



Labor market search and the dynamic effects of taxes and subsidies¹

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Abstract

This paper integrates the search model of unemployment into an intertemporal framework and examines the dynamic effects of a labor income tax, a capital income tax, an unemployment subsidy, a vacancy subsidy, and an investment tax credit. We also compute the marginal deadweight losses associated with these policies. The presence of unemployment reduces the relative welfare cost of capital income taxation to labor income taxation. With realistic parameter values, labor income taxation can even be more costly than capital income taxation. A vacancy subsidy is efficient, self-financed and shares many features with an investment tax credit. An unemployment subsidy is very inefficient. Alternative matching and wage determination schemes are analyzed. © 1999 Elsevier Science B.V. All rights reserved.

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1. Introduction

What are the dynamic effects and welfare costs of taxes and subsidies when there is unemployment? The answer to this question remains unsatisfactory in

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the literature. Despite the voluminous literature on factor income taxation, dynamic equilibrium models have typically omitted the important fact of unemployment.² The omission seriously undermines the predictive power of these models and delivers biased policy recommendations on tax reforms. On the other hand, models of unemployment do not have capital accumulation, with the exceptions of Merz (1995) and Andolfatto (1996) discussed below. The omission of capital accumulation oversimplifies agents' intertemporal decisions and makes the models incapable of evaluating the capital income tax. In this paper, we incorporate unemployment into an intertemporal framework and examine a labor income tax, a capital income tax, an unemployment subsidy, a subsidy to job vacancy, and an investment tax credit. With realistic parameter values, we analyze the dynamic responses of macroeconomic variables to changes in these policies and calculate the marginal deadweight losses.

As in earlier search models by Mortensen (1982) and Pissarides (1990), unemployment here is a result of the time-consuming matching process in the labor market. The basic modelling device resembles that in Merz (1995) and Andolfatto (1996), who integrate costly search into households' intertemporal choices.³ The integration endogenizes agents' reservation wage as the marginal rate of substitution between consumption and leisure. Our work differs from Merz (1995) and Andolfatto (1996) in three aspects. First, our objective is to examine taxes and subsidies, which are absent in Merz's and Andolfatto's models. Second, we directly characterize the decentralized equilibrium, while Merz and Andolfatto characterize the planner's allocation. A direct equilibrium characterization is necessary here because the equilibrium is distorted by taxes, subsidies and externalities in the labor market. There is little hope for the equilibrium to decentralize the planner's allocation. Third, we explore alternative labor market structures that endogenize the matching function and the wage determination scheme.

In comparison with a standard model without unemployment, such as Judd (1987), our model revises the effects of factor income taxation by (i) amplifying the negative effect of labor income taxation on employment, (ii) attenuating the negative effect of capital income taxation on employment, and (iii) increasing the relative welfare cost of labor income taxation to capital income taxation. A labor income tax has a large negative effect on employment because it acts like a tax on firms' profitability from hiring. By reducing the appeal of working, a labor income induces agents to substitute away from working and raises agents' reservation wage. The higher reservation wage reduces firms' profitability from hiring and reduces firms' incentive to hire. In contrast, a capital income

² Examples include Chamley (1981), Judd (1985), Judd (1987) and Auerbach and Kotlikoff (1987).

³ Mortensen (1992) has constructed a similar model. Shi and Wen (1997) have examined the analytical solution to a particular version of this integrated search model.

tax reduces the reservation wage, which mitigates the direct negative effect of the tax on firms' profitability from hiring.

With a set of parameter values used by Judd (1987), we find that the marginal deadweight loss of capital income taxation is below 50 cents in real income for a dollar increase in the tax revenue. In contrast, Judd has found that the loss easily exceeds a dollar and is 3 to 4 times as large as the welfare loss from labor income taxation. Compared with Judd (1987), the welfare cost of a labor income tax is also higher. For some realistic parameter values, the welfare cost of labor income taxation can even exceed that of capital income taxation.

The general equilibrium framework also enables us to evaluate other policies such as unemployment subsidy and vacancy subsidy. We have found that an unemployment subsidy resembles a labor income tax but is more inefficient. It reduces welfare and the governmental revenue. That is, cutting the unemployment subsidy increases governmental revenue and welfare. The marginal welfare gain from cutting the unemployment subsidy easily exceeds 50 cents. In contrast, a vacancy subsidy is very efficient. It not only increases the intertemporal utility but also raises the government revenue. A vacancy subsidy is self-financed because it increases the capital stock and employment sufficiently so that the increased tax revenue from these latter incomes exceeds the subsidy. The marginal welfare gain from a vacancy subsidy often exceeds 1.5 dollars. These results strongly suggest a policy shift *at the margin* from subsidizing unemployment to subsidizing job creation. A vacancy subsidy is efficient because it is a subsidy to investment in employment. For this reason, it shares many features with an investment tax credit, as illustrated in the text.⁴

We also examine the importance of labor market frictions. This is done in three directions. First, we check how the welfare costs of policies change with parameters underlying the matching function and the Nash wage bargaining scheme. Next, we use a sequential bargaining framework with insiders and outsiders to endogenize the bargaining weights. Finally, we use wage-posting to endogenize both the matching function and wage determination. Although the magnitudes of the welfare costs change as the environment changes, the welfare ranking between policies is robust. Analyzing wage-posting in a dynamic unemployment model is a novel deviation from the literature by itself.

