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Search, inflation and capital accumulation☆

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Abstract

This paper constructs a model to integrate the search monetary theory into a neoclassical growth model. With divisible goods and money, the model is used to examine the relationship between money growth and capital accumulation. The framework uncovers a distinct extensive effect that an increase in the money growth rate increases the frequency of successful trades by increasing the number of agents in the market. This positive extensive effect on the number of trades can dominate the conventional negative intensive effects of money growth on individuals' labor input and real money balance, in which case increasing the money growth rate increases aggregate capital and output. © 1999 Elsevier Science B.V. All rights reserved.

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☆This paper builds on, but substantially differs from, a section of an earlier paper circulated under the title "A simple divisible search model of fiat money". I am grateful to a referee and an associate editor (Stephen Williamson) for comments and suggestions that have led to significant improvements upon the first draft. I have also benefited from comments by Merwan Engineer and participants of the conference on "Exchanges in Search Equilibrium" at Penn State University. Financial support from the Social Sciences and Humanities Research Council of Canada is gratefully acknowledged. All errors are mine alone.

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1. Introduction

For monetary policies to have any robust effect on real output and economic growth, they must influence the accumulation of productive factors. Thus, as suggested by Johnson (1962), the question of how money growth affects capital is important in monetary economics. To address this question, monetary theorists have tried to integrate money into the neoclassical growth framework. Some well-known models are produced that differ in the treatment on money demand, ranging from *ad hoc* ones (Tobin, 1965) to intermediate treatments (Sidrauski, 1967; Brock, 1974; Stockman, 1981) and to more detailed accounts of the frictions that make money valuable (Wallace, 1980; Townsend, 1987). Despite existence of such well-known models, the question is far from being resolved for the following reason. Previous models assume that all exchanges are carried out in the Walrasian fashion. Since demand is always met with supply, what matters for price is the total amount of goods and money balances involved but not the extent of exchanges. Given the aggregate money balance used in exchange, whether the balance is used by, say, one million buyers or one billion buyers is not important for the price level. This means that, although the aggregate quantity of trade might be important, the number of trades is not important for the transactions cost in these models, which does not seem to be a good description of the transaction role of money.

The goal of the current paper is to integrate a non-Walrasian monetary model into the neoclassical growth framework and examine how money growth affects capital by affecting the extent of exchanges. Non-Walrasian exchanges are modelled using the search-theoretic framework of Kiyotaki and Wright (1991, 1993), where agents are randomly matched in pairs and trades in each match must be *quid pro quo*. Exchanges are costly because not everyone can immediately find a suitable match. The decentralized exchange process, together with restrictions on preferences and technology, implies that two randomly matched agents cannot have a double coincidence of wants in barter. This problem is alleviated by the use of fiat money.

As a necessary step to integrate the search model into the neoclassical growth framework, I introduce capital, whose presence as a predetermined variable also makes the dynamic stability analysis much more interesting than in previous search models. Also necessary are the extensions to make both goods and money divisible. In the original Kiyotaki–Wright model both goods and money are indivisible and so exchanges are one-for-one swaps of agents' holdings. In this paper, the buyer and the seller in a match can choose to exchange any non-negative quantities of goods and money, provided that the amount of money does not exceed the buyer's money holding. Divisible goods are incorporated following Shi (1995) and Trejos and Wright (1995); divisible money is incorporated by adapting my earlier work (Shi, 1997). Divisibility of both goods and money allows for price determination and a link of inflation to money growth.

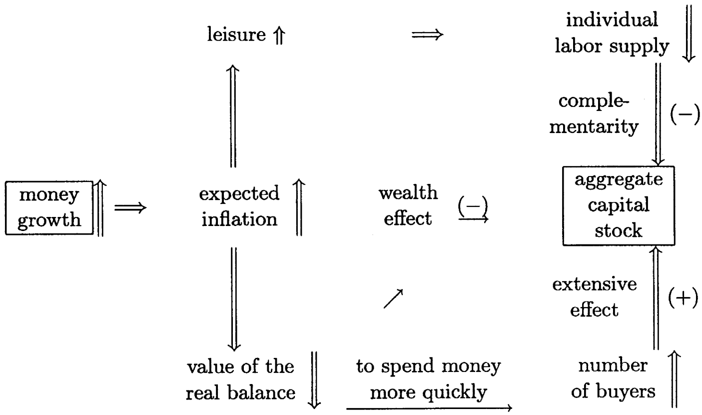

Fig. 1. Effects of money growth on aggregate capital.

A distinct feature of the non-Walrasian exchange process is that aggregate capital and output depend on both the quantity of goods exchanged in each match (the intensive margin) and on the number of matches (the extensive margin). Money growth affects aggregate capital and output on both margins. On the extensive margin, an increase in the money growth rate can increase the number of agents participating in the exchange and the number of matches. The mechanism for this positive extensive effect is simple. When the money growth rate increases, the expected inflation makes it costly to hold onto money and so it is rational for households to exchange money for goods more quickly. Since the only way to do so is to have more agents transact using money, the number of buyers increases and so does the number of matches. This extensive effect of money growth is illustrated by the bottom chain of effects in Fig. 1.

Although the extensive effect of money growth is new, the intensive effects of money growth accord well with the results in Walrasian models, as illustrated by the top and middle chains of effects in Fig. 1. First, in response to money growth, buyers are willing to accept a higher price of goods and higher inflation. Second, inflation induces agents to shift consumption from market goods to leisure and so each producer's (seller's) labor input decreases with money growth, which reduces each seller's incentive to employ capital since capital and labor are complementary in production. Third, inflation reduces individual buyer's real money balance. Since purchases on capital require cash, as in Stockman (1981), a lower real money balance represents a negative wealth effect that reduces capital accumulation. Despite these negative intensive effects, the positive extensive effect of money growth can dominate if the money growth rate is low. In this case, aggregate capital input and aggregate output increase with money growth, a result opposite to that in Stockman (1981) despite the similarity in the cash constraint.

To see the importance of non-Walrasian exchanges for the extensive effect, one may compare the current model with the Walrasian model by Jovanovic (1982), which is a general equilibrium version of Baumol (1952). Jovanovic shows that inflation shortens the time interval between which households take their goods to the market. Although this sounds like the extensive effect, it is fundamentally different. In any fixed interval of time, although aggregate quantities of trade in Jovanovic's model might be affected by inflation as individuals change the number of trips to the market, the aggregate number of trade is indeterminate and immaterial and so there is no benefit on the extensive margin. Increasing the number of trips to the market is then merely an attempt to economize on money balances, which reduces aggregate consumption and welfare.

In comparison with my earlier paper (Shi, 1997), the main contributions here are the introduction of capital and a more natural equilibrium concept. While households in Shi (1997) ignore their influence on the quantities of trade when they choose money balances, households in the current model fully take into account of such influence. This variation in modelling also illustrates the robustness of the extensive effect of money growth.[1]

The remainder of this paper is organized as follows. Section 2 describes the economy and the household's decision problem. Section 3 defines a symmetric monetary equilibrium, examines existence and stability of the equilibrium, and analyzes the effects of money growth. Section 4 concludes the paper and the appendices provide proofs.

