, which is an increasing function of the aggregate amount of IPO underpricing by all high-quality firms in the industry. A perceived high-quality firm benefits more from this component than a perceived low-quality firm. When the influence of the industry's publicity on each firm's expected earnings is weak, the only separating equilibrium is such that there is no underpricing. This no-underpricing equilibrium continues to exist when the influence of publicity is strong. But then there is also another separating equilibrium where all high-quality firms underprice their IPOs.

In the underpricing equilibrium, firms expect that the industry's publicity will be high. This expected publicity of the industry increases a low-quality firm's temptation to mimic a high-quality firm in the IPO market by increasing the payoff to mimicking. To deter mimicking and signal quality successfully, high-quality firms must incur a high cost such as underpricing in IPO. These firms have incentive to do so because the industry's publicity also raises market expectations on earnings and share prices for firms that successfully signal their high quality. Such an equilibrium response of the market price of shares to the industry's publicity magnifies the magnitude of underpricing and induces clustering. In a simple version of the model, underpricing can be as high as 100% of the market price! In turn, the large magnitude and clustering of underpriced IPOs fulfill the expectations of a high publicity for the industry.

Although it is well documented that underpriced IPOs cluster in particular industries at particular times (see Ritter 1984), there is very little effort in the literature to formally explain such a phenomenon. Instead, previous models have focused on a single firm's underpricing decision. With this focus, Allen and Faulhaber (1989), Welch (1989), and Grinblatt and Hwang (1989) construct signalling models. Rock (1986) argues that the winner's curse forces a firm to underprice IPO in order to induce uninformed investors to participate. Beatty and Ritter (1986), Benveniste and Spindt (1989) and others attribute underpricing to underwriters who try to build good reputations through the price gains. Mauer and Senbet (1992) motivate underpricing by IPO firms' concern for the liquidity in the secondary market. There is also a large collection of

empirical works on IPOs (see Michaely and Shaw 1994 for references).

We emphasize the interaction between many IPO firms through the market, using a signalling model. The interaction comes through the industry's publicity which affects all firms' expected earnings and it is critical for underpriced IPOs to cluster. Since the industry's publicity depends on many firms' underpricing decisions, the market price of shares is endogenous in equilibrium, although each firm takes it as given. This contrasts with previous IPO models, which assume that the market price of shares is exogenous in equilibrium once the firm's type is known.³

What can we gain from such an explicit model? First, we learn that even moderate differences between firms' intrinsic qualities can lead to large differences in market prices of shares and large IPO underpricing. We deliberately restrict the intrinsic earning differential between high-quality and low-quality firms to be small so that a firm will not underprice IPO in traditional models without publicity. It may underprice in our model because the market price of shares increases with the industry's expected publicity which increases the payoff to underpricing.

Second, the industry-wide component of earnings alone does not make underpriced IPOs cluster, in contrast to informal arguments. Among other necessary conditions firms' expectation of other firms' behavior is most important. Since the industry's publicity is endogenous, depending on firms' IPO actions, for the publicity to be high a firm must believe that other high-quality firms will underprice. If firms believe that other firms will not underprice, underpriced IPOs will not cluster no matter how large the industry-wide component of earnings is.

Third, information asymmetry is important for clustering. If firms' qualities were public information, underpriced IPOs would not cluster in our model no matter how high the industry's publicity is expected to be. With public information the industry's publicity merely serves as a "public good" and every firm tries to free-ride on such publicity, in which case there is no underpricing in equilibrium. The existence of private information overcomes the free-rider problem,

³Our model also differs from the "herd" behavior (Banerjee 1992 and Bikhchandani et al. 1992), where agents ignore their private information and follow previous agents' actions. In the current model, clustering occurs as firms try to signal their private information. Another difference is that herding occurs only when firms move sequentially but clustering can occur when firms move simultaneously.

because an individual high-quality firm must signal its quality successfully in order to benefit from the industry's publicity. This solution to the free-rider problem contributes to information economics. The importance of asymmetric information again shows that the existence of the industry-wide component in firms' earnings is not tautological to clustering.

We want to motivate two aspects of our model. The first is the assumption that the market's expectations on the industry-wide component of earnings are an increasing function of the average IPO underpricing in the industry. This is a reasonable assumption for new industries like the Internet industry. Since Internet firms offer products/services that have no resemblance to traditional businesses but nevertheless compete against the latter, there is very little guidance to predicting the demand for such products. These firms can do very well or fail as a group, depending in part on the public's awareness of their advantages relative to the traditional ones. Spectacular price gains in IPOs create publicity for the industry and may benefit firms in the industry, with good firms benefiting more than do mediocre firms. For example, Internet firms that sell books, auction goods, or provide market information on Internet compete directly against businesses that organize such activities in traditional ways. If these firms' IPOs have large price gains, the publicity may induce customers to switch from traditional firms to these new firms, e.g., switching from buying books in neighborhood bookstores to Internet book-selling firms.⁴

The second aspect is the use of a signalling model. Most criticisms of the signalling model have been based on the evidence that post-IPO performances of underpricing firms do not seem to exceed those of no-underpricing firms (see references above). For two reasons we view this as inconclusive evidence against the signalling model. First, the signalling model predicts that the magnitude of underpricing depends positively only on the part of future earnings that is private information prior to IPO. Most of the empirical tests do not distinguish this part of future earnings from the part that is publicly expected prior to IPO. Second, firms may take different post-IPO

⁴Firms may also use other methods to increase their publicity, e.g. advertisement. However, when the entire industry is new, those methods may not be as effective as the hard evidence of price gains in IPOs. Also, those advertising methods must be monitored in order to make them credible. In contrast, IPO price gains are publicly observed.

investment strategies that are not foreseen at the time of IPO. Since these investment strategies affect the firms' post-IPO performances, such performances may not be good indicators of the firms' conditional expected earnings at the time of IPO. Most of the empirical tests do not control for such a diversity in post-IPO investment strategies.

The signalling model has also been criticized for lacking some realistic institutional features such as the role of underwriters. We believe that such institutional features are important for explaining why individual firms' IPOs have different price gains, but not for why underpriced IPOs cluster in particular industries at particular times. If an underwriter underprices IPOs in order to build a good reputation through the price gains, it should underprice all IPOs at all times rather than underprice only those in particular industries at particular times. Also, Internet IPOs had large price gains in 1999 across a spectrum of underwriters, not only for particular underwriters. To explain such clustering of underpriced IPOs by underwriters' reputation is not more convincing than to explain city-wide increases in real estate prices by real estate agents' reputation. It also underestimates the rationality of firms' underpricing decisions.⁵

2. The Model

Consider an industry with $n \geq 2$ risk-neutral firms, each having a project that requires external financing of an amount normalized to one. The project's quality, denoted x , is the firm's private information. The public has the prior belief that the project is of high quality ($x = x_H$) with probability α and of low quality ($x = x_L$) with probability $1 - \alpha$, where $x_H > x_L > 0$ and $\alpha \in (0, 1)$. All n firms seek financing at the same time (see Section 5 for sequential decisions).

A firm can raise the required amount by initial public offering of its equity. As discussed in the introduction we abstract from the role of underwriters. Let the total number of a firm's shares be 1. The firm offers f shares to the public in IPO at an offer price s , where $f \in (0, 1]$. The firm's original owners keep $1 - f$ shares. The market price of shares is p . The shares are

⁵Despite the huge underpricing, Internet firms do not seem to lay blames on their underwriters. As one chief executive officer of a newly public Internet firm put it, "We don't second-guess what we left on the table. Our eyes are on the future in terms of building a great company." (Smith and Simon 1999). Loughran and Ritter (1999) employ prospect theory to explain such a phenomenon. They do not examine the clustering phenomenon.

underpriced if $s < p$ and the amount of underpricing is $d \equiv p - s$. The IPO revenue is $q \equiv sf$. If $q < 1$, the firm finances the remainder of the investment through alternative methods such as venture capital, loans, etc. The total expected cost of such alternative funds is $(1 + bx^{-1})(1 - q)$ for $q < 1$, where $b > 0$ is a constant. Thus, for each dollar of alternative funds, the additional expected cost is bx^{-1} , which is decreasing in the firm's quality. The linear cost function keeps the analysis simple, but all analytical results in this paper hold for a more general cost function $(1 + b/x)C(1 - q)$ that satisfies $C(0) = 0$, $C'(0) \geq 1$ and $C'' > 0$.

By assuming that the expected cost of alternative funds is a decreasing function of the firm's quality, we do not mean that the supplier of the alternative funds knows the firm's quality perfectly. Rather, the supplier has imperfect knowledge of the firm's quality and his/her knowledge is positively correlated with the firm's true quality. For example, suppose that the supplier screens the project before providing the funds. The screening technology may yield a noisy signal that is positively correlated with the firm's quality and the fund's supplier may charge a higher loan rate when the signal is low. Then the firm's expected cost of the alternative funds will be a decreasing function of the firm's true quality, as we assume here, although the fund's supplier does not know the true quality perfectly.

With this interpretation of the expected cost function of the alternative funds, we can even reverse the timing of activities by allowing firms to seek alternative funds before entering the IPO market. With such timing the IPO investors may observe the actual cost of the firm's alternative funds. This alleviates but does not eliminate the asymmetric information problem in the IPO market. Because the cost of alternative funds is not a perfect indicator of the firm's true quality, IPO investors still cannot tell a high-quality firm apart from a low-quality firm.⁶

The cost function does not imply that alternative funds are always more costly than equity. Since the firm's quality is private information, the firm must incur implicit cost in the equity market such as underpricing in order to overcome the informational problem. This trade-off

⁶The role of alternative financing methods links our model to James and Wier (1990) and Slovin and Young (1990). These authors have shown that IPOs of firms with previously established borrowing relationships are underpriced less than other IPOs. But, the former group of firms can still experience IPO underpricing.

between different costly financing methods endogenously determines the ratio between equity and non-equity funds.

A firm's earning is r_H if it is of high-quality and r_L if it is of low-quality. The earnings have two components. The first one, referred to as the firm's intrinsic earnings, is specific to the project and is $R_0 x_i / x_L$ for a quality i firm ($i = H, L$), where $R_0 > 0$. The firm knows this component and, once the project's quality is revealed, the market also knows it. The second component is uncertain to the firm and the market, even when the project's quality is observed. Let this component be m for a high-quality firm and, to simplify, 0 for a low-quality firm. Then, the earnings for a low-quality firm and a high-quality firm are, respectively,

$$r_L = R_0, \quad r_H = R_0 \frac{x_H}{x_L} + m. \quad (2.1)$$

The component m captures the industry-wide uncertainty in earnings and is at the heart of our analysis. As argued in the introduction, this is a realistic component for new industries like the Internet industry and market expectations on this industry-wide component are susceptible to the industry's publicity. To capture this effect of the industry's publicity, let D be the amount of IPO underpricing by a representative firm that is perceived as of high quality and assume that the number of firms (n) is large so that each firm takes D as given. If the market perceives a firm to be of high quality, market expectations on the firm's m conditional on D are

$$E(m|D, \text{ the firm's perceived quality} = x_H) = \rho D, \quad (2.2)$$

where $\rho \in (0, 1)$ is a constant that measures the influence of the industry's publicity on a high-quality firm's expected earnings. Conditional on D , a high-quality firm's own expectations on m are the same as market expectations. In this specification we have abstracted from the competition among firms within the industry in order to focus on the externality created by the industry's publicity. (In subsection 4.2 we will discuss the implications of the competition among firms.) We have also assumed that market expectations on m depend not on any single firm's but on all firms' IPO underpricing (see Section 5 for individual firm's publicity).⁷

⁷The assumption of a large n simplifies the analysis but is not necessary for the results (see Section 5 for cases

We want to emphasize two implications of the above specification. First, a firm's market value depends on the IPO decisions of all firms in the industry and hence it is endogenous in equilibrium. This is in contrast with previous IPO models which assume that a firm's market value is exogenous once its type is known. The endogenous market value is important for self-fulfilling multiple equilibria, as established later in Section 4. Second, if a firm can convince the market that it is of high quality, it benefits more from the industry's publicity than does a low-quality firm (since the expected value of m is positive). The differential benefit of the industry's publicity gives high-quality firms incentive to take costly actions to separate themselves out even when the intrinsic difference between high-quality and low-quality firms is not big.