This paper is organized as follows. Section 2 builds the model. Section 3 characterizes the equilibrium dynamic system and examines how taxes and subsidies affect the steady state. Section 4 parameterizes the model and analyzes the dynamic effects of the policies. Section 5 reports the welfare costs of the

⁴ We are grateful to a referee for persuading us to examine the effect of an investment tax credit. On vacancy subsidy, Millard and Mortensen (1994) also examine the welfare effect but they abstract from capital accumulation and the intertemporal consumption decision. This not only makes the model difficult to examine capital income taxation, but also omits the channel that tax policies affect other variables through the marginal rate of substitution between leisure and consumption.

policies. Section 6 examines alternative wage determination schemes and endogenizes the matching function. Section 7 concludes the paper and the appendix supplies necessary proofs.

2. Decentralized economy

2.1. Households

Consider an economy with many identical households. The measure of households is normalized to one. Each household has agents who are infinitely-lived and endowed with a flow of one unit of time. At any given point of time an agent can engage in only one of three activities: working for wage, searching for a job or enjoying leisure. We call an agent unemployed if he is searching for a job. Unemployed agents are randomly matched with job vacancies as described later. Each unemployed agent faces idiosyncratic risks in his job match. These idiosyncratic risks induce distributions of consumption and wealth across agents. Tracking these distributions in a dynamic model like ours seems an intractable exercise, although a significant progress has been made numerically in computing equilibria with heterogeneous agents (see Rios-Rull, 1995, for a survey). Since it is unclear how heterogeneity influences the dynamic behavior of aggregate variables, abstracting from such influence seems a good trade-off between realism and tractability. This is achieved here by assuming that each household consists of a continuum of agents with measure one and agents care only about the household's utility. Individual risks are completely smoothed within each household so that agents' decision problem can be simplified as a representative household's maximization problem. Assuming risk-smoothing is common in macroeconomics.⁵

The representative household's utility function is

$$U = \int_0^{\infty} u(c, 1 - s - n^s) e^{-\rho t} dt, \quad \rho > 0, \quad (2.1)$$

where $u(\cdot, \cdot)$ is increasing, concave and continuously differentiable, with a simplifying assumption $u_{12} = 0$. The variable c is the household's consumption, s the fraction of agents who are searching for jobs, and n^s the fraction of agents who are employed (the superscript s indicates 'supply'). The variable s conforms with the notion of unemployment and $s + n^s$ with the labor force participation. The unemployment rate is $s/(s + n^s)$. Since each agent can engage in only one

⁵ For example, in models with indivisible labor, Rogerson (1988), Hansen (1985), and Rogerson and Wright (1988) use employment lotteries to allow agents to smooth idiosyncratic risks in employment. Andolfatto (1996) applies the technique to a search model of unemployment. In a monetary model, Lucas (1990b) also assumes risk-sharing within each household.

activity at a time, n^s is also the household's hours in work and s the hours in search.

Different from a standard intertemporal model but in common with search models, employment is predetermined at each time and changes only gradually as workers separate from jobs or unemployed agents find jobs. A household's labor supply evolves as follows:

$$\dot{n}^s = ms - \theta n^s. \quad (2.2)$$

θ is a constant rate of job separation and m the rate at which unemployed agents find jobs. Although m depends on the number of aggregate job vacancies and unemployed agents in the economy, as discussed later, individual households take m as given.

A representative household chooses (c, s, n^s) and the supply of capital k^s to solve:

$$(PH) \quad \max_{(c, s, n^s, k^s)} U \text{ s.t.}$$

Eq. (2.2) holds;

$$\begin{aligned} \dot{k}^s &= (1 - \tau_k)(rk^s + \pi) + (1 - \tau_w)wn^s + \tau_u ws - c + L, \\ k^s(0) &= k_0, \quad n^s(0) = n_0 \text{ given.} \end{aligned} \quad (2.3)$$

Here r is the rental rate of capital, w the wage rate, and π the dividend defined later. τ_k is the capital income tax rate, τ_w the labor income tax rate, τ_u the rate of subsidy to unemployment, and L a lump-sum transfer from the government. For future use, denote $\tau_e = \tau_u/(1 - \tau_w)$ as the *effective rate of unemployment subsidy*.

It is worthwhile emphasizing that there are inflows into and outflows from the state of unemployment all the time, even in the steady state. Since the identities of unemployed agents are constantly changing, the unemployment duration, $1/m$, should be interpreted as the average duration of all unemployed agents. Similarly, τ_u should be interpreted as the average unemployment subsidy to all unemployed agents. Thus, τ_u can be constant over time even though in reality the subsidy to an unemployed individual typically falls with unemployment duration. For all the examinations, we will assume that the τ 's are piece-wise constant over time.