2. Description of the economy

Time is discrete. There are H types of households and H types of goods, where $H \geq 3$. There are a large number of households in each type, normalized to one, but I will refer to one arbitrary household of type $h \in H$ as household h. The decisions of household h will be described in detail, with lower-case letters denoting its decision variables. Capital-case variables denote other households' decision variables, which are taken as given by household h. Of course, in any

[1] Li (1994) has examined the effect of inflation on (non-productive) inventory in a search model with indivisible goods and money. Also, since indivisibility restricts the price level to unity, inflation in Li's model does not correspond to the usual notion of deteriorating purchasing power of money. Nor can it be tied to money growth, since the money supply must be constant in any stationary equilibrium.

symmetric equilibrium lower-case variables are equal to the corresponding capital-case variables.

Household h consumes only good h and produces only good $h+1$ (with modulus H). Good h is called household h's *consumption good*. A good can be stored and used as capital in production only by the households that consume the good. Thus, as in a standard one-sector neoclassical growth model, consumption good and capital good are physically the same good for each household; households accumulate capital for its productive power and its future use in consumption. In contrast to a standard growth model, output and consumption goods are not the same for a household; one's output is someone else's consumption good. Since non-consumption goods are perishable, no one will exchange for them and so goods including capital cannot be used as a medium of exchange, ruling out commodity money.[2] Without loss of generality, I assume that capital does not depreciate.

As in Kiyotaki and Wright (1991, 1993), agents are randomly matched in pairs in each period and every trade must be mutually beneficial to the two sides of the match. Since $H \geq 3$, two households are unable to both supply consumption goods to each other. There is no double coincidence of wants in barter between households. All exchanges are mediated through the use of an intrinsically useless object called money, which is perfectly storable by all agents and perfectly divisible. Monetary exchanges are possible for household h when it meets household $(h-1)$, in which case household h exchanges money for goods, or when it meets household $(h+1)$, in which case household h produces goods in exchange for money. In each of these two meetings, there is a single coincidence of wants in goods between the two households and money serves as a medium of exchange. The probability that a household has a single coincidence of wants with another randomly selected household is $\alpha \equiv 1/H$.

Random matching generates randomness in agents' consumption, capital/money stocks and labor input. In a stationary equilibrium there will be non-degenerate distributions of values of these variables across agents. Although some progress has been made to solve for the distribution of money balances in simpler search models (e.g. Green and Zhou, 1996), the analysis is not tractable in the presence of capital accumulation. To maintain tractability, I assume that each household consists of a continuum of members, who carry out different tasks but all share the same consumption and regard the household's utility as

[2] Allowing producers to be able to store their products introduces the possibility of commodity money and complicates the analysis considerably. Barring the possibility of commodity money, the existence of inventory will unlikely change the main qualitative results of the paper. As shown in Shi (1996), the existence of inventory does not eliminate the extensive effect of money growth. On the contrary, inventory can provide an additional propagation channel for money growth shocks to generate persistent, hump-shaped output responses.

Table 1
Members in a household

	Total		Desirable matches	
	Set	Size	Set	Size
Money holders	$\mathscr{B}$	b_t	$I_{mt} \subset \mathscr{B}$	$\alpha N b_t$
Producers	$\mathscr{N}$	n	$I_{pt} \subset \mathscr{N}$	$\alpha n B_t$
Leisure seekers	$(\mathscr{B} \cup \mathscr{N})^c$	$1 - n - b_t$	N.A.	

the common objective. Although each member may experience random outcomes in the matching process, his consumption and utility do not depend on his own luck, since idiosyncratic risks are smoothed within each household. This construction greatly simplifies the analysis because it permits the use of a representative household: Except for the differences in the types of goods they produce and consume, different households can have the same quantities of consumption, capital accumulation and leisure. It is then possible to focus on symmetric equilibria and aggregate effects of monetary policies.[3]

Members in each household are grouped into money holders (buyers), producers (sellers) and leisure-seekers, each performing one task at a time. A money holder tries to exchange money for consumption goods and a producer tries to produce goods for money. A leisure-seeker enjoys leisure and does not participate in the market. Table 1 lists the notation for the members of the representative household. The size of members together is normalized to one. The fraction of buyers, b_t, can vary over time but the fraction of sellers is fixed at a constant $n \in (0,1)$. Other households' fraction of sellers is denoted N, although $N = n$ in any symmetric equilibrium. Fixing the fraction of sellers facilitates the illustration and will be discussed in Shi (1998).

In each period t, agents are randomly matched in pairs. Only matches in which the buyer wants the seller's goods generate trade. These matches are termed *desirable matches*. For household h, its sellers in desirable matches are those who meet buyers in household $(h + 1)$. Denote this subset of buyers by I_{pt}, where p indicates that the agents can produce. Household h's buyers in desirable

[3] Assuming risk-sharing is common in macroeconomic models (e.g. Lucas, 1990). At the aggregate level, the continuum in each household can be alternatively interpreted as a unit interval of time endowed to a representative agent in a standard macroeconomic model. Although this alternative interpretation is more natural, implementing it in the current setting is considerably more difficult. First, it is difficult to construct a matching technology that eliminates aggregate uncertainty throughout the trading period. Second, the sequential decisions inherited in the dynamic interpretation are more difficult to detail.

matches are those who meet sellers in household ($h-1$). Denote this subset of sellers by I_{mt}, where m indicates that the agents hold money. Each seller meets a desirable seller with probability αB_t and so the size of I_{pt} is $\alpha B_t n$. Similarly, the size of I_{mt} is $\alpha N b_t$.[4]

In each match the buyer and the seller can trade any non-negative quantities of goods and money, provided that the amount of money exchanged does not exceed the buyer's money holding. Thus, both money and goods are divisible. The quantities of trade are prescribed by the household to the members in a fashion detailed later.

The household functions as follows. At the beginning of each period t, the household has k_t units of capital and m_t units of money. The household chooses the fraction of buyers, b_t, a uniform consumption level for all members, c_t, total money holdings for the next period m_{t+1} and total capital stock for the next period k_{t+1}. The household also prescribes the trading strategies for its members. Then the household evenly divides the money stock to its buyers and capital to its sellers; each buyer has a money balance m_t/b_t and each seller has a capital stock k_t/n. After the allocation the agents are matched. In desirable matches exchanges are carried out according to the prescribed strategies. After exchanges members bring back their receipts of goods and money and each member consumes c_t units of goods. Then the household receives a lump-sum monetary transfer, τ_t, and carries the stocks (m_{t+1}, k_{t+1}) to $t+1$.