Market expectations of a perceived high-quality firm are $R_H \equiv R_0 x_H / x_L + \rho D$. Since each individual firm takes D as given, it also takes R_H as given. Let I be the probability with which the market believes that a firm is of high-quality, after observing all n firms' IPO prices. Then the market expects the firm's earnings to be:

$$E(r|I) = R_I \equiv I \cdot R_H + (1 - I)R_0. \quad (2.3)$$

The game is as follows. First, firms simultaneously choose the number of shares f and the offer price s for IPO, taking R_H as given. After IPO the firm obtains alternative funds if the IPO revenue is not sufficient for the project. Finally, every firm carries out the project, realizes the earnings, repays the alternative funds first if there are any and then to the shareholders. Notice that at the time of IPO the public does not observe the firm's cost of alternative funds since the firm has not yet obtained such funds, although the public can infer the amount of such funds, $1 - q$. We also assume that the net risk-free rate is zero to simplify the algebra.

Let us isolate an arbitrary firm and express its decisions as $a \equiv (f, q)$ rather than (f, s) . By

of a small n). When a firm makes the IPO decision, it does not observe other firms' IPO decisions and so its expectations of its own m are $\rho D[1 - (1 - \alpha)^{n-1}]$, where $1 - (1 - \alpha)^{n-1}$ is the probability that there is at least one other high-quality firm in the IPO market. Such expectations are different from market expectations (which are made after observing all firms' IPO decisions) and the discrepancy by itself can make the offer price deviate from the market price. This discrepancy vanishes when n is large. We can achieve the same simplification by assuming D as the average of underpricing by other high-quality firms rather than by a representative high-quality firm.

choosing a , the firm intends to maximize the expected return to the original owners:⁸

$$V(f, q; R_I, x) \equiv (1 - f) \left[R_I - (1 + bx^{-1})(1 - q) \right]. \quad (2.4)$$

Note that the firm knows its own quality x . In contrast, investors do not know the firm's quality and are concerned only with the firm's expected return per share:

$$R_I - (1 + bE_I x^{-1})(1 - q),$$

where $E_I x^{-1} = Ix_H^{-1} + (1 - I)x_L^{-1}$.

Let p_I be the market price of a firm's share when the market belief is I . For investors to participate in IPO, the offer price cannot exceed the market price. Under rational expectations, the market price equals the expected return per share to shareholders and so the expected rate of return per share equals the gross risk-free rate (one). Thus

$$0 \leq s = q/f \leq p_I = R_I - (1 + bE_I x^{-1})(1 - q). \quad (2.5)$$

A high-quality firm may want to signal its quality. By refraining from a high IPO revenue, achieved by either underpricing and/or issuing fewer shares in IPO, a high-quality firm forces itself to meet the needs with alternative financing methods. This action signals to the market that the firm's quality is high enough to cover the cost of alternative funds. Since every firm wants the market to believe it is of high quality, a low-quality firm may want to mimic a high-quality firm. For a high-quality firm to signal successfully, a necessary condition is that it has greater incentive to signal than a low-quality firm. This is the well-known single-crossing property (see Fudenberg and Tirole 1993, p.259), satisfied in the current model in the following forms:

$$\frac{\partial}{\partial x} \left[-\frac{\partial V(f, q; R, x)}{\partial R} \bigg/ \frac{\partial V(f, q; R, x)}{\partial q} \right] = \frac{\partial}{\partial x} \left[-\frac{1}{1 + bx^{-1}} \right] < 0; \quad (2.6)$$

$$\frac{\partial}{\partial x} \left[-\frac{\partial V(f, q; R, x)}{\partial R} \bigg/ \frac{\partial V(f, q; R, x)}{\partial f} \right] = \frac{\partial}{\partial x} \left[\frac{1 - f}{R - (1 + bx^{-1})(1 - q)} \right] < 0. \quad (2.7)$$

⁸Throughout this paper the payoff to a firm refers to the payoff to the original owners of the firm after IPO, rather than the payoff to all shareholders.

These properties are illustrated in Figures 1a and 1b. The property (2.6) states that, for a fixed number of shares issued in IPO, a high-quality firm is willing to reduce the IPO revenue by more than does a low-quality firm in order to receive an increase in the expectations on earnings “rewarded” by the market. The property (2.7) states that, for a fixed IPO revenue, a high-quality firm increases the number of shares issued in IPO by less than does a low-quality firm in the event of an increased expected earnings. Both properties come directly from the assumption that a high-quality firm faces a lower expected cost of alternative funds than a low-quality firm.

Figures 1a and 1b here.

To focus on interesting cases, we now narrow our attention:

Assumption 1. 1A. *A high-quality firm, if its quality is publicly known, can make a positive return even when the investment all comes from alternative funds, i.e.,*

$$R_0 \frac{x_H}{x_L} - \left(1 + \frac{b}{x_H}\right) > 0.$$

1B. *A low-quality firm, if its quality is publicly known, cannot make a positive return when the investment all comes from alternative funds, i.e.,*

$$R_0 - \left(1 + \frac{b}{x_L}\right) < 0.$$

1C. *A low-quality firm has a positive payoff if the investment is 100% equity financed, even when the firm’s quality is publicly known. That is, $V(f, 1; R_0, x_L) > 0$ for some $f = 1/s$.*

1D. *The intrinsic earning difference between high-quality and low-quality firms is not too large:*

$$R_0 \left(\frac{x_H}{x_L} - 1\right) < \frac{b}{x_L}.$$

Assumption 1A provides a high-quality firm with the ability to signal its quality: Since it makes a positive return even with 100% non-equity funds, it can reduce the IPO revenue to signal its high quality. But the signalling attempt may or may not require underpricing. Assumptions 1B and 1C make it desirable for a low-quality firm to finance its investment through equity if its

quality is publicly known. Since the quality is not publicly known, however, a low-quality firm may try to use non-equity financing to mimic a high-quality firm.

Assumption 1D is a deliberate one that highlights the difference between our model and previous signalling models of IPOs. Under this assumption, there is no underpricing in previous signalling models but there is underpricing in our model (see Section 4).

We simplify Assumption 1C by obtaining the maximum payoff to a low-quality firm when the firm's quality is publicly known. With a known low quality, the market price of the firm's share is $p_L = R_0 - \left(1 + bx_L^{-1}\right)(1 - q)$. Since the number of shares issued in IPO must satisfy $f \geq q/p_L$, the payoff to a known low-quality firm satisfies

$$\begin{aligned} V(f, q; R_0, x_L) &\leq \left(1 - \frac{q}{p_L}\right) \left[R_0 - \left(1 + \frac{b}{x_L}\right)(1 - q)\right] \\ &= R_0 - \left(1 + \frac{b}{x_L}\right)(1 - q) - q. \end{aligned}$$

The last expression is increasing in q and so it is maximized at $q = 1$, generating a value $R_0 - 1$.

The payoff $R_0 - 1$ is also attainable for a low-quality firm through the actions $(q, f) = (1, 1/R_0)$.

Thus, Assumption 1C can be replaced by:

Assumption 1C'. $R_0 > 1$.

Before getting into the signalling game, it is important to note that the industry's publicity per se does not make underpricing cluster, as stated below:

Proposition 2.1. *No firm underprices in equilibrium if firms' qualities are public information.*

When firms' qualities are public knowledge, a low-quality firm cannot masquerade and so there is no need for a high-quality firm to signal its quality. Each firm maximizes its payoff by setting $q = 1$ and $s = p$. Since every high-quality firm wants to benefit from other high-quality firms' underpricing but is unwilling to underprice its own IPO, no firm underprices in equilibrium. This is a typical outcome of the free-rider problem, since the industry's publicity is a "public" good. The existence of private information is critical for overcoming this free-rider problem.

3. Signalling Equilibrium

For arbitrarily given R_H that satisfies Assumptions 1A – 1D, we characterize a firm’s strategy. The result is the firm’s best response to other firms’ strategies. For lack of an appropriate term, we refer to this single firm’s best response together with the market’s belief as a signalling equilibrium. The true equilibrium where R_H is also determined is called a market equilibrium, an object examined in the next section. For any given R_H that satisfies Assumptions 1A – 1D, a Bayesian perfect signalling equilibrium consists of market beliefs I and individual firms’ decisions (f, q) that satisfy the following conditions: (i) Given the beliefs, the choices (f, q) maximize the firm’s payoff $V(f, q; R_I, x)$; and (ii) The beliefs are rational according to Bayes updating given the firm’s choices.

As is well known, this definition permits a large set of equilibria since the beliefs off the equilibrium path are arbitrary. For example, a large set of pooling actions, which both high-quality and low-quality firms take, can be supported as equilibria if any deviation from these actions is viewed as a low-quality firm’s. In this paper we will employ the intuitive criterion by Cho and Kreps (1987) to refine the equilibrium and so the term “equilibrium” means an equilibrium that satisfies this criterion.

To apply the intuitive criterion, consider a pooling action $a_0 \equiv (f_0, q_0)$. Since the market does not gain any new information about a firm’s quality from observing this pooling action, the market’s belief after observing a_0 is the same as its prior, believing that the firm is of high-quality with probability $I = \alpha$. Denote a high-quality firm’s payoff from the pooling action as $V^0(x_H) \equiv V(f_0, q_0; R_\alpha, x_H)$ and a low-quality firm’s payoff as $V^0(x_L) \equiv V(f_0, q_0; R_\alpha, x_L)$. For the action and the belief to form a pooling equilibrium, the following necessary conditions must be met:

$$f_0, q_0 \in [0, 1]; \tag{3.1}$$

$$q_0/f_0 \leq p_\alpha = R_\alpha - (1 + bE_\alpha x^{-1})(1 - q); \tag{3.2}$$

$$V^0(x_L) \geq R_0 - 1. \tag{3.3}$$

The first condition is self-explanatory; the second condition requires the offer price to be at most the market price; the last condition requires a low-quality firm's payoff in the pooling equilibrium to be at least that from revealing the firm's type and choosing $(q, f) = (1, 1/R_0)$. The last condition also implies that a high-quality firm gets a higher payoff in the pooling equilibrium than from choosing $(q, f) = (1, 1/R_0)$ and being viewed by the market as a low-quality firm.

We rewrite (3.1) and (3.2). Since $q_0/f_0 \leq p_\alpha$, the restriction $f_0 \in [0, 1]$ is equivalent to $p_\alpha \geq q_0 \geq 0$. With the price expression in (3.2), (3.1) becomes:

$$1 \geq q_0 \geq Q_0 \equiv \max \left\{ 0, 1 - \frac{R_\alpha - 1}{bE_\alpha x^{-1}} \right\}. \quad (3.4)$$

Since $R_\alpha > R_0 > 1$ by Assumption 1C', $Q_0 < 1$. For $q_0 \geq Q_0$, (3.2) can be replaced by

$$f_0 \geq S_\alpha(q_0) \equiv q_0 / \left[R_\alpha - (1 + bE_\alpha x^{-1})(1 - q_0) \right]. \quad (3.5)$$

Lemma 3.1. $V^0(x_H), V^0(x_L) \leq R_\alpha - 1$.

The proof for this lemma is in Appendix A. Intuitively, the amount $R_\alpha - 1$ is the firm's payoff when the firm obtains the expected earnings R_α without incurring any cost of alternative financing. This is the best the original owners can get in a pooling equilibrium, since the pooling equilibrium may involve less than 100% equity financing.

In the set of actions that satisfy (3.3), (3.4) and (3.5), only those that do not leave any room for "credible" deviations by a high-quality firm satisfy the Cho-Kreps intuitive criterion. To describe a credible deviation by a high-quality firm, consider a deviation $(f, q) \neq (f_0, q_0)$ that satisfies the following conditions. First, the deviation is feasible for a high-quality firm, i.e., $f, q \in [0, 1]$ and the offer price does not exceed the implied market price:

$$0 \leq q/f \leq p_H = R_H - (1 + bx_H^{-1})(1 - q). \quad (3.6)$$

Second, if a low-quality firm makes the same deviation, it gets less than in the pooling equilibrium even when it is viewed as a high-quality firm:

$$(1 - f) \left[R_H - (1 + bx_L^{-1})(1 - q) \right] < V^0(x_L). \quad (3.7)$$

Third, the deviation generates a higher payoff to the high-quality firm than in the pooling equilibrium when the firm is viewed as a high-quality firm as a result of the deviation:

$$(1 - f) \left[R_H - (1 + bx_H^{-1})(1 - q) \right] > V^0(x_H). \quad (3.8)$$

Actions that satisfy (3.6), (3.7) and (3.8) are credible deviations by a high-quality firm.