Let Ω_H be the current-value shadow price of Eq. (2.2). The current-value shadow price of the constraint (2.3) can be computed as u_1 . The maximization problem generates:

$$\dot{c} = \frac{u_1}{u_{11}} [\rho - r(1 - \tau_k)], \quad (2.4)$$

$$\Omega_H = (u_2 - u_1 \tau_u w)/m, \quad (2.5)$$

$$\dot{\Omega}_H = (\theta + \rho)\Omega_H + u_2 - u_1(1 - \tau_w)w. \quad (2.6)$$

Eq. (2.4) is a standard condition. Eq. (2.5) requires the opportunity cost of search, u_2 , to be equal to the marginal benefit of search, $m\Omega_H + u_1\tau_u w$. Eq. (2.6) requires the capital value of employment to the household to be equal to the present value of cash flow, $u_1(1 - \tau_w)w - u_2$, plus the capital gain, $\dot{\Omega}_H$, with an effective discount rate $(\theta + \rho)$.

Note that $\Omega_H > 0$ implies $w < u_2/(\tau_u u_1)$. For $\Omega_H > 0$ around the steady state, Eq. (2.6) requires $w > u_2/[(1 - \tau_w)u_1]$. Thus, an equilibrium with $\Omega_H > 0$ exists only if $\tau_u < 1 - \tau_w$ (i.e., $\tau_e < 1$); otherwise agents would have no incentive to work. This condition is imposed throughout the paper.

2.2. Firms

There are many identical firms in the economy. Each firm chooses a number (v) of job vacancies. The unit cost of maintaining vacancy is a constant b . Job vacancies are matched with unemployed agents at a rate μ . Although μ depends on the labor market tightness, an individual firm takes μ as given (see Section 6 for an alternative approach). A firm's labor employment (n^d) evolves as

$$\dot{n}^d = \mu v - \theta n^d. \tag{2.7}$$

The firm receives a subsidy τ_v for each unit of incurred vacancy cost. Since the flow of new hiring is μv , the subsidy amounts to a hiring subsidy at a rate $\tau_v \mu / b$. According to Eq. (2.7), hiring can be viewed as the firm's gross 'investment' in employment and so the vacancy subsidy can be viewed as a tax credit to such investment. This draws a close link between the vacancy subsidy and the conventional tax credit to investment in physical capital, as analyzed later.

An individual firm takes as given the rental rate of capital r and the wage rate w offered by other firms. It chooses the number of vacancy v , the demand for capital (k^d) and employment (n^d) to maximize the present value:

$$(PF) \max_{(v, k^d, n^d)} \int_0^\infty \pi(t) e^{-\int_0^t (1 - \tau_k)r(\xi) d\xi} dt$$

s.t. Eq. (2.7) holds,

$$\begin{aligned} \pi &= F(k^d, n^d) - (r + \delta)k^d - wn^d - b(1 - \tau_v)v + \tau_I(\dot{k}^d + \delta k^d), \\ k^d(0) &= k_0, n^d(0) = n_0 \text{ given.} \end{aligned} \tag{2.8}$$

Here δ is the rate of capital depreciation and τ_I the investment tax credit. The production function $F(k, n)$ is assumed to be increasing, concave, differentiable and linearly homogeneous. Let Ω_F be the current-value shadow price of employment to the firm. Integrating by parts yields

$$\int_0^\infty \dot{k}(t) e^{-\int_0^t (1 - \tau_k)r(\xi) d\xi} dt = -k_0 + (1 - \tau_k) \int_0^\infty r(t)k(t) e^{-\int_0^t (1 - \tau_k)r(\xi) d\xi} dt.$$

Using this result, the maximization problem generates:

$$F_1 = r + \delta - \tau_l[\delta + r(1 - \tau_k)], \quad (2.9)$$

$$\Omega_F = b(1 - \tau_v)/\mu, \quad (2.10)$$

$$\dot{\Omega}_F = [\theta + r(1 - \tau_k)]\Omega_F + w - F_2. \quad (2.11)$$

Eq. (2.9) is a standard condition. Eq. (2.10) requires the effective marginal cost of vacancy, $b(1 - \tau_v)$, to be equal to the marginal benefit of vacancy, $\mu\Omega_F$. Eq. (2.11) requires the capital value of employment to be equal to the present value of the cash flow, $F_2 - w$, plus the capital gain, $\dot{\Omega}_F$, discounted at a rate $\theta + (1 - \tau_k)r$. We will refer to the difference, $F_2 - w$, as the firm's *marginal profitability from hiring*. Notice that a positive hiring cost implies $\Omega_F > 0$ and hence positive profitability from hiring around the steady state.

2.3. Matching and wage determination

We now describe the matching function and the wage determination scheme. Although both can be endogenized as described in Section 6, for the moment we assume that they are described exogenously as follows. The number of job matches is determined by a matching function. Let the number of vacancies per firm be \bar{v} and the number of unemployed agents per household be \bar{s} , distinguished from individual firm's and agent's choices (v, s). The flow of matches is

$$M(\bar{v}_t, \bar{s}_t) = A \cdot (\bar{v}_t)^\alpha (\bar{s}_t)^{1-\alpha}, \quad \alpha \in (0, 1), \quad A > 0. \quad (2.12)$$

The matching technology exhibits constant returns-to-scale, which is consistent with the observations by Pissarides (1986) and Blanchard and Diamond (1989). The Cobb–Douglas form is adopted for analytical simplicity. We call α the *elasticity of vacancy* in job matches. Denote $x = \bar{v}/\bar{s}$ as the tightness of the labor market; a smaller x represents a tighter market. With constant returns-to-scale, matches per unemployed agent (m) and per vacancy (μ) depend only on x :

$$m = m(x) = Ax^\alpha, \quad \mu = \mu(x) = m(x)/x. \quad (2.13)$$

Note that μ is a decreasing function of x . Note also that $m\bar{s} = \mu\bar{v}$.