To set up the household's decision problem, let me describe the production technology and preferences. For simplicity, the quantity of a producer's output (Q) is assumed to be a Cobb–Douglas function of the producer's labor input (l) and capital input (k/n):

$$Q = F(l,k/n) = F_0^{-\varepsilon} l^{\varepsilon} (k/n)^{1-\varepsilon}, \quad \varepsilon \in (0,1),$$

where F_0 is a positive constant. For given capital k/n, the labor input required to produce the quantity Q of goods can be found by inverting the above production function. That is,

$$l = F_0 Q^{1/\varepsilon} (k/n)^{(\varepsilon-1)/\varepsilon} \equiv \frac{k}{n} f\left(\frac{nQ}{k}\right). \tag{2.1}$$

The function f defined above satisfies:

$$f(0) = 0, \quad f'(\cdot) > 0, \quad f''(\cdot) > 0 \quad \text{and} \quad \frac{nq}{k} f' = \frac{f}{\varepsilon}. \tag{2.2}$$

[4] Strictly speaking, if each agent meets exactly another agent in a period, the probability with which a seller meets a desirable buyer should be $\alpha B_t/(N + B_t)$. Simplifying it to αB_t does not change the qualitative results much, because the simplification preserves the property that sellers' matching rate increases with the number of buyers in exchange, which is the one important for the extensive effect. The probability with which a buyer meets a desirable seller is simplified in a similar way.

These properties imply that having a higher capital stock saves the labor input required to produce a given quantity of goods, since capital and labor are complementary in production.

The utility of consumption in period t is $u(c_t)$ for each member. The function u is assumed to be linear with a marginal utility $u' > 0$ (but see Section 3.5). A leisure-seeker's utility of leisure is $\varphi > 1$; participating in the market reduces the utility of leisure to 1; producing goods using a labor input l_t reduces the utility of leisure further to $1 - \Phi(l_t)$. The disutility of labor is increasing and convex in the labor input, as captured by the following specific functional form:

$$\Phi(l) = \Phi_0 l^{\sigma}, \quad \sigma > 1, \;\; \Phi_0 \in (0,1). \tag{2.3}$$

If seller j in a desirable match inputs labor $l_t(j)$, the household's utility in t is

$$u'c_t + \;\; \varphi - \int_{j \in \mathcal{N} \cup \mathcal{B}} (\varphi - 1)\,\mathrm{d}j - \int_{j \in I_{pt}} \Phi(l_t(j))\,\mathrm{d}j.$$

The first integral is the sum of members' disutility of participating in the market, which is incurred by every agent in the market regardless of whether the agent has a desirable match. In contrast, the disutility of the labor input in production, captured by the second integral, is incurred only by sellers in desirable matches, i.e., those in the subset I_{pt}.

The household discounts future utility with a discount factor $\beta \in (0,1)$. The household's value function from period t onward can be expressed as a function of the capital stock and money holdings at the beginning of t, denoted $v(k_t, m_t)$. For future use, it is useful to define the household's period $(t + 1)$ marginal value of money, discounted to period t, as

$$\omega_t \equiv \beta v_m(k_{t+1}, m_{t+1}). \tag{2.4}$$

Similarly, other households' marginal value of future money is Ω_t.

I now describe the trading strategies that each household prescribes to its members. To ease exposition, I assume that the buyer in each desirable match makes a take-it-or-leave-it offer (a more general formulation is discussed in Section 3.5). The household prescribes a pair $(q_t(j), x_t(j))$ for each buyer in a desirable match ($j \in I_{mt}$) and a number $e_t(j) \in \{0,1\}$ for each seller in a desirable match ($j \in I_{pt}$). The quantity $q_t(j)$ is the amount of goods which the buyer asks the trading partner to produce and $x_t(j)$ is the money balance which the buyer gives up for the goods. For each of the household's sellers in a desirable match, the trading partner makes the offer (Q_t, X_t), which the seller takes as given and decides either to accept, i.e., $e_t(j) = 1$, or to reject, i.e., $e_t(j) = 0$.

The offer $(q_t(j), x_t(j))$ which the household's buyer j makes to the partner must give a non-negative surplus to the partner. To calculate the partner's surplus, note that if the partner (a seller) accepts the trade, the acquired money balance $x_t(j)$ will add to the partner household's money balance at the beginning of period $t+1$, whose value, discounted to period t, is $\Omega_t x_t(j)$. The cost associated with the trade is the disutility of the labor input in production, $\Phi(L_t(j))$, and so the seller's surplus is

$$\Omega_t x_t(j) - \Phi(L_t(j)), \quad \text{where } L_t(j) = \frac{K_t}{N} f\left(\frac{N q_t(j)}{K_t}\right).$$

For the producer to accept the trade, the above surplus must be non-negative, i.e.,

$$x_t(j) \geq \frac{1}{\Omega_t} \Phi\left(\frac{K_t}{N} f\left(\frac{N q_t(j)}{K_t}\right)\right), \quad j \in I_{mt}. \tag{2.5}$$

Also, since agents in matches are temporarily separated from other members of the household, they cannot borrow from other members during the match. Thus, a buyer cannot exchange more money than what he has, i.e.,

$$x_t(j) \leq \frac{m_t}{b_t}, \quad j \in I_{mt}. \tag{2.6}$$

For each t, the household chooses $(c_t, b_t, k_{t+1}, m_{t+1})$, $(q_t(j), x_t(j))_{j \in I_{mt}}$ and $(e_t(j))_{j \in I_{pt}}$ to solve the following dynamic programming problem:

$$\text{(PH)} \quad v(k_t, m_t) = \max\left\{ u' c_t - \int_{j \in I_{pt}} e_t(j) \Phi\left(\frac{k_t}{n} f\left(\frac{n Q_t}{k_t}\right)\right) \mathrm{d}j + \varphi - \int_{j \in \mathcal{N} \cup \mathcal{B}} (\varphi - 1)\, \mathrm{d}j + \beta v(k_{t+1}, m_{t+1}) \right\}$$

subject to

Eqs. (2.5) and (2.6) for every $j \in I_{mt}$,

$$c_t + k_{t+1} - k_t \leq \int_{j \in I_{mt}} q_t(j)\, \mathrm{d}j, \tag{2.7}$$

$$m_{t+1} \leq m_t + \tau_t + \int_{j \in I_{pt}} e_t(j) X_t\, \mathrm{d}j - \int_{j \in I_{mt}} x_t(j)\, \mathrm{d}j, \tag{2.8}$$

$$c_t, k_{t+1}, m_{t+1} \geq 0 \quad \text{and} \quad b_t \in [0, 1-n], \quad \text{for all } t.$$

The variables taken as given in the above problem are the state variables (k_t, m_t) and other households' choices (capital letters). In the objective function I have replaced $l_t(j)$ using Eq. (2.1). The constraint (2.7) looks like a standard budget constraint, except that what appears on the right-hand side is the total receipts of consumption goods rather than the household's output. Eq. (2.8) specifies the law of motion of the household's money balance, where τ_t is a lump-sum monetary transfer received after the transactions. The first integral on the right-hand side of Eq. (2.8) is the total amount of money obtained by the household's sellers and the last integral is the total amount of money given up in exchange by the household's buyers.