We can understand the credible deviations as follows. Deviations that satisfy (3.6) and (3.7) are feasible to the firms but yield lower payoffs to a low-quality firm than in the pooling equilibrium, even when the deviator is given the benefit of doubt and viewed as a high-quality firm. Thus, a low-quality firm will not make such deviations. When the deviations also satisfy (3.8), a high-quality firm wants to make such deviations if the market rewards the deviator by viewing it as a high-quality firm. Observing deviations that satisfy (3.6) – (3.8), the market should intuitively interpret the deviator as a high-quality firm. To satisfy this intuitive criterion, a pooling equilibrium cannot allow for deviations that satisfy (3.6)–(3.8). This restriction on beliefs off the equilibrium path eliminates a plethora of equilibria.⁹

Let us first examine the set of actions that satisfy (3.6) and (3.7). Under Assumption 1A, (3.6) can be rewritten as $f, q \in [0, 1]$ and

$$f \geq S_H(q) \equiv q / \left[R_H - (1 + bx_H^{-1})(1 - q) \right]. \quad (3.9)$$

To rewrite (3.7), define a critical level:

$$Q_1 \equiv 1 - \frac{R_H - V^0(x_L)}{1 + bx_L^{-1}}. \quad (3.10)$$

Note that Q_1 is less than one but is not necessarily greater than zero. If either $Q_1 < 0$ or $q \leq Q_1$ then (3.7) is satisfied for all $f \in [0, 1]$. For $q \geq \max\{0, Q_1\}$, (3.7) can be rewritten as

$$f > IND_L(q) \equiv 1 - V^0(x_L) / \left[R_H - (1 + bx_L^{-1})(1 - q) \right]. \quad (3.11)$$

Figures 2a and 2b depict the two curves $f = S_H(q)$ and $f = IND_L(q)$ for the two cases $Q_1 < 0$ and $Q_1 > 0$, respectively. The curve $f = S_H(q)$ is the full-price curve for a high-quality

⁹In the current context, separating equilibria that satisfy the intuitive criterion are the Riley (1979) outcomes.

firm, above which underpricing occurs. The curve $f = IND_L(q)$ is the set of actions to which a deviation by a low-quality firm generates the same payoff as the pooling action (f_0, q_0) when the firm is viewed as a high-quality firm after the deviation. Actions above the curve $f = IND_L(q)$ generate strictly lower payoffs to a low-quality firm even if the firm is viewed as a high-quality firm after a deviation to such actions. Thus, the shaded area in each diagram is the set of actions that satisfy (3.9) and (3.11) (i.e., (3.6) and (3.7)).

Figures 2a and 2b here.

The following lemma formally states the properties of the two curves $S_H(q)$ and $IND_L(q)$ in Figures 2a and 2b (see Appendix A for a proof):

Lemma 3.2. (i) Under Assumptions 1A – 1C, $IND_L(q)$ is an increasing and concave function for all $q > Q_1$; $S_H(q)$ is an increasing and concave function for all $q > 0$. (ii) If $Q_1 < 0$, then $IND_L(q) > S_H(q)$ for all $q \geq 0$; If $Q_1 \geq 0$, then there is a unique solution to $IND_L(q) = S_H(q)$ in the range $q \geq Q_1$, denoted Q_A , and $IND_L(q) > S_H(q)$ if and only if $q > Q_A$. (iii) A high-quality firm's payoff is an increasing function of q along $f = S_H(q)$ and a decreasing function of q along $f = IND_L(q)$.

Let us explain (iii) since it is important for our arguments below. A high-quality firm's payoff is an increasing function of q along the full-price curve $f = S_H(q)$ because, as the firm raises a higher revenue through IPO without underpricing, the firm economizes on the cost of alternative funds and so the expected profit is higher. To explain why a high-quality firm's payoff is a decreasing function of q along $f = IND_L(q)$, recall that a high-quality firm's desire to increase the number of shares issued in IPO is weaker than a low-quality firm's (see (2.7)). As actions move upward along the curve $f = IND_L(q)$, the IPO revenue increases and such actions are increasingly more enticing to a low-quality firm. To keep a low-quality firm indifferent between these actions and the pooling action, the number of shares issued in IPO must increase sharply, which is not desirable to a high-quality firm.

Now consider a deviation from the supposed pooling equilibrium to actions in the shaded areas in Figures 2a and 2b. As argued before, the market should intuitively view such deviations as coming from a high-quality firm and attach a belief $I = 1$ to the deviator. Similarly, a high-quality firm should consider only deviations that maximize its payoff. That is, any deviation in the shaded area that is not the best cannot be an equilibrium that satisfies the Cho-Kreps criterion, since further deviations from this action to the best actions do not change the market's belief ($I = 1$) but increase the firm's payoff.

The best deviation by a high-quality firm from the supposed pooling equilibrium is arbitrarily close to and above the action depicted by point A in Figure 2a if $Q_1 < 0$ and in Figure 2b if $Q_1 > 0$. To see this, note that the firm's payoff increases when actions move southeast in Figures 2a and 2b and so the best deviations are located arbitrarily close to and above the lower boundaries of the shaded areas. Moreover, since a high-quality firm's payoff is an increasing function of q along the full-price curve $f = S_H(q)$ and a decreasing function of q along the curve $f = IND_L(q)$ (see Lemma 3.2), the best deviation is arbitrarily close to and above point A in Figures 2a (or Figure 2b). The limit of this deviation is point A , described by:¹⁰

$$(f_b, q_b) \equiv \begin{cases} \left(1 - \frac{V^0(x_L)}{R_H - 1 - bx_L^{-1}}, & 0 \right), & \text{if } Q_1 \leq 0 \\ (S_H(Q_A), & Q_A), & \text{if } Q_1 > 0. \end{cases} \quad (3.12)$$

Lemma 3.3. *A high-quality firm gets a higher payoff from the deviation (f_b, q_b) than from any pooling action (f_0, q_0) iff $(1 - f_b)R_H > (1 - f_0)R_\alpha$.*

This lemma, proved in Appendix B, describes the condition necessary and sufficient for (3.8). When the deviation (f_b, q_b) dominates the pooling action under the Cho-Kreps intuitive criterion, there is no pooling equilibrium and so the best action for a low-quality firm is $(f, q) = (1/R_0, 1)$, yielding a payoff $R_0 - 1$. Replacing $V^0(x_L)$ by $R_0 - 1$, the condition $Q_1 \leq 0$ becomes $R_H \geq R_0 +$

¹⁰In the borderline case $Q_1 = 0$ (where point A coincides with the origin of the plane), the best deviation is $f = \varepsilon > 0$ and $q = 0$, where ε is sufficiently small. Since this case involves underpricing, it can be grouped with the case $Q_1 < 0$.

bx_L^{-1} and we denote the corresponding values of (f_b, q_b) by (f^*, q^*) . That is, for $R_H \geq R_0 + bx_L^{-1}$,

$$f^* = 1 - \frac{R_0 - 1}{R_H - (1 + bx_L^{-1})}, \quad q^* = 0; \quad (3.13)$$

and for $R_H < R_0 + bx_L^{-1}$,

$$\begin{cases} f^* & = & S_H(q^*); \\ \frac{q^*}{R_H - (1 + bx_H^{-1})(1 - q^*)} & = & 1 - \frac{R_0 - 1}{R_H - (1 + bx_L^{-1})(1 - q^*)}. \end{cases} \quad (3.14)$$

We have the following propositions (see Appendix B for a proof):

Proposition 3.4. *Under the Cho-Kreps intuitive criterion, there is a unique separating signalling equilibrium for given R_H . In this equilibrium, a high-quality firm takes actions (f^*, q^*) , characterized by (3.13) when $R_H - R_0 \geq bx_L^{-1}$ and by (3.14) when $R_H - R_0 < bx_L^{-1}$. A low-quality firm takes actions $(f, q) = (1/R_0, 1)$.*

Proposition 3.5. *A pooling signalling equilibrium for given R_H exists only when $Q_1 < 0$ and $q_0 < 1 - R_\alpha/R_H$. There exist $\bar{\alpha}, \underline{\alpha} \in (0, 1)$ such that, if $\alpha > \bar{\alpha}$, pooling equilibria exist for suitably restricted actions and beliefs. If $\alpha < \underline{\alpha}$, no pooling equilibrium exists.*

Some pooling equilibria survive the Cho-Kreps refinement because the extent to which a high-quality firm can signal is limited by the requirement that the IPO revenue be non-negative. When the expected earning of a high-quality firm is sufficiently higher than that of a low-quality firm, the high-quality firm must incur a sufficiently high signalling cost in order to prevent a low-quality firm from mimicking. This becomes difficult when there is a lower bound on the IPO revenue and so some pooling equilibria with a small IPO revenue can survive. In particular, when the prior for a high-quality firm, α , is large, the difference between R_α and R_0 is large and so the benefit from mimicking is large. Even with a very low q , a low-quality firm may get a higher payoff from mimicking than from taking the separating action, in which case the pooling equilibrium exists. When α is small, however, it does not pay to mimic and so the pooling equilibrium does not exist.

A high-quality firm has a preference over the two ways to signal. One is to reduce the number of shares issued in IPO and the other is to underprice IPO. Both methods reduce the IPO revenue

and serve as signals of high quality. However, it is preferable to reduce the number of shares than to underprice, if the former alone can achieve the signalling purpose successfully. By reducing the number of shares in IPO without underpricing, the firm keeps a larger stake of the firm and hence of its future earnings. In contrast, if the firm underprices its IPO it must give up a large number of shares to the public in order to raise the same amount of funds and hence give up a larger claim on the firm's future earnings.

But there is a limit to which a high-quality firm can signal successfully by reducing the number of IPO shares. Even by reducing f to zero the firm can only signal an expected earning of $R_0 + bx_L^{-1}$.¹¹ For expected earnings higher than this level, the firm must sacrifice even more in order to prevent a low-quality firm from mimicking and this entails underpricing. When the firm underprices IPO, the number of shares underpriced increases with the level of expected revenue that the firm wants to signal. That is, f increases with R_H in the underpricing region.

Therefore, the number of shares issued in IPO has a U -shaped relationship with the firm's expected earnings, as depicted in Figure 3. When a high-quality firm's expected earnings increase from low levels, the number of shares issued to the public decreases, while IPO is at the full market price. This continues until the number of shares issued to the public reaches a minimum, which is zero in this version of the model. Then the number of IPO shares increases with the expected earnings (see subsection 5.1 for more discussions). The absence of a monotonic (negative) relationship between f and underpricing is consistent with the empirical finding of Michaely and Shaw (1994).

Figure 3 here.

4. Market Equilibrium and Clustering

4.1. Characterization

Now we determine the market equilibrium by solving for expected earnings R_H . From now on we consider only the separating equilibrium. Since such an equilibrium is unique for any given R_H ,

¹¹This is obtained by setting $(f, q) = (0, 0)$ and $V^0(x_L) = R_0 - 1$ in the equality form of (3.11).

the multiplicity of market equilibria in this section has nothing to do with the usual multiplicity associated with signalling equilibria; instead, it arises because multiple values of R_H can be supported by rational expectations.

A symmetric market equilibrium is a pair (d, D) such that $d = D$ and that d is a best response of a high-quality firm to D , given implicitly by Proposition 3.4. To write the best response explicitly, denote

$$D_0 \equiv \frac{1}{\rho} \left[\frac{b}{x_L} - R_0 \left(\frac{x_H}{x_L} - 1 \right) \right]. \quad (4.1)$$

Then $D_0 > 0$ (Assumption 1D). Also, $R_H - R_0 \geq bx_L^{-1}$ if and only if $D \geq D_0$. A high-quality firm's best response to other firms' decisions is:

$$d = \begin{cases} 0, & \text{if } D < D_0 \\ p_H = \rho D + R_0 x_H / x_L - 1 - bx_H^{-1}, & \text{if } D \geq D_0. \end{cases} \quad (4.2)$$

This is the underpricing curve in Figure 4. An important feature is that each high-quality firm responds positively to other firms' underpricing (D), which is the key to clustering.