Once an unemployed agent is matched with a vacancy, the agent and the firm decide the agent's current and future wages. The wages are assumed to maximize a weighted Nash product of the agent's and the firm's surpluses. To be precise, let t_0 be the time when the match is created. Denote by $\{\hat{w}(t)\}_{t \geq t_0}$ the path of wage rates to be determined for the new worker conditional on the continuation of the worker's employment. Then having an additional member working at the wage schedule increases the household's utility at each time $t \geq t_0$ by

$[(1 - \tau_w(t))\hat{w}(t)u_1(t) - u_2(t)] dn$. Hiring an additional worker dn with the wage schedule increases the firm's current-valued surplus at time $t \geq t_0$ by $[F_2(t) - \hat{w}(t)] dn$. With normalization, the Nash bargaining solution solves

$$\max_{\hat{w}(t)} [F_2(t) - \hat{w}(t)]^{1-\lambda} \left[\hat{w}(t) - \frac{u_2(t)}{(1 - \tau_w(t))u_1(t)} \right]^\lambda \quad \text{for } t \geq t_0.$$

The parameter $\lambda \in (0,1)$ is the worker's bargaining weight.⁶ Solving the Nash problem yields:

$$\hat{w}(t) = \lambda F_2(t) + (1 - \lambda) \frac{u_2(t)}{(1 - \tau_w(t))u_1(t)}. \quad (2.14)$$

Since firms are identical, they offer the same wage rate in any symmetric equilibrium. Also, since the wage formula is independent of when the wage is negotiated, two workers who are hired at different times must be paid the same wage at any given time. Therefore $\hat{w} = w$.

Several remarks follow. First, the quantity $u_2/[u_1(1 - \tau_w)]$ can be interpreted as the agent's reservation wage. Since the reservation wage directly affects the firm's profitability from hiring, taxes and subsidies directly affect the labor demand decision by changing consumption and leisure in addition to their effects on the marginal product of labor. The assumption $\tau_e < 1$ ensures that the wage rate is between the reservation wage and the marginal product of labor. Second, since there is only one agent on each side of the match, both sides have some monopoly power and so any division of the total match surplus which gives each side a non-negative surplus is feasible. Each choice of λ gives a particular division of the surplus. We will first characterize the equilibrium with a fixed $\lambda \in (0,1)$ and then examine the policies when λ is assigned a realistic value. In Sections 5.3 and 6, we check the robustness of the results by changing the value of λ and changing the wage determination schemes.

Third, the wage rate associated with an arbitrary λ is in general not efficient. As analyzed by Mortensen (1982) and Hosios (1990), inefficiency arises because individuals ignore the externalities created by their choices. When there are no distortionary policies, there is a particular division of the match surplus that induces efficient allocations. Such a division rewards the two sides of the match by their contributions to the match formation so that the positive and negative externalities exactly cancel out. These efficiency conditions are (see Hosios, 1990):

$$vM_1(v,s)/M = 1 - \lambda, \quad sM_2(v,s)/M = \lambda.$$

⁶ The Nash formulation maintains tractability. In the steady state the Nash bargaining formulation generates the same outcome as some noncooperative sequential bargaining games (Wolinsky, 1987). For a comparison between the two in a non-steady state environment, see Coles and Wright (1994).

Under the Cobb–Douglas matching function, the two conditions are equivalent to a single condition, $\lambda = 1 - \alpha$. With distortionary taxes and subsidies, this condition does not ensure efficiency in the matching process, as discussed in Section 5.3. In any case, the equilibrium is characterized for any arbitrarily fixed (λ, α) and the sensitivity analysis is conducted later.

2.4. Government

The government faces the following budget constraint:

$$L \leq \tau_k(rk + \pi) + \tau_w wn - \tau_u w \bar{s} - b\tau_v \bar{v} - \tau_f(\dot{k} + \delta k) - g. \quad (2.15)$$

Government spending, g , is assumed to be constant. Any change in the revenue caused by changes in taxes and subsidies is rebated to households through the lump-sum transfer L . Government bonds are abstracted from the model for simplicity. Also, all changes in taxes are assumed to be permanent and marginal (see Section 4 for a discussion).

2.5. Search equilibrium

The following is a definition of a symmetric search equilibrium.

Definition 2.1. A search equilibrium is a sequence of the household's choice $(c(t), s(t), n^s(t), k^s(t))$, firm's choice $(v(t), n^d(t), k^d(t))$, factor prices $(r(t), w(t))$, profit $\pi(t)$, aggregate variables $(\bar{v}(t), \bar{s}(t))$ (and the induced matching rates $m(t)$ and $\mu(t)$ in Eq. (2.13)) such that

1. symmetry holds: $(v, s) = (\bar{v}, \bar{s})$;
2. given factor prices, profit and matching rates, the household choice solves (PH);
3. given factor prices and matching rates, the firm's choice solves (PF);
4. the rental rate r clears the capital market: $k^s = k^d$;
5. $w(t)$ and $\pi(t)$ satisfy Eqs. (2.14) and (2.8) respectively;
6. the government budget constraint (2.15) is satisfied.