Note that the household directly chooses the quantities of trade (q, x) and so the household directly takes into account of how the household's other choices (c, b, k, m) affect the quantities of trade. This approach seems more reasonable than that in Shi (1997), where the household ignores how its choices on (c, b, k, m) affect the quantities of trade.

Before characterizing the optimality conditions for (PH), let me simplify the problem. Note first that Eq. (2.8) must hold with equality if money is valued in the future, i.e., if $v_m(k_{t+1}, m_{t+1}) > 0$, which must hold for any monetary equilibrium. For the same reason Eq. (2.5) must hold with equality. Thus, m_{t+1} can be substituted by the equality of Eq. (2.8) and $x_t(j)$ by the equality of (2.5) throughout (PH). Similarly, one can substitute c_t by the equality of (2.7). Note also that other households' choices (Q, X) must satisfy a condition similar to Eq. (2.5). That is, the representative household's sellers in desirable matches get a nonnegative surplus by accepting the terms of trade put forward by their trading partners. Thus, $e_t(j) = 1$ for all $j \in I_{pt}$ in any symmetric equilibrium.[5] After substituting (m_{t+1}, x_t, c_t, e_t), the remaining choices are (b_t, k_{t+1}) and $(q_t(j))_{j \in I_{mt}}$. It can be verified that the maximand in (PH) is concave in (k_{t+1}, q_t).

Let $\lambda_t(j)$ be the shadow price of Eq. (2.6), expressed in period-t utility. The first-order condition for k_{t+1} is a standard one: $u' = \beta v_k(k_{t+1}, m_{t+1})$, which equates the opportunity cost of capital accumulation to the discounted future value of capital. The envelope condition for k_t, the first order condition for $q_t(j)$ and the envelope condition for m_t are as follows:

$$\frac{1}{u'}\int_{j \in I_{pt}} \Phi'(l_t(j))\left(\frac{-\partial l_t(j)}{\partial k_t}\right) \mathrm{d}j = \beta^{-1} - 1, \tag{2.9}$$

$$u' = [\omega_t + \lambda_t(j)]\frac{\Phi'(L_t(j))\partial L_t(j)}{\Omega_t \quad \partial q_t(j)}, \tag{2.10}$$

$$\beta^{-1}\omega_{t-1} = \omega_t + \frac{1}{b_t}\int_{j \in I_{mt}} \lambda_t(j)\, \mathrm{d}j. \tag{2.11}$$

Eq. (2.9) states that the net rate of return to capital must be equal to the marginal rate of substitution between consumption in two adjacent periods,

[5] Although mixed strategies $(0 < e_t < 1)$ are consistent with the equality of (2.5), they are not robust. The buyer can always induce the seller to trade with probability one by asking for a quantity of goods that is δ amount less than what is described by the equality of (2.5), where δ can be arbitrarily close to zero.

which is equal to the subjective discount rate when utility is linear in consumption. Note that the net return to capital derives from its role in saving the labor input in production. An important feature of the return to capital is that it is an increasing function of the number of (other households') buyers in the market, which enters the return to capital through the measure of the subset $I_{pt}, \alpha B_t n$. When there are more buyers in the market, a seller is more likely to meet a desirable buyer and so more sellers have desirable matches, i.e., the measure of the subset I_{pt} is larger. As more sellers produce and exchange, more capital is utilized to economize on the use of labor, increasing the return to capital.

Eq. (2.10) states that, for a buyer in a desirable match, the marginal utility of getting a higher quantity of goods from the seller must be equal to the opportunity cost of money that must be used to purchase these additional goods. To see this, recall that the amount of money that the buyer uses in exchange for $q_t(j)$ units of goods is $\Phi(L_t(j))/\Omega_t$, as shown in Eq. (2.5). If the buyer asks for an additional unit of good from the seller, he must increase the amount of money paid to the seller in order to compensate for the seller's increasing labor cost, with a magnitude equal to the product of the second and third terms on the right-hand side of Eq. (2.10). Increasing the monetary payment has two costs to the buyer: he must give up the future value of money and face a tighter constraint (2.6). The sum of these costs is $\omega_t + \lambda_t(j)$ and so the right-hand side of Eq. (2.10) measures the marginal cost to the buyer of obtaining a larger quantity of goods in exchange.

Eq. (2.11) is the envelope condition for the money balance at the beginning of t, m_t. To understand it, note that the balance can be used in two ways. One is to alleviate the money constraint (2.6) for buyers in desirable matches, the benefit of which is given by the second term on the right-hand side of Eq. (2.11). The second use of the money balance is to increase the money balance at the beginning of $t+1$, the value of which is ω_t. Eq. (2.11) requires that the value of m_t, expressed in period-t value, must be equal to the sum of these benefits.

Finally, taking the derivative of the maximand with respect to b_t yields:

$$b_t \begin{cases} \in [0, 1-n], & \text{if } q_t = \dfrac{\sigma(\varphi - 1)}{(\sigma - \varepsilon)\alpha N u'}, \\ = 1 - n, & \text{if } q_t > \dfrac{\sigma(\varphi - 1)}{(\sigma - \varepsilon)\alpha N u'}. \end{cases} \tag{2.12}$$

Therefore, if the household chooses an interior fraction of members to be buyers, the quantity of goods traded in each match must be constant. This is not a critical feature of the model. It arises from the combination of the linear disutility function of participation in the market and the convex disutility function of labor. Since the marginal disutility of labor is increasing, households try to smooth the labor input by fixing the quantity of production.

3. Symmetric search equilibrium

3.1. Definition of equilibrium

I will focus on monetary equilibria where households are symmetric, defined below.

Definition 1. A symmetric monetary equilibrium is a sequence of a household's choices

$$(b_t, c_t, k_{t+1}, m_{t+1}, (q_t(j), x_t(j))_{j \in I_{mt}}, (e_t(j))_{j \in pt})_{t=0}^{\infty},$$

the implied shadow prices $(\omega_t, (\lambda_t(j))_{j \in I_{mt}})_{t=0}^{\infty}$, and other households' choices such that

(i) given other households' choices and shadow prices, each household's choices solve (PH);
(ii) $(q_t(j), x_t(j)) = (q_t, x_t)$ for all $j \in I_{mt}$ and $e_t(j) = e_t$ for all $j \in I_{pt}$;
(iii) the choices (and shadow prices) are the same across households;
(iv) $0 < b_t \leq 1 - n$ and $0 < \omega_t m_t < \infty$ for all t;
(v) $\tau_t = m_{t+1} - m_t = (\gamma - 1)m_t$, where $\gamma > 0$.