To characterize market equilibria, denote

$$\underline{\rho} \equiv \frac{bx_L^{-1} - R_0(x_H/x_L - 1)}{b(x_L^{-1} - x_H^{-1}) + R_0 - 1}. \quad (4.3)$$

Then $0 < \underline{\rho} < 1$ (Assumptions 1A and 1D). Imposing the equilibrium requirement $d = D$ on (4.2), we can verify the following proposition (the proof is omitted):

Proposition 4.1. *Under Assumptions 1A – 1D, there is a market equilibrium for all $0 \leq \rho < 1$ where no firm underprices IPO. A market equilibrium with IPO underpricing exists if and only if $\underline{\rho} \leq \rho < 1$. Thus, when $0 \leq \rho < \underline{\rho}$, only the no-underpricing equilibrium exists; when $\underline{\rho} \leq \rho < 1$, the underpricing equilibrium and the no-underpricing equilibrium co-exist. In the underpricing equilibrium, the amount of underpricing increases with ρ .*

Figure 4 depicts the case $\underline{\rho} < \rho < 1$. The no-underpricing equilibrium is at point EN and the underpricing equilibrium is at point EU .¹² In both equilibria high-quality firms successfully

¹²When $\underline{\rho} \leq \rho < 1$, there might also be a mixed-strategy equilibrium at $D = D_0$ if we allow each high-quality firm to underprice with a probability in $(0, 1)$. The mixed-strategy equilibrium does not add much to the issues that we focus on in this paper, such as clustering of underpricing.

separate themselves from low-quality firms. In the no-underpricing equilibrium they do so by reducing the number of IPO shares and, in the underpricing equilibrium, by underpricing.

Figure 4 here.

The above proposition has several noteworthy aspects. First, the no-underpricing equilibrium is the only equilibrium when $0 \leq \rho < \underline{\rho}$, in which case the level D_0 is large and the entire underpricing curve lies below the 45-degree line in Figure 4. Thus, no firm underprices its IPO if the industry's publicity does not have a strong positive externality on each firm's earnings. Similarly, there is no underpricing if there is only one firm in the industry.¹³

Second, the no-underpricing equilibrium exists for all $\rho \in [0, 1)$. Even when the industry's publicity has a strong externality on each firm's earnings (i.e., when $\rho > \underline{\rho}$), there may not be any underpricing if firms expect the industry's publicity to be low. With such expectations, the difference in expected earnings between a high-quality firm and a low-quality firm is small, as maintained by Assumption 1D. Then low-quality firms' temptation to mimic is weak, in which case a high-quality firm can separate itself from a low-quality firm by reducing the number of shares in IPO without underpricing. The absence of underpricing in turn supports the expectations that the industry's publicity is low.

Third, the two equilibria both exist when the externality is sufficiently strong (i.e., when $\underline{\rho} \leq \rho < 1$). The coexistence is an outcome of self-fulfilling expectations. We have already explained why a firm will not underprice if it expects that other firms will not underprice. On the other hand, if a high-quality firm expects that other high-quality firms will underprice, the difference in expected earnings between a high-quality firm and a low-quality firm is large, due to the influence of the industry's publicity. A low-quality firm's temptation to mimic is strong in this case and so a high-quality firm must underprice in order to separate itself from a low-quality firm. Since all high-quality firms underprice IPOs in this case, there is clustering. The clustering

¹³As noted before, Assumption 1D is important for this result. When the intrinsic earning difference between a high-quality and a low-quality firm is large enough to violate Assumption 1D, then $\underline{\rho} < 0$ and there is a need for a high-quality firm to underprice anyway. In fact, only the underpricing equilibrium exists in this case.

of underpriced IPOs in turn fulfills the expectations that the industry's publicity is high. The coexistence of the no-underpricing equilibrium with the underpricing equilibrium illustrates the fragility of the clustering of underpriced IPOs.

Finally, the model is capable of producing large underpricing. In the underpricing equilibrium high-quality firms offer their shares free of charge! When the expected earning difference between a high-quality firm and a low-quality firm passes the critical level bx_L^{-1} , the amount of underpricing jumps discontinuously from zero to 100% of the market price and the offer price drops to zero. (Of course, a zero offer price is unrealistic. In subsection 5.1 we extend the model to generate a positive offer price in the underpricing equilibrium.) Considering that the intrinsic difference between high-quality and low-quality firms is assumed to be small (Assumption 1D), the large magnitude of underpricing in our model is remarkable. If the same assumption is imposed in previous signalling models without publicity, there will be no underpricing.

The large magnitude and the multiplicity of equilibria rely on the feature that the market price of shares is endogenous and depends on the IPO decisions of all firms in the industry. If all high-quality firms take the action of no-underpricing, the industry's publicity is low and so the share price is low. But if all high-quality firms take the action of underpricing, the industry's publicity is significantly higher and so the share price is high. In the latter case the small difference between firms' intrinsic earnings is magnified by the industry's publicity into a large difference in the market price of shares, leading to the large magnitude and clustering of underpricing. In contrast, previous signalling models assume that the market price of shares is exogenous in equilibrium once the firm's type is known, and so they cannot generate underpricing when the difference between firms' intrinsic earnings is small as restricted in Assumption 1D.

4.2. Testable Implications

The results on clustering are novel and we list them below.

Implication 1: Large underpricing tends to cluster in time.

This implication simply restates the meaning of an underpricing equilibrium: In such an equilibrium, it is optimal for a high-quality firm to underprice its own IPO in response to the price cut by other high-quality firms.

Implication 2: Large underpricing tends to cluster in specific industries where there is a great uncertainty about the performance of the industry as a whole.

This implication comes from the role of the industry-wide component of earnings, m . Firms cluster their IPO underpricing decisions because the industry-wide component of earnings is uncertain and its expected value varies with the industry's publicity. If the industry's performance is less uncertain, the industry's publicity has a smaller effect on each firm's earning (i.e., ρ is smaller) and hence firms are less likely to underprice. In this sense, the clustering of underpriced IPOs may be only a temporary phenomenon in new industries such as the Internet industry where publicity is likely to yield a large benefit initially. As the industry becomes established, competition against other firms in the same industry becomes more important than against firms in traditional sectors. In this case one firm's underpricing may hurt rather than benefit other firms in the same industry and so IPOs with large underpricing are less likely to cluster.¹⁴

Despite the obvious role of the industry's publicity for clustering, we want to repeat that the publicity alone suppresses rather than induces clustering, by creating a free-rider problem (Proposition 2.1). The presence of asymmetric information is important for overcoming the free-rider problem. Because a high-quality firm must convince the market of its quality in order to capture the benefit of the industry's publicity, such a firm is willing to take costly actions such as underpricing to achieve the signalling purpose.

Implication 3: The frequency and the magnitude of clustering are likely to be higher in economic expansions than in downturns. Likewise, the clustering is more likely to occur in an economy

¹⁴This case corresponds to $\rho < 0$. Since underpricing is the best response to other firms' underpricing only if $\rho D > bx_L^{-1} - R_0(x_H/x_L - 1)$ (see (4.1) and (4.2)), under Assumption 1D there cannot be an underpricing equilibrium when $\rho < 0$.

with an easy access to the credit market than in an economy with a difficult access, other things being equal between the two economies.

To see this, first note that the cost of alternative funds is likely to be lower and the average expected earnings among firms are likely to be higher in economic expansions than in downturns. That is, b is lower and R_0 is higher in expansions than in downturns. Similarly, b is lower and R_0 is higher in an easy credit market than in a tight credit market. Note that the lowest value of ρ for an underpricing equilibrium to exist, i.e., $\underline{\rho}$ in (4.3), increases in b and decreases in R_0 . Thus, for any given ρ firms are more likely to underprice when the cost of alternative funds is lower and/or when the average earnings are higher.

The explanation is as follows. When the cost of alternative funds is low, it is less costly to underprice IPO since the firm can easily find alternative funds to make up for the shortfall in the IPO revenue. This makes it less costly for a low-quality firm to mimic, and so it is more likely that high-quality firms resort to very costly actions such as underpricing to separate themselves out. If high-quality firms underprice, the amount of underpricing is also higher for any given ρ . Similarly, when R_0 is high, the intrinsic earning difference between a high-quality and a low-quality firm is high. The temptation to mimic is high for low-quality firms and, again, high-quality firms are more likely to underprice IPOs in order to signal quality successfully.

5. Extensions and Robustness

In this section we extend the model to impose a lower bound on equity financing, to allow a firm to create its own publicity in addition to the industry's publicity, and to examine sequential IPO decisions. These extensions all illustrate the robustness of the results and improve the model's quantitative implications. For example, by imposing a lower bound on equity financing, we show that the offer price can be positive in the underpricing equilibrium.

5.1. Lower Bound on Equity Financing

A firm may not be able to obtain as much non-equity fund as it likes and so must obtain a minimum IPO revenue. Let this minimum be $Q_b s/p$, where $Q_b \in (0, 1)$. The specification incorporates the idea that the suppliers of alternative funds may be more willing to supply funds to a firm whose IPO has a larger price gain. Substituting the market price of shares in a separating equilibrium, we can rewrite the constraint $q \geq Q_b s/p$ for a high-quality firm as:

$$f \geq \frac{Q_b}{R_H - (1 + bx_H^{-1})(1 - q)} \equiv LB(q). \quad (5.1)$$

With this constraint, the separating action depicted by point A in Figures $2a$ and $2b$ may no longer be feasible to a high-quality firm. A scenario is depicted in Figure 5 for the case $Q_1 > 0$, in which the best separating action is given by point B rather than A . Because point B lies above the full-price curve $f = S_H(q)$, the IPO is underpriced. Such an underpricing equilibrium exists if and only if the curve $f = LB(q)$ crosses the curve $f = IND_L(q)$ before crossing $f = S_H(q)$. Equivalently, this requires $IND_L(Q_b) > S_H(Q_b)$, which is satisfied when Q_b is sufficiently close to 1.

Figure 5 here.

Two properties of the separating equilibrium here are in contrast with the simple model. First, an underpricing firm's offer price can be positive, as is at point B in Figure 5. Second, the number of shares issued in IPO does not necessarily increase with earnings in the underpricing equilibrium. In Figure 5, for example, when R_H increases, the curve $f = IND_L(q)$ shifts up but the curve $f = LB(q)$ shifts down. These two forces change f in opposite ways and so analytically the effect of R_H on f is ambiguous in the underpricing equilibrium. When the externality is sufficiently strong, however, f is likely to increase with R_H . In this case firms underprice greatly, as in the simple model.

5.2. A Firm's Own Influence on Publicity

A firm may directly benefit from its underpricing in addition to the industry's publicity. To allow for this benefit, let us return to the simple model and modify market expectations on m as

$$E(m|D, d, \text{perceived quality of the firm} = x_H) = \rho(\gamma d + D),$$

where d is the firm's own underpricing and $\gamma > 0$ is the relative impact of the firm's own underpricing on its expected earnings. A firm benefits more from its own publicity than from the industry's publicity if $\gamma > 1$. The simple model examined before corresponds to $\gamma = 0$.

If a firm is perceived as of high-quality, its expected earning is $R_H = R_0 x_H / x_L + \rho(\gamma d + D)$. In contrast to previous sections, now a firm cannot take R_H as given because R_H depends on the firm's own decisions. Denote the part that the firm takes as given by $W = R_0 x_H / x_L + \rho D$. The market price of the firm's shares under the belief I is

$$p_I = \frac{1}{1 - I\rho\gamma} \left[IW + (1 - I)R_0 - I\rho\gamma q/f - (1 + bE_I x^{-1})(1 - q) \right]. \quad (5.2)$$

For the price to be positive, we restrict $0 \leq \rho < 1/\gamma$. The constraint $s \leq p_H$ can be written as

$$f \geq q \left[W - (1 + b x_H^{-1})(1 - q) \right]. \quad (5.3)$$

Proposition 5.1. *There exist $\gamma_1 > 0$ and $\rho_1 \in (0, 1/(1 + \gamma))$ such that an underpricing equilibrium exists if $\rho \in (\rho_1, 1/(1 + \gamma))$ and $\gamma \leq \gamma_1$. There exist $\gamma_2 > 0$ and $\rho_2 \in (0, 1/\gamma)$ such that a no-underpricing market equilibrium exists if $0 \leq \rho < \rho_2$ and $\gamma \leq \gamma_2$. Moreover, $\rho_1 < \rho_2$ and so the two market equilibria coexist when $\rho \in (\rho_1, \rho_2)$ and $\gamma \leq \min\{\gamma_1, \gamma_2\}$.*

The proof of this proposition is in Appendix C. This proposition shows that the qualitative results in this extended environment are similar to those in the simple model. In particular, IPO underpricing can cluster when the industry's publicity has a strong effect on individual firms' expected earnings. This may be the case even when $\gamma > 1$. That is, even when a firm benefits more from its own publicity at the margin than from the industry's publicity, it may still be optimal for the firm to respond to other firms' underpricing by underpricing its own IPO.