Missing from the above definition is the labor market clearing condition. However, since the flows of workers in and out of employment are equal to each other in any symmetric equilibrium, i.e., $m(x)s = \mu(x)v$, Eqs. (2.2) and (2.7) indicate that the demand for labor indeed equates the supply of labor. That is, the labor market clearing condition does not pin down the wage rate as in a standard framework. Instead, the wage is given by the wage Eq. (2.14).

Under the symmetry condition (i), we will suppress the bar over (v, s) . Under the market clearing condition (iv), we will suppress the superscripts s and d on (k, n) . Also, it is easier to work with the variable x than with v so we substitute v by x . The existence of an equilibrium amounts to a convergent path of the equilibrium dynamic system described below.

3. Dynamic system and steady state

3.1. Equilibrium conditions and steady state

The equilibrium conditions are (2.2)–(2.6), (2.8)–(2.11) and (2.13)–(2.15). After suitable substitutions, they yield a system of differential equations that characterize the dynamics of $Y \equiv (c, s, x, n, k)^T$, as follows:

$$\dot{c} = \frac{u_1}{u_{11}} \left[\rho - (1 - \tau_k) \frac{F_1 - \delta(1 - \tau_I)}{1 - \tau_I(1 - \tau_k)} \right], \tag{3.1}$$

$$\begin{aligned} \dot{s} = & -\dot{n} - \frac{1}{u_{22}} \{ (\theta + \rho + \dot{m}/m)(u_2 - u_1\tau_u w) + \tau_u w u_{11} \dot{c} \\ & + m[u_2 - u_1(1 - \tau_w)w] + u_1\tau_u \dot{w} \}, \end{aligned} \tag{3.2}$$

$$\begin{aligned} \dot{x} = & \frac{1}{1 - \alpha} \left\{ x \left[\theta + (1 - \tau_k) \frac{F_1 - \delta(1 - \tau_I)}{1 - \tau_I(1 - \tau_k)} \right] \right. \\ & \left. - \frac{(1 - \lambda)m}{b(1 - \tau_v)} \left(F_2 - \frac{u_2}{(1 - \tau_w)u_1} \right) \right\}, \end{aligned} \tag{3.3}$$

$$\dot{n} = m(x)s - \theta n, \tag{3.4}$$

$$\dot{k} = F - \delta k - bsx - c - g. \tag{3.5}$$

Eq. (3.2) is derived from Eqs. (2.5) and (2.6); Eq. (3.3) from Eq. (2.11); Eq. (3.5) from Eqs. (2.3), (2.8) and (2.15). Denote $\tau \equiv (\tau_w, \tau_k, \tau_u, \tau_v, \tau_I)^T$. The dynamic system can be written as

$$\dot{Y} = h(Y, \tau). \tag{3.6}$$

The initial conditions of this system are $k(0) = k_0$ and $n(0) = n_0$.

Denote the steady state of the equilibrium dynamic system by $Y^* = (c^*, s^*, x^*, n^*, k^*)^T$. It is given by the solution to $h(Y^*, \tau) = 0$ and characterized by the following equations:

$$F_1 = \delta(1 - \tau_I) + \rho \left(\frac{1}{1 - \tau_k} - \tau_I \right), \tag{3.7}$$

$$\lambda x^* + (\theta + \rho) [1 - (1 - \lambda)\tau_e] \frac{x^*}{m(x^*)} = \frac{(1 - \lambda)(1 - \tau_e)}{b(1 - \tau_v)} F_2, \tag{3.8}$$

$$s^* = \frac{\theta n^*}{m(x^*)}, \tag{3.9}$$

$$c^* = \left(\frac{F - \delta k^*}{n^*} - \frac{b\theta x^*}{m(x^*)} \right) n^* - g, \tag{3.10}$$

$$\frac{u_2}{(1 - \tau_w)u_1} = a(\tau_e, x^*) F_2, \tag{3.11}$$

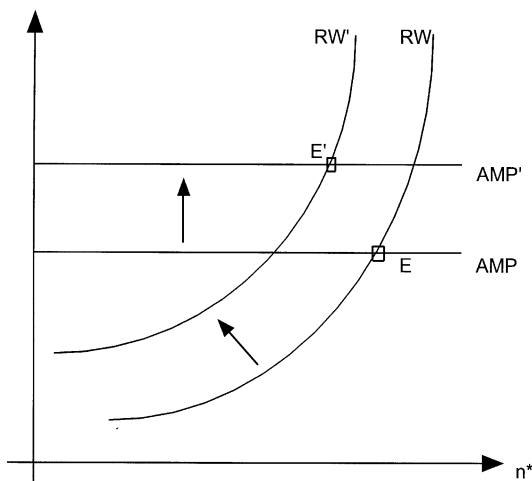


Fig. 1. The steady state employment.

where

$$a(\tau_e, x^*) \equiv \frac{\lambda[(\theta + \rho)\tau_e + m(x^*)]}{(\theta + \rho)[1 - (1 - \lambda)\tau_e] + \lambda m(x^*)}, \quad a_1 > 0, \quad a_2 > 0. \quad (3.12)$$

In particular, Eqs. (3.8) and (3.11) are derived from the equations $\dot{x} = 0$ and $\dot{s} = 0$.