Part (i) of the definition requires each household's choices to be the best response to other households' choices. Part (iii) requires the equilibrium to be a symmetric solution to such best response correspondence, while Part (ii) requires symmetry within each household among members of the same type and match. Part (iv) requires that money be held ($b_t > 0$), that money be valued ($\omega_t m_t > 0$), and that the total value of money be bounded above ($\omega_t m_t < \infty$). The requirement $\omega_t m_t < \infty$ is necessary for the household's maximization problem (PH) to have a non-trivial solution (if $\omega_t m_t = \infty$ for some t, then the optimal choice for the household is to use the money in t to finance an infinite c_t). Part (v) specifies the monetary policy, where γ is the gross rate of money growth.

Under symmetry, capital-case variables are equal to and replaced by the corresponding lower-case variables. Using Eq. (2.1) and its counterpart for L_t to calculate $\partial l_t / \partial k_t$ and $\partial L_t / \partial q_t$, expressing (Φ', f') in terms of (Φ, f) and imposing symmetry, Eqs. (2.9)–(2.11) can be rewritten as

$$b_t \Phi\left(\frac{k_t}{n} f\left(\frac{nq_t}{k_t}\right)\right) = \frac{\varepsilon(\beta^{-1} - 1)u'}{\alpha n \sigma(1 - \varepsilon)} k_t, \tag{3.1}$$

$$\lambda_t = \omega_t\left(\frac{\varepsilon}{\sigma} \frac{u' q_t}{\Phi_t} - 1\right), \tag{3.2}$$

$$\omega_t = \beta(\omega_{t+1} + \alpha n \lambda_{t+1}). \tag{3.3}$$

I will focus on the case where b is strictly between 0 and $1-n$. One reason for not emphasizing the case $b = 1-n$ is that the corresponding steady state is dynamically unstable (see Section A.3).[6] With $b \in (0,1-n)$, Eq. (2.12) implies that q_t is constant for all t:

$$q_t = q_A \equiv \frac{\sigma(\varphi - 1)}{(\sigma - \varepsilon)\alpha n u'}. \tag{3.4}$$

Finally, the equality forms of Eqs. (2.5) and (2.8) can be rewritten as

$$x_t = \Phi_t/\omega_t, \quad \text{with } \lambda_t\left(\frac{m_t}{b_t} - x_t\right) = 0, \tag{3.5}$$

$$m_{t+1} = \gamma m_t. \tag{3.6}$$

Eqs. (3.1)–(3.6) characterize equilibrium dynamics of the key variables.

The trading restriction (2.6) may bind ($\lambda > 0$) or may not bind ($\lambda = 0$). However, it can be shown that the only case for $\lambda = 0$ is when the money supply shrinks at a pace equal to the discount rate, i.e., when $\gamma = \beta$ (see Shi, 1998). In this case the price level falls over time at the discount rate and there are a continuum of monetary equilibria that differ only in the initial nominal price. Since real quantities in such equilibria can be approached by taking the limit $\gamma \to \beta$ on the equilibrium where Eq. (2.6) binds, the remaining analysis will focus on the case $\lambda > 0$.

3.2. Existence and stability

To establish existence of the equilibrium with $\lambda > 0$, note that $\lambda_t > 0$ implies $x_t = m_t/b_t$. Substituting Φ from Eq. (3.5) into Eq. (3.1) yields:

$$\omega_t m_t = k_t \frac{\varepsilon u'(\beta^{-1} - 1)}{\alpha n \sigma(1 - \varepsilon)}. \tag{3.7}$$

Thus, the equilibrium restriction $\omega_t m_t \in (0, \infty)$ is satisfied if and only if $k_t \in (0, \infty)$. Substituting ω from the above equation and λ from Eq. (3.2) into Eq. (3.3) yields:

$$k_t = \frac{\beta}{\gamma}\left[(1 - \alpha n)k_{t+1} + \frac{\alpha n \varepsilon u' q_A}{\sigma} k_{t+1} \Big/ \Phi\left(\frac{k_{t+1}}{n} f\left(\frac{n q_A}{k_{t+1}}\right)\right)\right]. \tag{3.8}$$

This is a dynamic equation that involves only the capital stock. Once the dynamics of the capital stock are determined from this equation, the dynamics of other variables can be recovered and the equilibrium is determined.

[6] Another reason for not emphasizing the equilibrium with $b = 1-n$ is that it can be eliminated by assuming that the cost of participating in the market is a function $\varphi(b+n)$, with $\varphi'(1) = \infty$.

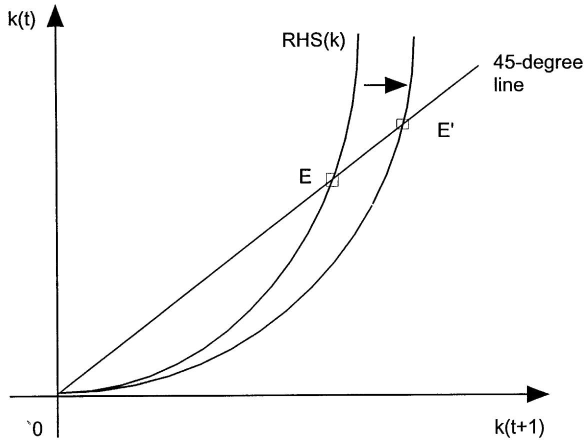

Fig. 2. The positive steady state and and its response to an increase in the money growth rate.

Fig. 2 depicts Eq. (3.8), where $RHS(k)$ temporarily denotes the right-hand side. The dynamic equation (3.8) has a zero steady state and a non-zero steady state. The zero steady state is not interesting and will be omitted. The non-zero steady state, denoted with k^*, can be obtained by setting $k_t = k_{t+1} = k^*$ in Eq. (3.8), which yields:

$$\Phi\left(\frac{k^*}{n} f\left(\frac{nq_A}{k^*}\right)\right) = \frac{\varepsilon(\varphi - 1)}{(\sigma - \varepsilon)(\gamma\beta^{-1} - 1 + \alpha n)}. \tag{3.9}$$

The following proposition (see Section A.1 for proof) describes existence and stability.

Proposition 1. *There exist numbers* $\varphi_A > 1$ *and* γ_A, *both defined in Appendix A (Section A.1), such that the equilibrium with* $\lambda > 0$ *and* $b < 1 - n$ *exists if and only if* $\varphi \in (1, \varphi_A)$ *and* $\gamma \in (\beta, \gamma_A)$. *The steady state of this equilibrium is dynamically stable.*

The requirement $\gamma > \beta$ ensures $\lambda > 0$. If $\gamma = \beta$, instead, there would be sufficient deflation for households to hoard some money, which would produce $\lambda = 0$. The money growth rate cannot be too high either. If $\gamma > \gamma_A$, for example, the resulted inflation would be too high for holding money to be rational. The upper bound γ_A is a decreasing function of the disutility of participation in the market, φ. When such disutility is high, households choose to let some of its members incur the disutility by entering the market holding money only when inflation is low. Therefore, for the number of money holders to be positive,

φ cannot be too high. The requirement $\varphi \in (1, \varphi_A)$ is necessary and sufficient for the interval (β, γ_A) to be non-empty.