5.3. Sequential Decisions

In the simple model we have assumed that different firms go to the IPO market at the same time. By this we do not mean that firms in reality literally make their IPO decisions at the same date but rather that some firms' IPO dates may be close to each other so that one firm cannot change the IPO decision to take into account of observed actions by other firms. Although this interpretation is appealing, one may still want to know what happens if firms can modify their IPO decisions upon observing other firms' actions. We analyze this sequential game now and show that firms still tend to cluster their IPO underpricing decisions.

Consider only two firms, firm 1 and firm 2. Firm 1 goes to the IPO market at date 1 and firm 2 at date 2. To simplify matters, we assume that both firms have earnings only at date 2 and there is no time discounting. Let d_i be the amount of underpricing by firm $i = 1, 2$. Market expectations on firm i 's m , conditional on the perception that the firm is of high quality, are given by (2.2) with D being replaced by $d_{i'}$ ($i' \neq i$).

Given d_1 , firm 2's pricing decision is analogous to that analyzed in the simple model. That is, if firm 2 is a low-quality firm, then $d_2 = 0$; if firm 2 is a high-quality firm, then

$$d_2 = \begin{cases} 0, & \text{if } d_1 < D_0 \\ \rho d_1 + R_0 x_H / x_L - 1 - b x_H^{-1}, & \text{if } d_1 \geq D_0, \end{cases}$$

where D_0 is defined in (4.1). Note that firm 2 responds to firm 1's underpricing positively.

Firm 1 anticipates such influence of its pricing decision on firm 2's. Given firm 1's prior on firm 2's quality, the expected amount of firm 2's IPO underpricing is

$$\alpha \chi_{(d_1 > D_0)} (\rho d_1 + R_0 x_H / x_L - 1 - b x_H^{-1}),$$

where $\chi_{(d_1 > D_0)} = 1$ if $d_1 > D_0$ and 0 otherwise. Suppose firm 1 chooses $d_1 < D_0$. Then $d_2 = 0$ and there is no publicity from which firm 1 can benefit. In this case firm 1's best decision is $d_1 = 0$ and the payoff to both firms is identical to that in the no-underpricing equilibrium in the simple model. This can be a market equilibrium in the current case if and only if the payoff to firm 1 is not lower than that generated by the action $d_1 \geq D_0$.

Now suppose firm 1 chooses $d_1 \geq D_0$. If the market believes that the firm is of high-quality with probability I , the expected earning of the firm is

$$R_I = (1 - I)R_0 + I \left[R_0 x_H / x_L + \rho \alpha (\rho d_1 + R_0 x_H / x_L - 1 - b x_H^{-1}) \right].$$

Slightly change the earlier notation to denote $W = (1 + \rho \alpha) R_0 x_H / x_L - \rho \alpha (1 + b x_H^{-1})$. The market price of such a firm under the belief I is

$$p_I = \frac{1}{1 - I \alpha \rho^2} \left[I W + (1 - I) R_0 - I \alpha \rho^2 q / f - (1 + b E_I x^{-1}) (1 - q) \right].$$

This is similar in form to the market price in the last subsection, with $\alpha \rho^2$ replacing $\rho \gamma$, and so firm 1's offer price decision can be analyzed analogously. To ensure a positive share price, we restrict $0 \leq \rho < \alpha^{-1/2}$. The proof of the following proposition is in Appendix D:

Proposition 5.2. *There exist $\rho_3, \rho_4 \in (0, \alpha^{-1/2})$ such that firm 1 underprices IPO if and only if it is of high-quality and $\rho \in (\rho_3, \rho_4)$. Firm 2 underprices IPO if and only if it is of high-quality and if firm 1 underprices IPO.*

Example 5.3. *The interval (ρ_3, ρ_4) can be non-empty: When $\alpha = 0.1$, $b = 0.2$, $x_L = 1$, $x_H = 1.18$ and $R_0 = 1.02$, $\rho_3 = 1.75 < \rho_4 = 1.91$.*

As the example shows, both firms underprice IPOs in some cases. More importantly, when firm 1 underprices IPO, firm 2 will do so as well if it is of high quality. Since such underpricing would not occur if there were only one firm in the industry or if publicity had no effect on expected earnings, the result shows that the interaction between firms is important for the clustering of underpriced IPOs, just as in the case of simultaneous decisions. It is not surprising then that the underpricing equilibrium here also requires the externality to be strong (i.e., $\rho > \rho_3$).¹⁵

In contrast to the case of simultaneous moves, too strong an externality (i.e., $\rho > \rho_4$) will destroy the underpricing equilibrium in the current case. This is because underpricing is costly

¹⁵Firm 1's underpricing is not always echoed by firm 2, since firm 2 may turn out to be a low-quality firm. This uncertainty is eliminated in the case of simultaneous moves with the assumption of a large number of firms. As a result, the amount of underpricing is larger there than here.

and, when the externality is very strong, the amount of underpricing is too large to be desirable for firm 1 as the first mover in the game.

Multiplicity of equilibria disappears with sequential moves. However, this is an artifact of the exogenously fixed order of moves by the two firms. Being a first mover is costly in the current setup. Firm 1 must underprice sufficiently in order to entice firm 2 to underprice. If firms can choose when to go to the IPO market, they may go to the market at dates that are very close to each other in order to explore the great externality. Then, the multiplicity analyzed in the simple model would reappear.¹⁶

6. Conclusion

In this paper we have shown that industry-wide uncertainty in earnings, together with private information and self-fulfilling expectations, can induce many firms to underprice IPOs at the same time. When the industry's publicity greatly affects market expectations on the industry's performance and when firms' qualities are private information, each high-quality firm finds it optimal to underprice its own IPO in response to other firms' underpricing. In this case underpricing clusters and is large in magnitude. The industry's publicity induces a high-quality firm to underprice because it increases the temptation for a low-quality firm to mimic, making underpricing a necessary action for a high-quality firm to separate itself out.

Private information and expectations are critical for the story. If firms' qualities are public information, the publicity only induces firms to free ride on other firms' underpricing, which eliminates underpricing altogether in equilibrium. Underpricing is not inevitable. Whenever there is an underpricing equilibrium, there is also another equilibrium without underpricing. Thus, even when the industry's publicity has a strong effect on firms' expected earnings, firms may not underprice when they do not expect other firms to underprice.

Our emphasis on the clustering is a marked shift from the literature's emphasis on a single firm's underpricing. Three conclusions of our model seem to be general. First, the clustering of

¹⁶Tambanis and Bernhardt (1999) explicitly model the possibility that firms can delay the timing of their equity issue. However, they do not analyze IPO underpricing.

large IPO underpricing is an industry-wide phenomenon. Industries that have a very uncertain outlook and that are susceptible to the influence of publicity are more likely to exhibit the clustering of underpriced IPOs. Second, the clustering of underpriced IPOs is both fragile and specific — fragile because the clustering is unlikely to occur if firms do not expect other firms to underprice, specific because important for the story is some publicity from which firms collectively benefit. Third, IPO underpricing is more likely to cluster when the marginal cost of alternative funds is low or when the average return to firms is high. Thus, even for established industries, more firms will underprice IPOs when the economy is in good times than in bad times.

Once we accept that the industry-wide uncertainty is an important determinant of the clustering of underpriced IPOs, it is possible to view the phenomenal “hot-issues” market in 1999 as a rational outcome. Observing that prices of individual Internet stocks are well above the corresponding firms’ expected earnings per share, some commentators have argued that bubbles exist in these stocks. This argument overlooks the possibility that the high market prices might not be so much an indication of specific firms’ future earnings but rather of how the Internet industry as a whole will perform in the future. The high share prices can be rational if the total expected future earnings of the Internet industry match the total value of Internet stocks available today. Nevertheless, the clustering of hot IPOs is fragile. Adverse news about even a single firm can greatly affect all IPO performances in that industry, since the news may induce investors to switch the expectations from the underpricing equilibrium to the no-underpricing equilibrium.¹⁷

¹⁷An example is the Biotech industry that experienced large underpricing in IPOs at the beginning of the 1990s. The heat over biotech stocks cooled down considerably when the Food and Drug Administration rejected several promising drugs such as Centocor Inc.’s Centoxin, a medicine meant to fight a deadly bacteria infection common in surgery patients.

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Appendix

A. Proofs of Lemmas 3.1 and 3.2

For Lemma 3.1, we have:

$$\begin{aligned}
V^0(x_H) &= (1 - f_0) \left[R_\alpha - \left(1 + bx_H^{-1}\right) (1 - q_0) \right] \\
&\leq \left\{ 1 - q_0 / \left[R_\alpha - \left(1 + bE_\alpha x^{-1}\right) (1 - q_0) \right] \right\} \left[R_\alpha - \left(1 + bx_H^{-1}\right) (1 - q_0) \right] \\
&\leq \left\{ 1 - q_0 / \left[R_\alpha - \left(1 + bx_H^{-1}\right) (1 - q_0) \right] \right\} \left[R_\alpha - \left(1 + bx_H^{-1}\right) (1 - q_0) \right] \\
&= R_\alpha - \left(1 + bx_H^{-1}\right) (1 - q_0) - q_0 \\
&\leq R_\alpha - 1.
\end{aligned}$$

The first inequality follows from substituting the lower bound on f_0 in (3.5); the second inequality follows because the preceding expression is increasing in x ; and the last inequality follows because the preceding expression is increasing in q_0 . The same procedure establishes $V^0(x_L) \leq R_\alpha - 1$.

For Lemma 3.2, we can verify the monotone and concavity features of $S_H(q)$ and $IND_L(q)$ directly. To prove other properties in the lemma, note that

$$S_H(1) = 1/R_H < 1/R_\alpha < 1 - (R_\alpha - 1)/R_H < 1 - V^0(x_L)/R_H = IND_L(1).$$

The third inequality above follows from Lemma 3.1.

Consider first the case $Q_1 < 0$ (see Figure 2a). In this case the relevant range of q is $q \in [0, 1]$.

Since $Q_1 < 0$, we have

$$S_H(0) = 0 < 1 - V^0(x_L)/(R_H - 1 - bx_L^{-1}) = IND_L(0).$$

Thus, the curve $IND_L(q)$ lies above the curve $S_H(q)$ at both ends. If we can show that $IND_L(q)$ crosses $S_H(q)$ always from below if they ever cross each other in the positive quadrant, then the two curves never cross each other and so $S_H(q) < IND_L(q)$ for all $q \in [0, 1]$. To show the crossing property, suppose the two curves cross each other at $q_c \in [0, 1]$, i.e.,

$$1 - V^0(x_L) / \left[R_H - (1 + bx_L^{-1})(1 - q_c) \right] = q_c / \left[R_H - (1 + bx_H^{-1})(1 - q_c) \right]. \quad (\text{A.1})$$

Computing the derivatives $IND'_L(q)$ and $S'_H(q)$ and substituting $V^0(x_L)$ from (A.1), we can show

that $IND'_L(q_c) - S'_H(q_c)$ has the same sign as that of the following expression:

$$\left[R_H - (1 + bx_L^{-1})(1 - q_c) \right] q_c b x_H^{-1} + \left[R_H - (1 + bx_H^{-1})(1 - q_c) - q_c \right] \times \left\{ (1 + bx_L^{-1}) \left[R_H - (1 + bx_H^{-1})(1 - q_c) \right] - \left[R_H - (1 + bx_L^{-1})(1 - q_c) \right] \right\}.$$

The expression in $\{.\}$ is clearly positive. Also, Assumption 1A implies

$$R_H - (1 + bx_H^{-1})(1 - q_c) - q_c > R_H - (1 + bx_H^{-1}) > 0.$$

Since $Q_1 < 0$, then $R_H - (1 + bx_L^{-1})(1 - q_c) > V^0(x_L) > 0$. Thus, $IND'_L(q_c) > S'_H(q_c)$. That is, the curve $IND_L(q)$ is steeper than the curve $S_H(q)$ whenever the two cross each other. This is the desired result and so $S_H(q) < IND_L(q)$ for all $q \in [0, 1]$ in this case.

Consider now the case $Q_1 > 0$. Since $IND_L(q) < 0$ and $S_H(q) > 0$ if $0 \leq q < Q_1$, the two curves cannot cross each other in this range. Thus, consider only the range $q \geq Q_1$. In this range the above proof for the crossing property between $IND_L(q)$ and $S_H(q)$ goes through. Moreover, $IND_L(Q_1) = 0 < S_H(Q_1)$. Therefore, there is a unique crossing between the two curves.