The Eqs. (3.7), (3.8), (3.9), (3.10) and (3.11) determine a unique steady state. Denote the steady state capital-labor ratio as $\kappa^* = k^*/n^*$. Since F_1 is only a function of κ^* , Eq. (3.7) solves for a unique $\kappa^* = \kappa(\tau_k, \tau_l)$, with $\kappa_1 < 0$ and $\kappa_2 > 0$. Eq. (3.8) solves for $x^* = x(\tau_e, \tau_v, \kappa^*)$, with $x_1 < 0$, $x_2 > 0$ and $x_3 > 0$. Eq. (3.9) gives $s^* = s(n^*, x^*)$, with $s_1 > 0$ and $s_2 < 0$. Eq. (3.10) gives $c^* = c(n^*, x^*, \kappa^*)$, with $c_1 > 0$, $c_2 < 0$ and $c_3 > 0$. Substituting these functions into Eq. (3.11) yields:

$$\frac{u_2(c(n^*, x^*, \kappa^*), 1 - n^* - s(n^*, x^*))}{(1 - \tau_w)u_1(c(n^*, x^*, \kappa^*), 1 - n^* - s(n^*, x^*))} = a(\tau_e, x^*)F_2(\kappa^*). \quad (3.13)$$

Since x^* and κ^* have already been solved, the only unknown in this equation is n^* .

Fig. 1 depicts Eq. (3.13). The straight line AMP stands for ‘adjusted marginal product of labor’ and depicts the right-hand side of Eq. (3.13). According to the above solution procedure, (x^*, κ^*) are solved independently of n^* so that the right-hand side of Eq. (3.13) is independent of n^* . The curve RW stands for the ‘reservation wage’ and depicts the left-hand side of Eq. (3.13). One can verify

that the left-hand side of Eq. (3.13) is an increasing function of n^* and so the curve RW is upward sloping. The intersection, point E, gives the unique solution for steady state employment. This being done, s^* , c^* and k^* can be recovered.

Before analyzing the steady state effects of policies, notice that there is no natural way to reduce the current equilibrium to the one in Judd (1987) with a Walrasian labor market. When the labor market is Walrasian, the wage rate, the reservation wage and the marginal product of labor are all equal to each other. To make the reservation wage approach the wage rate, the effective unemployment subsidy must approach one (see Eq. (3.11)). This would not only eliminate agents' incentive to work but also give a large subsidy to a part of time agents spend away from employment, which does not exist in Judd's model. On the other hand, for the wage rate to approach the marginal product of labor, either $\lambda \rightarrow 1$ or $b(1 - \tau_v) \rightarrow 0$. These two approaches to the limit have opposite implications on the tightness of the labor market (see Eq. (3.8)) and it is difficult to pick one way over the other as the true limit. For these reasons, we will directly compare the results here with Judd's model rather than trying to nest Judd's model as a special case. Nevertheless, we will try in Sections 5.3 and 6 to check how our results change with the labor market friction.

3.2. Steady state effects of policies

We now analyze the steady-state effects of the policies, with a focus on their effects on (x^*, n^*) . All the policies have unambiguous effects on x^* . This can be confirmed through (3.8), which solves for $x^* = x(\tau_e, \tau_v, \kappa^*)$ with $x_1 < 0$, $x_2 > 0$ and $x_3 > 0$. First, a labor-income tax and an unemployment subsidy both make the labor market tighter. They do so by increasing the effective rate of unemployment subsidy τ_e , which raises the opportunity cost of working, raises the reservation wage and reduces firms' profitability from hiring. Second, a capital-income tax makes the labor market tighter in the steady state by reducing the capital-labor ratio and the marginal product of labor. In contrast, a vacancy subsidy τ_v and an investment tax credit both reduce the tightness of the labor market. A vacancy subsidy reduces the effective vacancy cost and encourages firms to maintain more job vacancies; an investment tax credit makes investment more attractive, increases the capital-labor ratio and raises the marginal product of labor.

The effects of the policies on steady state employment can be analyzed with the aid of Fig. 1, where the initial steady state is point E. The effect of a vacancy subsidy on employment is the easiest to analyze. Since a vacancy subsidy makes the labor market less tight, it increases the fraction a of the marginal product of labor paid to the worker. This shifts the line AMP upward. Also, for given n^* , the increase in x^* reduces $s(n^*, x^*)$ and $c(n^*, x^*)$. That is, the reservation wage falls for given n^* and the curve RW shifts down (not drawn in Fig. 1). Both effects increase steady state employment.

An unemployment subsidy τ_u has analytically ambiguous effect on steady state employment. On the one hand, an unemployment subsidy induces agents to substitute search for work, which reduces employment. On the other hand, when more agents are searching for jobs, firms have more matches per vacancy. In Fig. 1, the effect of τ_u on the reservation wage is represented by an upward shift of the RW curve to RW'; the effect of τ_u on the number of matches is represented by an upward shift of the AMP curve to AMP'. The new steady state employment level is given by point E'. The unemployment subsidy reduces steady state employment if and only if it increases the reservation wage by more than it increases AMP (as depicted in Fig. 1).