3.3. Steady state effects of money growth

I now turn to the effects of an unanticipated increase in γ on the steady state, which are summarized below (see Appendix A (Section A.2) for a proof).

Corollary 1. *In the equilibrium with $\lambda > 0$ and $b < 1 - n$, an increase in γ increases the number of buyers, aggregate factor inputs and aggregate output in the steady state. The steady state inflation rate rises in the same magnitude as γ does. The labor input in each match falls but the quantity of goods exchanged in each match is unaffected.*

An important feature of the equilibrium is that the increase in the money growth rate increases the number of money holders. Intuitively, a higher money growth rate increases the expected inflation rate and reduces the value of the real money balance. Facing a lower value of money, each household tries to exchange money away more quickly by having more of its members enter the market with money. To support this intuitive argument, note that the shadow value of the real money balance is $\omega_t p_t$, where p_t is the price level at time t given by $p_t = m_t/(q_A b_t)$. Eqs. (3.7) and (3.1) imply that $\omega_t p_t = \Phi^*/q_A$, which indeed decreases with γ (see Eq. (3.9)).

The positive response of the number of buyers to money growth induces the positive response of the capital stock. Specifically, the increase in the number of buyers increases the number of desirable matches for each seller, αb. As capital is now more likely to be utilized to economize on labor, the return to accumulating capital is higher. In response to this higher return, households accumulate more capital. This positive response of the capital stock to money growth is worth noting because in the current model both capital and consumption goods must be bought with money. With a similar cash requirement but with Walrasian markets, Stockman (1981) has obtained the opposite result that money growth reduces the aggregate capital stock.

I will term the effect of money growth on the number of agents participating in the exchange the *extensive effect* of money growth. It is clear from the above explanation that the positive extensive effect is key to the positive effect of money growth on aggregate capital and output. This extensive effect is a distinct feature of non-Walrasian exchanges. It is precisely because a successful exchange cannot be guaranteed for every agent in the exchange does the number of desirable matches depend on the number of agents in the market. If markets were Walrasian in nature, agents would be costlessly assigned to desirables matches and the number of desirable matches would be independent of the money growth rate.

The result that the number of buyers increases in response to a higher money growth rate may give a false impression that a higher money growth increases the appeal of money relative to capital. Contrary to this impression, a higher money growth increases the inflation rate and widens the gap between rates of return to capital and money, as in most Walrasian models. Money growth motivates more agents to transact with money not because money growth increases the value of money but, on the contrary, because money growth reduces the value of real money balances. As intuition suggests, higher inflation induces agents to circulate money faster. In a Walrasian model, such a desire to 'dump' money into the market leads to a higher price of goods. In the current model, households must not only accept a higher price of goods but also transact more frequently with money in order to achieve the 'dumping'.

Since Walrasian monetary models, such as Sidrauski (1967) and Stockman (1981), are unable to isolate the extensive effect of money growth, their implications mainly reflect the *intensive* effects of money growth – the responses of equilibrium quantities in each trade. On the intensive margin, a higher money growth induces each producer to reduce the labor input, although aggregate labor input increases. Note, however, that an increase in γ increases the aggregate real money balance $m/p\ (=bq)$. This effect is unrealistic and not robust. In Section 3.5, I will argue that the real money balance can fall with γ in suitable extensions of the model. In contrast, the positive extensive effect is robust.

3.4. Steady state welfare and optimal money growth

I now briefly examine the money growth rate that maximizes steady state utility. Denote the steady state welfare as $v^*(\gamma)$ to emphasize its dependence on the money growth rate. Substituting b from Eq. (3.1) and q_A from Eq. (3.4) into the objective function of (PH) yields:

$$v^*(\gamma) = \frac{1}{1-\beta}\left\{\varphi - (\varphi - 1)n + \left(\frac{\varepsilon u' q_A}{\sigma \Phi^*} - 1\right)k^* \frac{\varepsilon u'(\beta^{-1} - 1)}{\sigma(1-\varepsilon)}\right\}.$$

Note that $\varepsilon u' q_A/(\sigma\Phi^*) = 1 + (\lambda/\omega)^*$ and that $\lambda \to 0$ when $\gamma \to \beta$. Thus, when the money growth rate follows the Friedman rule $\gamma = \beta$, the steady welfare level is strictly lower than that generated by any money growth rate in the range (β,γ_A).

In fact, since k^* is an increasing function of γ and Φ^* is a decreasing function of γ (see Eq. (3.9)), $v^*(\gamma)$ is an increasing function of γ. The money growth rate that maximizes the steady state welfare level is the maximum level that does not destroy the equilibrium, i.e., $\gamma \to \gamma_A$, which induces $b \to 1 - n$. This result is dramatic and depends on the assumption that the opportunity cost of participating in the market is constant. If the marginal cost of participation is sufficiently high when b is close to $1-n$, the optimal money growth rate will be in the interior of (β,γ_A). In any case the Friedman rule is unlikely to maximize steady state welfare in the presence of the positive extensive effect of money growth.

3.5. Discussion

The above model has a number of special assumptions, among which are the linear utility of consumption and the take-it-or-leave-it offer by buyers. Both might have been important for the strong extensive effect. The linear utility function of consumption allowed consumption to adjust in a large magnitude and the take-it-or-leave-it offer by buyers encouraged households to increase the number of buyers. Moreover, the model produced an unrealistic result that money growth increases the aggregate real money balance. It is important to find how sensitive the extensive effect is to these assumptions and to eliminate the unrealistic response of the real money balance.

Allowing for concave utility of consumption is straightforward, but allowing for a non-trivial split of the match surplus between the buyer and the seller is much more involved. One way to generate such a non-trivial split is to assume that bargaining proceeds in a sequential fashion as in Rubinstein and Wolinsky (1985) and in each round one side of the match is randomly chosen by nature to make the offer. In this case, each household must prescribe two sets of the quantities of trade, one for buyers to propose in desirable matches and the other for sellers to propose. The details on how this works are supplied in Shi (1998) and here I only describe the main results.

The main difference is that both the buyer's and the seller's shares of the match surplus are endogenous, depending on the ratio λ/ω, which measures the cost of the liquidity constraint, Eq. (2.6). The endogeneity of the shares is also in sharp contrast with the exogenous Rubinstein shares and the difference can be attributed to the presence of Eq. (2.6). When both sides of the match have chance to make offers, the seller must share the cost of the constraint. In particular, when it is the seller's turn to make the offer, the seller must ensure that the offer does not violate the money constraint. As long as the constraint is binding, i.e., $\lambda/\omega > 0$, the seller gets less than the corresponding Rubinstein share and the buyer gets more than the Rubinstein share. Moreover, the seller's share decreases with λ/ω and the buyer's share increases with λ/ω. Of course, this dependence would not arise if buyers could make take-it-or-leave-it offers.