Along $f = IND_L(q)$, a high-quality firm's payoff is

$$[1 - IND_L(q)][R_H - (1 + bx_H^{-1})(1 - q)] = V^0(x_L) \cdot \frac{R_H - (1 + bx_H^{-1})(1 - q)}{R_H - (1 + bx_L^{-1})(1 - q)},$$

which is a decreasing function of q . Along $f = S_H(q)$, a high-quality firm's payoff is

$$[1 - S_H(q)] \left[R_H - (1 + bx_H^{-1})(1 - q) \right] = R_H - (1 + bx_H^{-1})(1 - q) - q,$$

which is an increasing function of q .

QED

B. Proofs of Lemma 3.3 and Propositions 3.4 and 3.5

We prove Lemma 3.3 first. When out-of-equilibrium beliefs satisfy the Cho-Kreps intuitive criterion, the deviation (f_b, q_b) in (3.12) generates the following gain to a high-quality firm relative to a pooling equilibrium:

$$\begin{aligned} & (1 - f_b) \left[R_H - (1 + bx_H^{-1})(1 - q_b) \right] - V^0(x_H) \\ = & (1 - f_b) \left[R_H - (1 + bx_H^{-1})(1 - q_b) \right] - (1 - f_b) \left[R_H - (1 + bx_L^{-1})(1 - q_b) \right] \\ & + \left\{ (1 - f_b) \left[R_H - (1 + bx_L^{-1})(1 - q_b) \right] - V^0(x_L) \right\} + [V^0(x_L) - V^0(x_H)] \\ = & b(x_L^{-1} - x_H^{-1})(1 - f_b)(1 - q_b) - b(x_L^{-1} - x_H^{-1})(1 - f_0)(1 - q_0) \\ = & \frac{b(x_L^{-1} - x_H^{-1})}{1 + bx_L^{-1}} [(1 - f_b)R_H - (1 - f_0)R_\alpha]. \end{aligned}$$

The first equality follows from adding and subtracting the same terms; the second equality follows from the fact that the term in $\{\cdot\}$ is zero by the definitions of (f_b, q_b) ; the third equality follows from substituting the definitions of q_b and q_0 . Then Lemma 3.3 is evident.

For Propositions 3.4 and 3.5, we locate the position of the pooling action (f_0, q_0) . Since the pooling action must satisfy (3.5), it must lie on or above the curve $f = S_\alpha(q)$. Also, we can verify that $IND_L(q_0) > f_0$ and so the point (f_0, q_0) must lie below the curve $f = IND(q)$. This implies $f_0 > f_b$ in the case $Q_1 > 0$ (see Figure 2b).

Consider first the case $Q_1 > 0$ (Figure 2b). Since $f_b < f_0$ in this case and $R_H > R_\alpha$, the gain to a high-quality firm from the deviation to (f_b, q_b) is strictly positive. Thus there cannot be a pooling equilibrium in this case. The only equilibrium is a separating equilibrium (f^*, q^*) defined by (3.14). The condition corresponding to this case, $Q_1 > 0$, becomes $R_H - R_0 < bx_L^{-1}$.

Now consider the case $Q_1 \leq 0$, where the separating actions are given by (3.13). These actions may not necessarily generate a higher payoff to a high-quality firm than in the pooling equilibrium. In fact, since

$$\begin{aligned} (1 - f_b)R_H - (1 - f_0)R_\alpha &= \frac{V^0(x_L)}{R_H - (1 + bx_L^{-1})} R_H - (1 - f_0)R_\alpha \\ &= \frac{1 - f_0}{R_H - (1 + bx_L^{-1})} [R_\alpha - R_H(1 - q_0)], \end{aligned}$$

the gain to a high-quality firm from deviating from the pooling action to (f_b, q_b) is strictly positive if and only if $q_0 > 1 - R_\alpha/R_H$. Thus, (f^*, q^*) form a unique separating equilibrium against pooling actions with q_0 sufficiently close to 1. In this case the corresponding condition ($Q_1 \leq 0$) becomes $R_H - R_0 \geq bx_L^{-1}$. This completes the proof of Proposition 3.4.

For Proposition 3.5, we know from the above that a pooling action satisfies the Cho-Kreps intuitive criterion if and only if (3.3), (3.4), (3.5), $Q_1 \leq 0$ and $q_0 \leq 1 - R_\alpha/R_H$ are all satisfied. From the definition of Q_0 in (3.4) we have $Q_0 > 0$ if and only if $R_\alpha - 1 - bE_\alpha x^{-1} < 0$, i.e., iff

$$\alpha < \alpha_0 \equiv \frac{1 + bx_L^{-1}}{R_H + b(x_L^{-1} - x_H^{-1})}.$$

Note that $\alpha_0 \in (0, 1)$ under Assumption 1A. Consider the case $\alpha > \alpha_0$ and so $Q_0 < 0$, in which case all $q_0 \in (0, 1 - R_\alpha/R_H]$ satisfy (3.4). For any such q_0 , let f_0 solve (3.5) as an equality and

note $f_0 \in (0, 1)$. The payoff to a low-quality firm from this pooling action is

$$\left[1 - \frac{q_0}{R_\alpha - (1 + bE_\alpha x^{-1})(1 - q_0)} \right] \left[R_\alpha - (1 + bx_L^{-1})(1 - q_0) \right].$$

Both terms of the product are increasing functions of q_0 (for $q_0 > 0 > Q_0$). Thus the payoff is maximized by setting $q_0 = 1 - R_0/R_H$. If this maximum pooling payoff satisfies (3.3) with strict inequality, then there exist q 's lower than but close to q_0 that satisfy (3.3) as well. After substituting $R_\alpha = R_0 + \alpha(R_H - R_0)$ and $E_\alpha x^{-1} = x_L^{-1} - \alpha(x_L^{-1} - x_H^{-1})$, we can verify that the maximum pooling payoff satisfies (3.3) with strict inequality iff

$$\alpha - \frac{1 - \alpha}{R_H - 1 - bx_L^{-1} + b\alpha(x_L^{-1} - x_H^{-1})} + \frac{R_H - R_0(1 + bx_L^{-1})}{(R_H - R_0)(R_H - 1 - bx_L^{-1})} > 0.$$

The left-hand side of the above inequality is an increasing function of α . When $\alpha = 0$, its value is negative. When $\alpha = 1$, its value has the same sign as

$$(R_H - R_0)(R_H - 1 - bx_L^{-1}) + R_H - R_0(1 + bx_L^{-1}).$$

This is positive, since $R_H \geq R_0 + bx_L^{-1}$ (as $Q_1 \leq 0$) and the above expression has a value 0 when $R_H = R_0 + bx_L^{-1}$. Therefore there exists $\underline{\alpha} \in (0, 1)$ such that (3.3) is satisfied with strict inequality for the above described (f_0, q_0) if $\alpha > \underline{\alpha}$. Define $\bar{\alpha} \equiv \max\{\alpha_0, \underline{\alpha}\}$. For $\alpha > \bar{\alpha}$, there exist pooling actions (f_0, q_0) that satisfy the Cho-Kreps intuitive criterion. These actions can be supported as pooling equilibria for given R_H by the following beliefs: Any deviation from such (f_0, q_0) is believed to be coming from a low-quality firm.

On the other hand, if $\alpha < \underline{\alpha}$, no pooling action satisfies the Cho-Kreps criterion. QED.

C. Proof of Proposition 5.1

Let V_L^0 be the payoff to a low-quality firm from a pooling action (f_0, q_0) . As in the simple model, we find separating actions that generate lower payoffs to a low-quality firm than in a pooling equilibrium. Then we choose the best among these actions as a candidate for the action of a high-quality firm in a separating equilibrium. If a low-quality firm deviates from the pooling

action to an action (f, q) and is perceived as a high-quality firm, the payoff is

$$(1-f) \left[W + \rho\gamma(p_H - s) - (1 + bx_L^{-1})(1-q) \right] = \frac{1-f}{1-\rho\gamma} [W - \rho\gamma q/f - (1+z)(1-q)],$$

where $W = R_0x_H/x_L + \rho D$ and $z = \rho\gamma b/x_H + (1-\rho\gamma)b/x_L$. This payoff is less than that in the pooling equilibrium if and only if

$$\begin{aligned} q < G(f) &\equiv \frac{1+z-W+(1-\rho\gamma)V_L^0/(1-f)}{1+z-\rho\gamma/f}, & \text{for } f > \frac{\rho\gamma}{1+z}; \\ q > G(f), & & \text{for } f < \frac{\rho\gamma}{1+z}. \end{aligned}$$

Figures 6a and 6b here.

Let us divide the proof into two cases.

Case 1: $W > (1+z)[1 + (1-\rho\gamma)V_L^0/(1+z-\rho\gamma)]$. This case is depicted in Figure 6a. Let $S_H(q)$ now denote the right-hand side of (5.3) and let its inverse be S_H^{-1} . It can be shown that there exists $\gamma_1 > 0$ such that $G(f) > S_H^{-1}(f)$ in the region $f < \rho\gamma/(1+z)$ if $\gamma \leq \gamma_1$, as depicted in Figure 6a. Restrict attention to $\gamma \leq \gamma_1$. In this case the relevant region is $f > \rho\gamma/(1+z)$ and the shaded area is the set of actions that yield lower payoff to a low-quality firm but may yield higher payoff to a high-quality firm than in the pooling equilibrium. We can verify the following properties for the segment of $G(f)$ with $f > \rho\gamma/(1+z)$:

(1a) $G(f) > 0$ iff $f > 1 - (1-\rho\gamma)V_L^0/(W-1-z)$ (i.e., iff f is higher than point A).

(1b) $G'(f) > 0$ for all $f > 1 - (1-\rho\gamma)V_L^0/(W-1-z)$.

(1c) The payoff to a high-quality firm from taking actions along $q = G(f)$ is decreasing in f .

These properties imply that, if $\gamma \leq \gamma_1$, the best deviation for a high-quality firm from a pooling equilibrium is point A in Figure 6a. In this case, $q = s = 0$ and there is underpricing as in the corresponding case in the simple model.

Case 2: $W < (1+z)[1 + (1-\rho\gamma)V_L^0/(1+z-\rho\gamma)]$. In this case, the best deviations for a high-quality firm in the region $f < \rho\gamma/(1+z)$ lie on the curve $f = S_H(q)$ and, by property (2c) below, they are strictly dominated by the action at point A in Figure 6b. Thus, it suffices to consider only the region $f > \rho\gamma/(1+z)$. The curve $q = G(f)$ for $f > \rho\gamma/(1+z)$ is depicted

by Figure 6b, where the shaded area is the set of deviations that are feasible to a firm (when perceived as a high-quality firm as a result of deviation) and that generate lower payoffs to a low-quality firm than in the pooling equilibrium. A lengthy exercise can establish the following properties, some of which are depicted in Figure 6b:

(2a) There exists a level $f_c \in (\rho\gamma/(1+z), 1)$ such that the curve $q = G(f)$ is decreasing in f for $f \in (\rho\gamma/(1+z), f_c)$ and increasing in f for $f \in (f_c, 1)$.

(2b) $S_H(1) = 1/W > \rho\gamma/(1+z)$ and $G(1/W) < 1$. That is, the intersection between the curve $f = S_H(q)$ and $q = 1$ lies in the region $q > G(f)$ and $f > \rho\gamma/(1+z)$. Since the curve $f = S_H(q)$ starts outside this region when q is small, there is at least one intersection between $f = S_H(q)$ and $q = G(f)$, as depicted by point A in Figure 6b.

(2c) A high-quality firm's payoff from actions along the curve $f = S_H(q)$ increases in q .

(2d) A high-quality firm's payoff from actions along the curve $q = G(f)$ (for $f > \rho\gamma/(1+z)$) decreases in f for all $f \geq (\rho\gamma/W)^{1/2}$.

(2e) There exists $\gamma_2 > 0$ such that, if $\gamma \leq \gamma_2$, the intersection (point A) has $f \geq (\rho\gamma/W)^{1/2}$.

These properties imply that, if $\gamma \leq \gamma_2$, the payoff to a high-quality firm from deviating from the pooling action is maximized at the intersection between the curve $f = S_H(q)$ and $q = G(f)$, such as point A in Figure 6b. There is no underpricing in this case.