Compared with τ_u , a labor-income tax has a larger negative effect on steady state employment. This is because a labor-income tax increases the reservation wage directly, as well as indirectly through its positive effects on x^* as in the case of an unemployment subsidy. The direct effect shifts the curve RW up further, generating further fall in steady state employment.

In contrast to a labor income tax, a capital-income tax can raise steady state employment. This is because a capital income tax reduces the reservation wage, which mitigates the fall in firms' profitability from hiring caused by a low marginal product of labor. To see this, notice that a capital-income tax reduces the capital-labor ratio and the net output per worker, $(F - \delta k^*)/n^*$. Thus, for given n^* consumption c^* falls.⁷ The reservation wage falls and the curve RW shifts downward. Although the line AMP also shifts down because F_2 and a fall, the capital income tax can increase steady state employment if it reduces the reservation wage by more than it reduces the product aF_2 . Numerical exercises in Section 4 confirm this possibility with realistic parameters.

An investment tax credit has the most complicated effects on steady state employment. First, like a vacancy subsidy, an investment tax credit increases x^* for given (n^*, κ^*) , which shifts up the AMP curve and shifts down the RW curve. Second, an investment tax credit induces a higher steady state capital-labor ratio. This increases the marginal product of labor and shifts the AMP curve up further. For given n^* , the higher capital-labor ratio also induces a higher net output per worker and hence higher steady state consumption, which shifts up the RW curve. Overall, the AMP curve shifts up, while the RW curve is likely to shift up as well. Steady state employment increases if the effect through the marginal product of labor is predominant.

In summary, the effects of the five policies on steady state employment can be ranked in a descending order as follows: vacancy subsidy τ_v , capital-income tax τ_k , investment tax credit τ_I , unemployment subsidy τ_u and labor-income tax τ_w . A vacancy subsidy increases steady state employment, a labor-income tax

⁷ A capital income tax also reduces x^* and saves some resources from hiring. However, it can be verified that the fall in net output is far greater than the savings from the hiring cost.

reduces employment, and the rest falls in between. In comparison with an intertemporal model without unemployment, a labor income tax has a larger negative effect on employment since it increases the reservation wage; a capital income tax has a smaller negative effect on employment since it reduces the reservation wage. This suggests that a labor income tax is more costly and a capital income tax is less costly here than in a standard model. However, to evaluate the policies it is necessary to examine their dynamic effects.

4. Dynamic effects of policies

4.1. Locally stable path

To begin, let us model policy changes as a shift at time 0 from the initially constant path τ_0 to a path $\tau(t) = \tau_0 + \Delta \cdot \tau_1(t)$, where Δ is chosen to be a small number so that the tax changes are marginal. Since we only examine permanent policy changes, $\tau_1(t) = \tau_1 \equiv (\tau_{w1}, \tau_{k1}, \tau_{u1}, \tau_{v1}, \tau_{I1})^T$ for all $t \geq 0$. A capital income tax change, for example, can be modelled by setting $\tau_{k1} = 1$ and $\tau_{w1} = \tau_{u1} = \tau_{v1} = \tau_{I1} = 0$. All tax changes are unexpected. Given the form of the policy changes, the results obtained below must be interpreted with care. In particular, the welfare cost calculated later for each policy is a marginal one – it may not necessarily apply to large policy changes. Also, unanticipated and permanent tax changes may not necessarily be the most realistic types of policy changes. They are analyzed here because they are the simplest and the results provide a sharp contrast with a standard model. The current framework can be extended to study large tax changes as in Lucas (1990a) and anticipated temporary changes as in Judd (1987). Such analyses are omitted to economize space.

Denote $Y_\Delta(t) = dY(t)/d\Delta$ as the responses of variables $Y(t)$ to the policy change and $Y_\Delta^* = dY^*/d\Delta$ as steady state responses. When $\Delta = 0.01$ and $\tau_{k1} = 1$, for example, $n_\Delta(t)/n^*$ is roughly the percentage change in employment responding to one percentage point increase in the capital income tax rate. This will be the interpretation used later when we report the results. Since the policy changes are small and permanent, we focus on local dynamics of $Y_\Delta(t)$, which can be derived by differentiating the dynamic system (3.6) with respect to Δ and evaluating the result at $\Delta = 0$. This procedure creates:

$$\dot{Y}_\Delta(t) = J \cdot [Y_\Delta(t) - Y_\Delta^*]. \quad (4.1)$$

J is a 5×5 matrix defined by $J = h_Y(Y^*, \tau_0)$ and $Y_\Delta^* = -J^{-1} \cdot h_\tau(Y^*, \tau_0) \cdot \tau_1$. The initial conditions of this linear differential system are $k_\Delta(0) = n_\Delta(0) = 0$. Since there are two predetermined variables in Eq. (4.1), stability of the system requires that matrix J have two stable eigenvalues and three unstable