Endogenous shares of the match surplus induce multiple steady states when households can choose the number of sellers. There are a high-activity steady state and a low-activity one. In the high-activity steady state, the degree of the liquidity constraint, measured by λ/ω, is lower than in the low-activity steady state but aggregate consumption, capital stock and labor are all higher. The reason for multiplicity is as follows. If households expect the purchasing power of money to be high, the real money balance will be high and the liquidity constraint will be less binding (i.e., lower λ/ω). In this case a seller's share of the match surplus will be high and so households will allocate more members to be sellers, which increases the demand for money and fulfills the expectation of a high purchasing power of money. Similarly, the belief that money has a low purchasing power can be supported by a tighter liquidity constraint.

An increase in the money growth has opposite intensive effects in the two equilibria. In the high-activity steady state, an increase in γ reduces each buyer's real money balance q^* and aggregate real money balance m/p ($= bq^*$). Each seller's capital k^*/n^* and labor input l^* also fall. In contrast, in the low-activity equilibrium, the three variables ($q^*, k^*/n^*, l^*$) all increase in response to γ. These differences in responses are intuitive. In the low-activity equilibrium, agents are severely liquidity constrained. An increase in the money growth rate provides additional liquidity, which outweighs the negative effect of inflation, and so each seller is willing to produce more than before to trade for the liquidity. In contrast, in the high-activity equilibrium agents are not very severely liquidity constrained; the inflation consequence of a high money growth rate dominates and so each seller is only willing to produce less for money.

Despite the differences in the intensive effects between the two steady states, money growth has similar positive extensive effects. In both steady states, an increase in money growth increases the number of sellers and the number of trades. For low money growth rates, this extensive effect dominates in both equilibria and so aggregate capital and output increase with the money growth rate. For high money growth rates, aggregate responses can be different between the two equilibria. Moreover, in contrast to the simple model, increasing the money growth rate can decrease the aggregate real money balance.

4. Conclusion

This paper has examined the relationship between money growth and capital in a search monetary model with divisible goods and money. It is shown that an increase in the money growth rate increases the number of trades by increasing the number of agents in the market. This positive extensive effect can dominate the conventional negative intensive effects of money growth on individuals' labor input and real money balance, in which case increasing the money growth rate increases aggregate capital and output. The extensive effect is a non-Walrasian feature. If markets were Walrasian in nature, agents would be costlessly assigned to desirable matches and the number of desirable matches would be independent of the money growth rate.

Care must be taken when interpreting the results established here, since some of them do not have apparent counterparts in Walrasian monetary models to which monetary theorists have been accustomed when thinking about policy effects. In particular, the result that money growth increases the number of buyers in the current model does not mean that inflation increases the rate of return to money relative to other assets. On the contrary, the increase in the number of buyers/sellers generated by money growth widens the gap between rates of return to capital and money. The proper interpretation of the increase in the number of buyers is that increases in inflation induce agents to circulate money more quickly, which seems consistent with evidence.

Incorporating capital accumulation into the search monetary model has brought the search model a step closer to a standard neoclassical growth model and the framework may have future use in analyzing growth in a monetary economy with decentralized exchanges. To conclude this paper, let me discuss the empirical evidence between per capita income growth and inflation. Using data for over 100 countries and over the 1960–1989 period, Levine and Zervos (1993) have found no robust relationship, positive or negative, between growth and inflation. Bruno and Easterly (1998) have found that growth is slower during inflation crises and higher after inflation crises, defined as episodes in which annual inflation rates exceed 40% for two years or more. These pieces of evidence seem to suggest that the relationship between aggregate capital/output and inflation is non-monotonic: Aggregate capital increases with inflation when inflation is initially low and decreases with inflation when inflation is initially high. The extension in Section 3.5 delivers this non-monotonic relationship. When the medium inflation rate in the sample exceeds the critical inflation rate, a cross-country regression naturally finds a negative relationship between growth and inflation. In any case, the casual relationship established in this paper from inflation to aggregate capital and output cannot be refuted simply by regression results.

Appendix A. Proofs

A.1. Proof of Proposition 1

The properties of $f(\cdot)$ and $\Phi(\cdot)$ imply that $RHS(k)$, the right-hand side of Eq. (3.8), is increasing and convex with $RHS(0)=0$ and $\lim_{k\to\infty} RHS(k)/k=\infty$. Thus, Eq. (3.8) has a zero steady state and a non-zero steady state. The non-zero steady state, determined by Eq. (3.9), is positive if and only if the right-hand side of Eq. (3.9) is positive, i.e., if and only if $\gamma > \beta(1-\alpha n)$, which holds under $\gamma > \beta$.

Dynamic stability of the steady state can be established as follows. For any given initial capital stock $k_0 > 0$, the sequence of the capital stock, $\{k_t\}_{t=0}^{\infty}$, is given successively by $k_{t+1} = RHS^{-1}(k_t)$, where $RHS^{-1}(\cdot)$ is the inverse function of $RHS(\cdot)$. Since $RHS(k)$ is increasing and convex, $RHS^{-1}(k)$ is increasing and concave, implying that $k_{t+1} > k_t$ for all $k_t < k^*$ and $k_{t+1} < k_t$ for all $k_t > k^*$. Thus, there is global convergence to the steady state for any $k_0 > 0$.

Since $k^* \in (0,\infty)$, it is clear that $\omega_t m_t \in (0,\infty)$ around the steady state, as argued in the text. For the equilibrium to satisfy $\lambda > 0$, it is necessary and sufficient that $\Phi < \varepsilon u' q_A/\sigma$ (see Eq. (3.2)). This is satisfied near the steady state if and only if $\Phi^* < \varepsilon u' q_A/\sigma$, which is equivalent to $\gamma > \beta$ under Eq. (3.9). The equilibrium should also satisfy $b_t < 1 - n$, which requires $b^* < 1 - n$ near the steady state. To verify this requirement, define a function $k_A(\gamma)$ as follows:

$$k_A(\gamma) \equiv \frac{\alpha n(1-n)\sigma(1-\varepsilon)(\varphi-1)}{(\sigma-\varepsilon)(\beta^{-1}-1)(\gamma\beta^{-1}-1+\alpha n)u'}. \tag{A.1}$$

Clearly, $k_A'(\gamma) < 0$ for any $\gamma \geq \beta$. Then, using Eqs. (3.1) and (3.9) to substitute for b^* yields that $b^* < 1 - n$ iff $k^* < k_A(\gamma)$. Since $\Phi((k/n)f(nq_A/k))$ is a decreasing function of k, $k^* < k_A(\gamma)$ iff

$$\Phi\left(\frac{k_A(\gamma)}{n} f\left(\frac{nq_A}{k_A(\gamma)}\right)\right) < \Phi\left(\frac{k^*}{n} f\left(\frac{nq_A}{k^*}\right)\right).$$