When α is sufficiently small, in both case 1 and case 2 one can also show that the payoff at point A to a high-quality firm is higher than the payoff in the pooling equilibrium, provided that the market views such deviation as coming from a high-quality firm. Thus, the action given by point A is the separating equilibrium that satisfies the Cho-Kreps criterion. Substituting $W = R_0 x_H/x_L + \rho D$ and noting that the payoff to a low-quality firm is $R_0 - 1$ in the absence of pooling (thus V_L^0 in the above analysis is replaced by $R_0 - 1$), we have,

$$d = p_H = \frac{1}{1-\rho\gamma} [\rho D + R_0 x_H/x_L - 1 - b/x_H], \quad (C.1)$$

$$\text{if } R_0 \frac{x_H}{x_L} + \rho D > (1+z) \left[1 + \frac{(1-\rho\gamma)(R_0-1)}{1+z-\rho\gamma} \right],$$

$$d = 0 \quad \text{if } R_0 \frac{x_H}{x_L} + \rho D < (1+z) \left[1 + \frac{(1-\rho\gamma)(R_0-1)}{1+z-\rho\gamma} \right]. \quad (C.2)$$

To solve for market equilibria, impose symmetry $d = D$. Doing so for case 1 we get:

$$d = D = \frac{R_0 x_H / x_L - 1 - b/x_H}{1 - \rho(1 + \gamma)}.$$

Thus, $d > 0$ only if $\rho < 1/(1 + \gamma)$. Also, (C.1) must be satisfied in order to have $D > 0$, i.e.,

$$R_0 \frac{x_H}{x_L} + \rho \frac{R_0 x_H / x_L - 1 - b/x_H}{1 - \rho(1 + \gamma)} > (1 + z) \left[1 + \frac{(1 - \rho\gamma)(R_0 - 1)}{1 + z - \rho\gamma} \right]. \quad (\text{C.3})$$

Note that z and $(1 - \rho\gamma)/(1 + z - \rho\gamma)$ are decreasing functions of ρ and so is the right-hand side of the above inequality. The left-hand side is an increasing function of ρ . Since the inequality is satisfied for $\rho = 1/(1 + \gamma)$ and violated for $\rho \rightarrow 0$, there exists a critical level $\rho_1 \in (0, 1/(1 + \gamma))$ such that the above inequality is satisfied if and only if $\rho > \rho_1$. Therefore, an underpricing equilibrium exists if $\rho_1 < \rho < 1/(1 + \gamma)$ and $\gamma \leq \gamma_1$.

For the no-underpricing equilibrium, impose $d = D = 0$ in case 2. The equilibrium exists if

$$R_0 \frac{x_H}{x_L} < (1 + z) \left[1 + \frac{(1 - \rho\gamma)(R_0 - 1)}{1 + z - \rho\gamma} \right]. \quad (\text{C.4})$$

The right-hand side of this inequality is a decreasing function of ρ . The inequality is satisfied when $\rho \rightarrow 0$ and violated when $\rho \rightarrow 1/\gamma$. Thus, there exists $\rho_2 \in (0, 1/\gamma)$ such that the inequality is satisfied for $0 < \rho < \rho_2$. If $\gamma \leq \gamma_2$, in addition, the no-underpricing equilibrium exists.

Comparing (C.3) and (C.4) we can immediately show $\rho_1 < \rho_2$. Therefore, the underpricing equilibrium and the no-underpricing equilibrium coexist if $\rho \in (\rho_1, \rho_2)$ and $\gamma \leq \min\{\gamma_1, \gamma_2\}$. This completes the proof of Proposition 5.1. QED

D. Proof of Proposition 5.2

We have already argued in the text that firm 2 underprices only if firm 1 underprices sufficiently (i.e., if $d_1 \geq D_0$). Analogous to the derivation of (C.1) in Appendix C, we have:

$$d_1 = \frac{1}{1 - \alpha\rho^2} (W - 1 - b/x_H), \quad (\text{D.1})$$

$$\text{if } W > (1 + \alpha\rho^2 b x_H^{-1} + (1 - \alpha\rho^2) b x_L^{-1}) \left[1 + \frac{(1 - \alpha\rho^2)(R_0 - 1)}{1 + \alpha\rho^2 b x_H^{-1} + (1 - \alpha\rho^2) b x_L^{-1} - \alpha\rho^2} \right], \quad (\text{D.2})$$

where $W = (1 + \rho\alpha)R_0x_H/x_L - \rho\alpha(1 + bx_H^{-1})$. The underpricing equilibrium has $q = G(f) = 0$.

With V_L^0 being set to $R_0 - 1$, $G(f) = 0$ implies:

$$f = 1 - \frac{(1 - \alpha\rho^2)(R_0 - 1)}{W - [1 + \alpha\rho^2bx_H^{-1} + (1 - \alpha\rho^2)bx_L^{-1}]}.$$
 (D.3)

For firm 1 to underprice, d_2 must also be positive and so we need $d_1 \geq D_0$, i.e.

$$W - 1 - b/x_H \geq \frac{1 - \alpha\rho^2}{\rho} \left[\frac{b}{x_L} - R_0 \left(\frac{x_H}{x_L} - 1 \right) \right].$$
 (D.4)

Note that W increases in ρ and the right-hand side of (D.2) decreases in ρ . Moreover, (D.2) is satisfied when $\rho \rightarrow \alpha^{-1/2}$ and is violated when $\rho \rightarrow 0$. Then, there exists $\rho_a \in (0, \alpha^{-1/2})$ such that (D.2) is satisfied if and only if $\rho \in (\rho_a, \alpha^{-1/2})$. Similarly, there exists $\rho_b \in (0, \alpha^{-1/2})$ such that (D.4) is satisfied if and only if $\rho \in [\rho_b, \alpha^{-1/2})$. Let $\rho_3 = \max\{\rho_a, \rho_b\}$. Then both (D.2) and (D.4) are satisfied if and only if $\rho \in (\rho_3, \alpha^{-1/2})$.

In addition to the requirement $\rho \in (\rho_3, \alpha^{-1/2})$, the payoff to firm 1 (when it is high-quality) must be higher with $d_1 > 0$ than with $d_1 = 0$ in order for the firm to underprice. With $d_1 = 0$, the payoff to high-quality firm 1 is

$$\begin{aligned} & (1 - f^*) \left[R_0 \frac{x_H}{x_L} - \left(1 + \frac{b}{x_H} \right) (1 - q^*) \right] = R_0 \frac{x_H}{x_L} - \left(1 + \frac{b}{x_H} \right) (1 - q^*) - q^* \\ & = (R_0 - 1) [R_0 x_H/x_L - (1 + b/x_H)(1 - q^*)] / [R_0 x_H/x_L - (1 + b/x_H)(1 - q^*)] \end{aligned}$$

where the inequalities come from substituting the definitions of (f^*, q^*) in (3.14). When $d_1 > 0$ in (D.1), $q = 0$ and f is given by (D.3). The total return to shareholders is $(W - 1 - bx_H^{-1})/(1 - \alpha\rho^2)$ and the payoff to high-quality firm 1 from underpricing is

$$\frac{(R_0 - 1)(W - 1 - bx_H^{-1})}{W - [1 + \alpha\rho^2bx_H^{-1} + (1 - \alpha\rho^2)bx_L^{-1}]}.$$

Substituting W and simplifying, we can show that the firm's payoff is higher with underpricing than without if and only if

$$\frac{1 - \alpha\rho^2}{1 + \alpha\rho} > \frac{(1 - q^*)(R_0 x_H/x_L - 1 - b/x_H)}{R_0 x_H/x_L - (1 + b/x_H)(1 - q^*)}.$$

There exists $\rho_4 \in (0, \alpha^{-1/2})$ such that the above condition is satisfied if and only if $0 \leq \rho < \rho_4$.

The level ρ_4 is not necessarily greater than ρ_3 . Only when $\rho_4 > \rho_3$ and $\rho \in (\rho_3, \rho_4)$ does high-quality firm 1 underprice IPO. QED

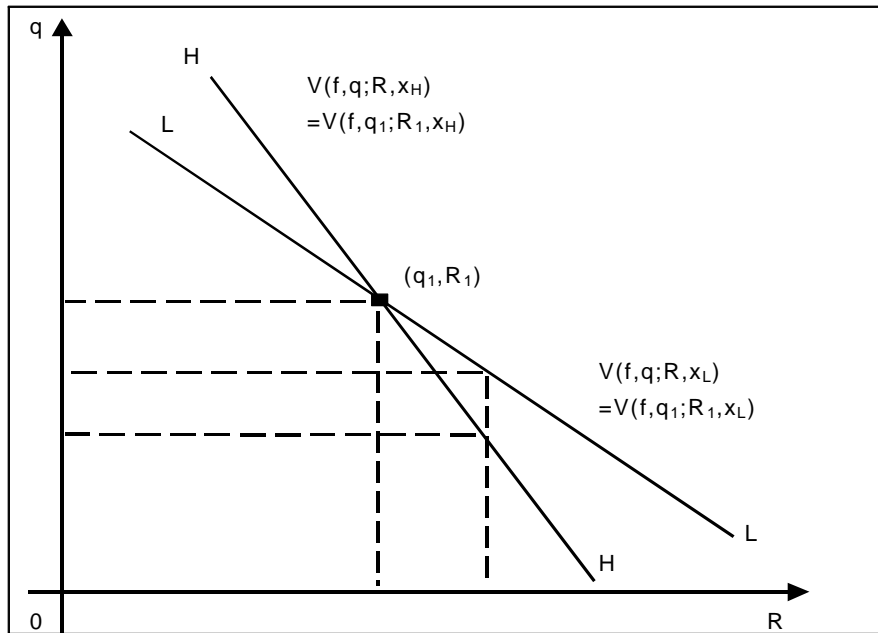


Figure 1a Firms' relative incentive to change q for fixed f in response to an increase in R

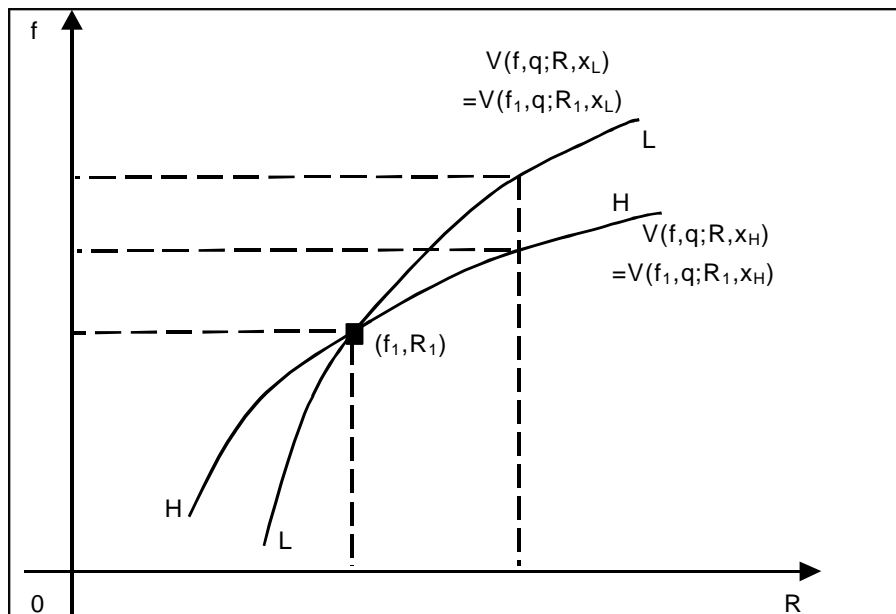


Figure 1b Firms' relative incentive to change f for fixed q in response to an increase in R

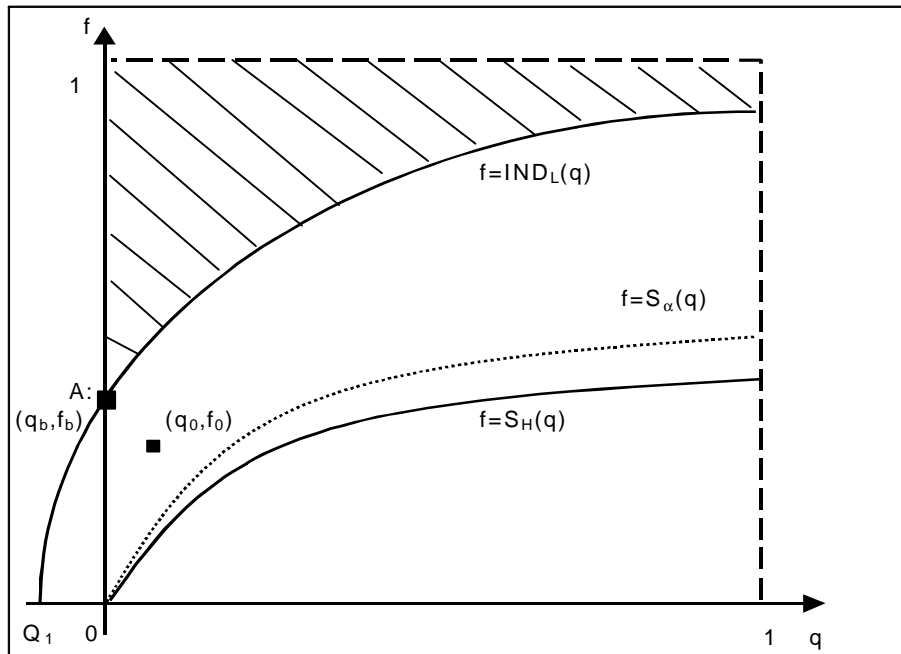


Figure 2a Deviations by a high-quality firm: $Q_1 < 0$

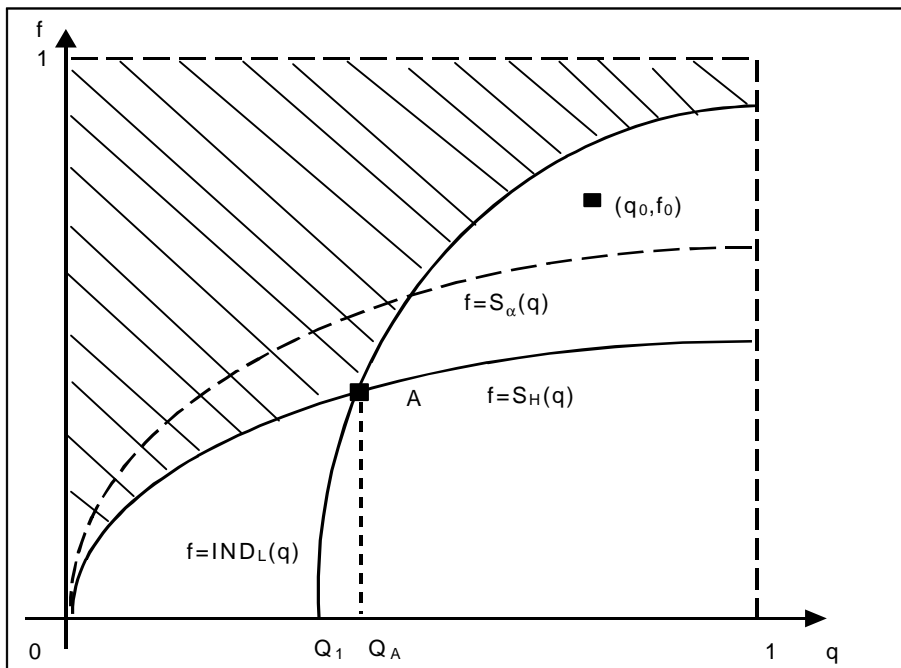


Figure 2b Deviations by a high-quality firm: $Q_1 > 0$

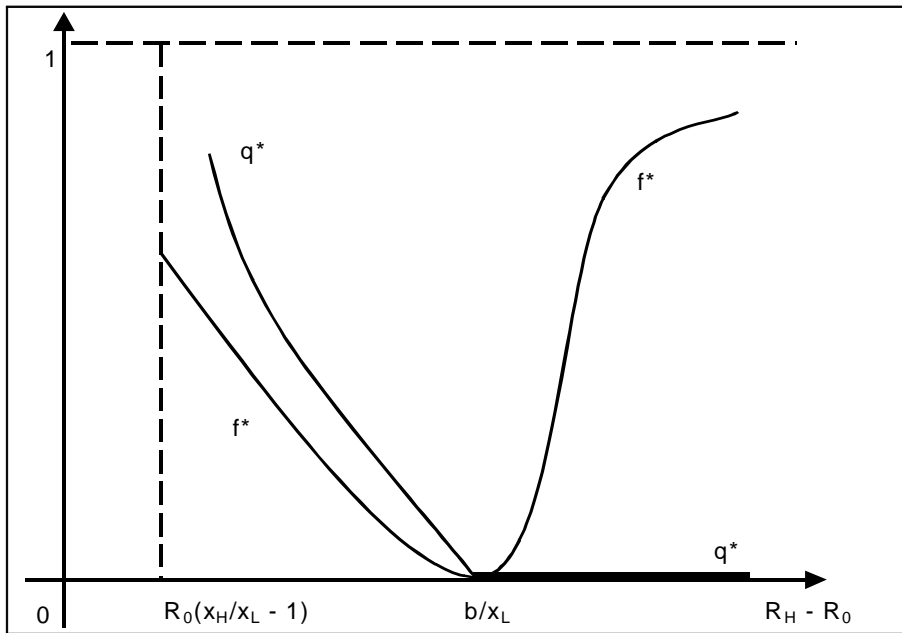


Figure 3 Dependence of (f, q) on the earnings difference between a high-quality and a low-quality firm in the separating equilibrium

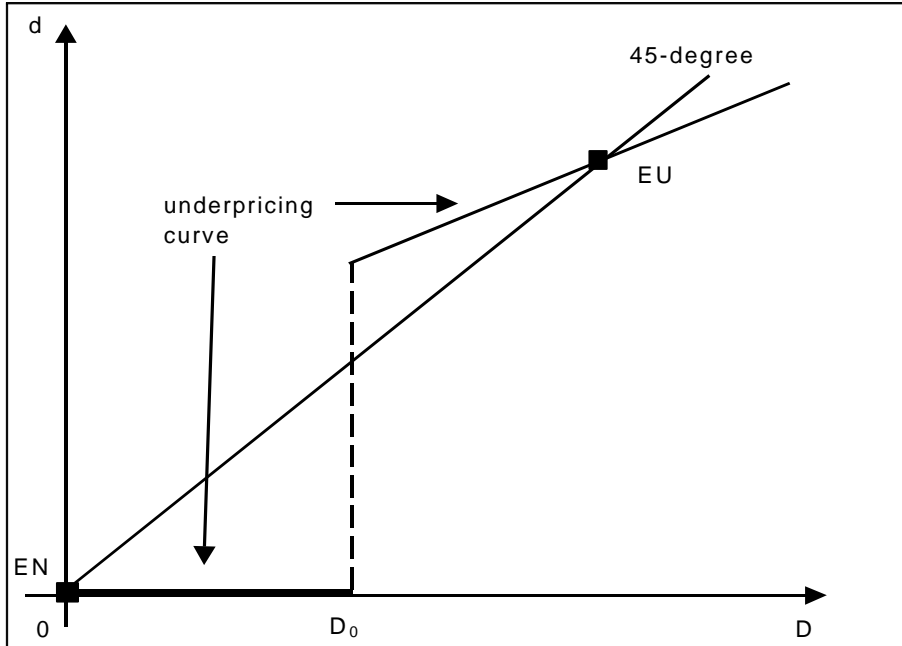


Figure 4 Market equilibria

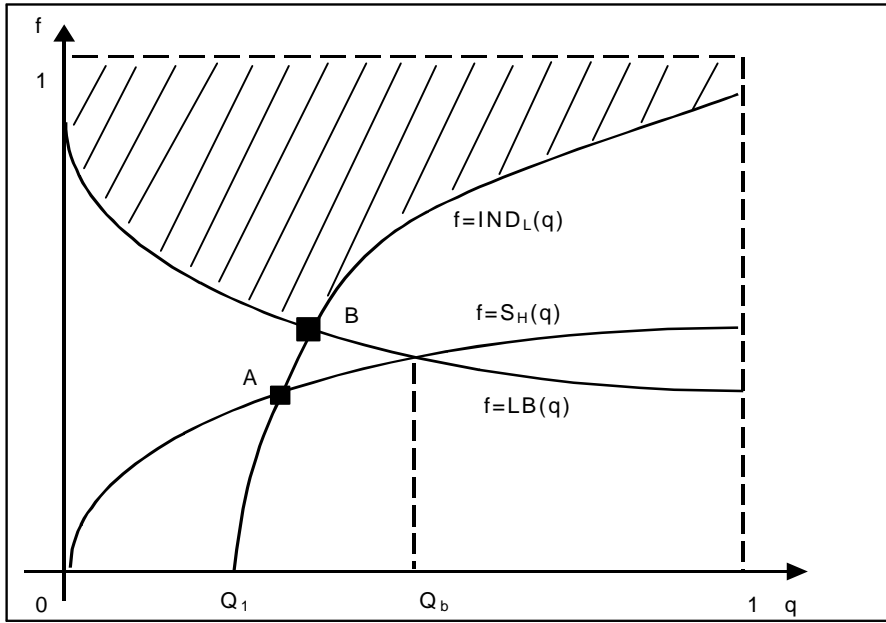


Figure 5 A separating equilibrium when there is a lower bound on the amount of equity financing

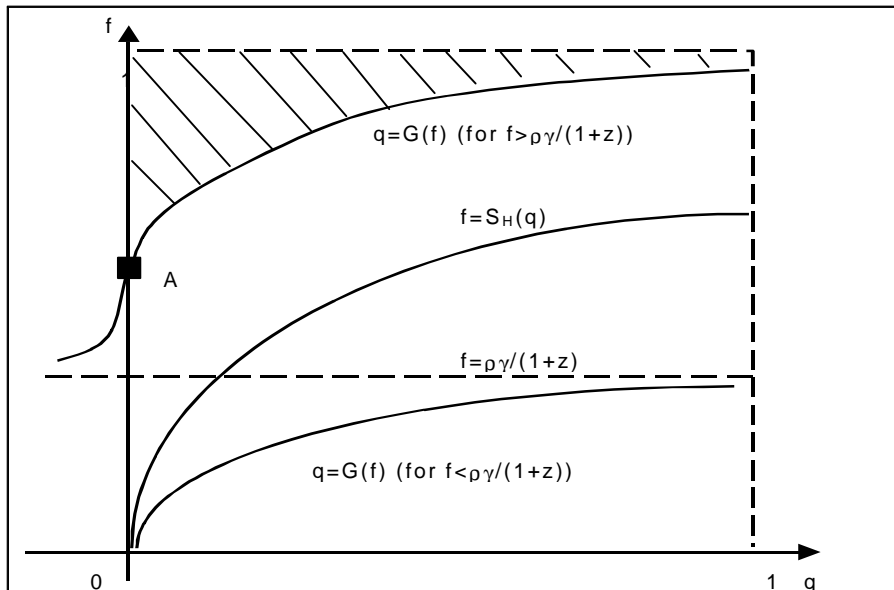


Figure 6a When a high-quality firm has its own influence on publicity: Case 1 (large W)

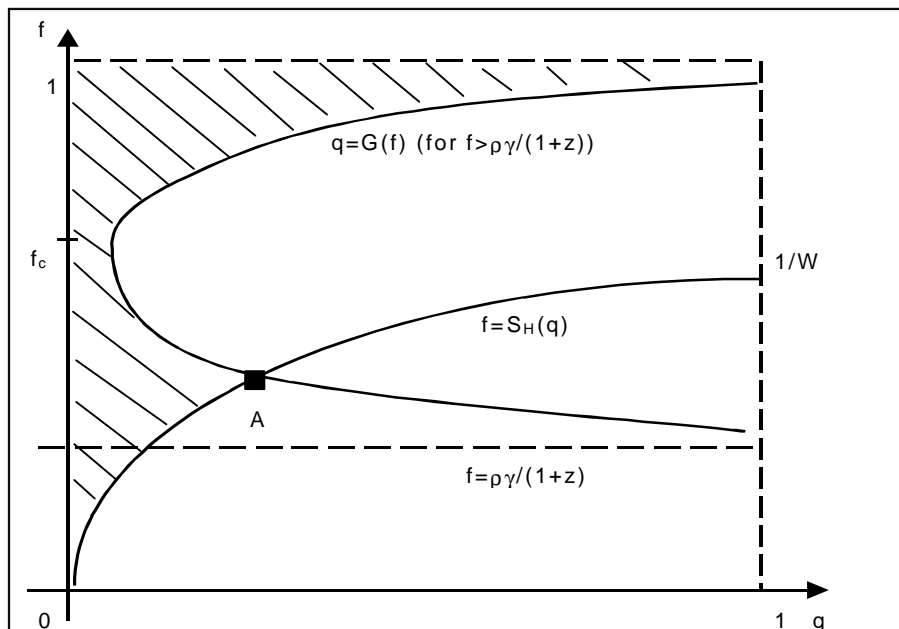


Figure 6b When a high-quality firm has its own influence on publicity: Case 2 (small W)