Using Eq. (3.9) to substitute for Φ^* and rearranging terms yields that $b^* < 1 - n$ iff

$$\frac{1}{k_A(\gamma)} \Phi\left(\frac{k_A(\gamma)}{n} f\left(\frac{nq_A}{k_A(\gamma)}\right)\right) < \frac{\varepsilon(\beta^{-1}-1)u'}{\alpha n(1-n)\sigma(1-\varepsilon)}. \tag{A.2}$$

The left-hand side of Eq. (A.2) is a decreasing function of k_A and, since $k_A'(\gamma) < 0$, it is an increasing function of γ. One can verify that $k_A(\beta(1-\alpha n)) = \infty$ and $k_A(\infty) = 0$, which imply:

$$LHS(\text{A.2})|_{\gamma=\beta(1-\alpha n)} = 0 \quad \text{and} \quad LHS(\text{A.2})|_{\gamma=\infty} = \infty.$$

Therefore, there is a unique level of γ, denoted γ_A, that makes Eq. (A.2) as an equality. That is,

$$\frac{1}{k_A(\gamma_A)} \Phi\left(\frac{k_A(\gamma_A)}{n} f\left(\frac{nq_A}{k_A(\gamma_A)}\right)\right) = \frac{\varepsilon(\beta^{-1}-1)u'}{\alpha n(1-n)\sigma(1-\varepsilon)}. \tag{A.3}$$

It is clear from the above argument that $\beta(1-\alpha n) < \gamma_A < \infty$ and that $b_t < 1 - n$ around the steady state if and only if $\gamma < \gamma_A$.

Since the monetary equilibrium requires both $\gamma > \beta$ and $\gamma < \gamma_A$, the parameter region for existence is non-empty if and only if $\gamma_A > \beta$. Since the left-hand side of Eq. (A.3) is an increasing function of γ_A, $\gamma_A > \beta$ if and only if replacing γ_A in Eq. (A.3) by β changes the equality into an inequality " < ", i.e., if and only if

$$\frac{\alpha n(\sigma-\varepsilon)}{\varepsilon(\varphi-1)} \Phi\left(\frac{(1-n)\sigma(1-\varepsilon)(\varphi-1)}{n(\sigma-\varepsilon)(\beta^{-1}-1)u'} f\left(\frac{\beta^{-1}-1}{\alpha(1-n)(1-\varepsilon)}\right)\right) < 1. \tag{A.4}$$

The left-hand side of this inequality is an increasing function of the parameter $(\varphi - 1)$. Let us define φ_A as such that makes the above inequality as an equality, i.e.,

$$\frac{\alpha n(\sigma-\varepsilon)}{\varepsilon(\varphi_A - 1)}\Phi\left(\frac{(1-n)\sigma(1-\varepsilon)(\varphi_A-1)}{n(\sigma-\varepsilon)(\beta^{-1}-1)u'}f\left(\frac{\beta^{-1}-1}{\alpha(1-n)(1-\varepsilon)}\right)\right) = 1. \tag{A.5}$$

Clearly, φ_A is well-defined and $\varphi_A \in (1, \infty)$. Also, $\gamma_A > \beta$ if and only if $\varphi < \varphi_A$.

A.2. Proof of Corollary 1

From Eq. (3.4), it is clear that an increase in γ has no effect on the quantity of goods exchanged in each match, q. To see that k^* increases with the money growth rate, note that an increase in γ lowers the curve $RHS(k)$ in Fig. 2, changes the steady state from point E to point E' and increases the steady-state capital stock. Since $k/\Phi((k/n)f(nq/k))$ is an increasing function of k, Eq. (3.1) then implies that b^* increases with γ. Aggregate capital input, αnb^*k^*, and aggregate output, αnb^*q_A, both increase with the money growth rate. The labor input in each desirable match is $l^* = (k^*/n)f(nq_A/k^*)$. Since $dl^*/dk^* < 0$ and $dk^*/d\gamma > 0$, then $dl^*/d\gamma < 0$. However, aggregate labor input, αnb^*l^*, increases with γ, since differentiating Eqs. (3.1) and (3.9) yields:

$$\frac{d(b^*l^*)}{d\gamma} \sim 1 + (\sigma-1)(1-\varepsilon)/\varepsilon > 0.$$

Finally, the price level in period t is $p_t = m_t/(q_t b_t)$ and steady-state inflation is $p_{t+1}/p_t = m_{t+1}/m_t = \gamma$.

A.3. Equilibrium with $b = 1 - n$ in Section 3

Let us examine the equilibrium with $\lambda_t > 0$ and $b_t = 1 - n$ for $t \geq 0$ and focus on its stability feature. For $b_t = 1 - n$, Eq. (2.12) requires $q_t > q_A$, where q_A is defined in Eq. (3.4). Substituting $b_t = 1 - n$ into Eq. (3.1), one can solve for $q_t = g(k_t)$, where

$$g(k) \equiv \frac{k}{n}f^{-1}\left(\frac{n}{k}\Phi^{-1}\left(\frac{\varepsilon(\beta^{-1}-1)u'}{\alpha n(1-n)\sigma(1-\varepsilon)}k\right)\right). \tag{A.6}$$

The functions f^{-1} and Φ^{-1} are the inverse functions of f and Φ, respectively. It can be verified that $g(k)$ is an increasing and concave function with $g(0) = 0$ and

$$\frac{d}{dk}\left(\frac{g(k)}{k}\right) = -\frac{g(k)}{k^2}\varepsilon\left(1-\frac{1}{\sigma}\right) < 0, \quad \left.\frac{g(k)}{k}\right|_{k=0} = \infty .$$

Now following the same procedure that led to Eq. (3.8), one can obtain:

$$k_t = \frac{\beta}{\gamma}\left[(1-\alpha n)k_{t+1} + \frac{\alpha^2 n^2 (1-n)(1-\varepsilon)}{\beta^{-1}-1} g(k_{t+1})\right]. \tag{A.7}$$

Denote the right-hand side of this equation temporarily by $RHS2(k_{t+1})$ and its inverse by $RHS2^{-1}(\cdot)$. Given any $k_0 > 0$, the path $\{k_t\}_{t=0}^{\infty}$ is given successively by $k_{t+1} = RHS2^{-1}(k_t)$.

As in the equilibrium with $b < 1-n$, Eq. (A.7) has a steady state $k = 0$ and a non-zero steady state, say k^{**}. The non-zero steady state is positive if and only if $\gamma > \beta(1-\alpha n)$. In contrast to the equilibrium with $b < 1-n$, $RHS2(k)$ is a concave function and so $RHS2^{-1}(k)$ is a convex function. Thus, $k_{t+1} < k_t$ if $k_t < k^{**}$ and $k_{t+1} > k_t$ if $k_t > k^{**}$. For any $k_0 \neq k^{**}$, the sequence $\{k_t\}_{t=0}^{\infty}$ diverges away from k^{**}.

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