Table 1
Base values of parameters

ρ	σ	η	γ	δ	A	β	θ	g
0.01	-2.0	3.5	0.25	0.0107	1.0	3.5468	0.05	0.2754
α	λ	b	τ_{k0}	τ_{w0}	τ_{u0}	τ_{r0}	τ_{l0}	
0.6	0.4	1.1636	0.30	0.30	0.45	0	0	

eigenvalues.⁸ Let the stable eigenvalues be ω_1 and ω_2 and denote $Z_i = (Z_{i1}, Z_{i2}, Z_{i3}, Z_{i4}, Z_{i5})^T$ to be the eigenvector of J that corresponds to ω_i . The stable path of Eq. (4.1) is

$$Y_A(t) - Y_A^* = (Z_1, Z_2) \begin{pmatrix} a_1 e^{\omega_1 t} \\ a_2 e^{\omega_2 t} \end{pmatrix}, \quad (4.2)$$

where a_1 and a_2 are uniquely determined by the two initial conditions $k_A(0) = n_A(0) = 0$. Because of the large dimension of the dynamic system, the above stable path cannot be obtained analytically. We thus resort to numerical exercises.

To compare our results with Judd's (1987), we choose the same parameter values as in Judd whenever it is possible. For parameters related to labor market search that are absent in Judd's model, we choose the values to match the steady state behavior of the model with observations. The base values of parameters are summarized in Table 1. First, the rate of time preference is $\rho = 0.01$. Interpreting a period as a quarter, the steady state annual interest rate is roughly 4%. The instantaneous utility function is

$$u(c, 1 - n - s) = \frac{c^\sigma}{\sigma} - \beta \frac{(n + s)^\eta}{\eta}.$$

The base value of σ is -2.0, which gives a relative risk aversion 3 that is consistent with estimates in Hansen and Singleton (1983). The base value of η is set at 3.5 to obtain a labor supply elasticity $\varepsilon = 1/(\eta - 1) = 0.4$, which falls in the range in Killingsworth (1983). We also allow σ to take values in $(-0.1, -5.0)$ and η in $(1.1, 6)$. The value of β , found through Eq. (3.11) by matching steady state labor force participation $n^* + s^*$ with the realistic value 0.68, differs from that in Judd (1987).

⁸ A stable eigenvalue has a negative real part and an unstable eigenvalue has a positive real part. In addition to the correct numbers of stable and unstable eigenvalues, local stability of the original nonlinear system also requires a regularity condition, as specified in Scheinkman (1976) and Epstein (1987). This regularity condition and the conditions on eigenvalues are met in later numerical exercises.

Second, on the production side, the production function is $F(k,n) = k^\gamma n^{1-\gamma}$ with $\gamma = 0.25$ and the rate of capital depreciation δ is such that capital consumption allowance in the steady state is 12% of the net output.⁹ Third, on government policies, government spending is chosen to be 20% of steady state output. The investment tax credit τ_{I0} is zero as in Judd in order to interpret τ_{k0} as the effective capital income tax. Judd chooses $\tau_{k0} = \tau_{w0} = 0.3$ as the factor income tax rates, which are consistent with King and Fullerton (1984). He also allows for $\tau_{k0} = 0.5$ and $\tau_{w0} = 0.4$ to check the sensitivity of the results. We follow the same practice.

There are seven parameters related to search, $(A, \theta, b, \alpha, \lambda, \tau_{u0}, \tau_{v0})$, that are absent in Judd (1987). We identify them as follows. First, normalize $A = 1$ and set $\theta = 0.05$. The value of θ resembles the quarterly rate of transition from employment to unemployment in Mortensen and Pissarides (1993). This value of θ also implies that the steady state duration of unemployment is $1/m(x^*) = 1.27$, which roughly matches the average unemployment duration (a quarter) in postwar US data (Layard et al., 1991).¹⁰ Second, the unit cost of vacancy, b , is determined through Eq. (3.8) by matching the steady state unemployment rate $s^*/(s^* + n^*)$ with the realistic value 0.06. Third, the base value of α is 0.6 as indicated by Blanchard and Diamond (1989) and the base value of λ is 0.4. In later sensitivity analysis we let (α, λ) take different values 0.2, 0.4, 0.6 and 0.8. Fourth, the base value of the unemployment subsidy τ_{u0} is 0.45 and a higher value $\tau_{u0} = 0.55$ is also explored. Since vacancy subsidies are rare in reality, $\tau_{v0} = 0$.

Let $nw = (1 - \tau_w)w$ be the after-tax wage rate. For the base values of parameters listed in Table 1, Figs. 2–6 plot the percentage changes in the variables (c, s, n, k, v, nw) when there is one percentage point increase in the tax rates $\tau_u, \tau_w, \tau_k, \tau_v$ and τ_I , respectively. We compare four pairs of policies: (τ_u, τ_w) , (τ_k, τ_w) , (τ_v, τ_u) and (τ_I, τ_v) .

4.2. Comparison between τ_u and τ_w

An unemployment subsidy τ_u and a labor income tax τ_w are similar in many aspects, as suggested in Section 3 and illustrated in Figs. 2 and 3. In particular, both policies raise the reservation wage and the actual wage to compensate for the increased opportunity cost of working ($w_A(t) > 0$). As a result, both policies discourage hiring ($v_A < 0$), reduce employment ($n_A < 0$) and induce a tighter

⁹ This value of γ used in Judd (1987) implies a labor share 0.722 here, which seems high when compared with the estimate (0.64) in Christiano (1988). We have also done the numerical exercise with the figure 0.64 and found that the same welfare ranking among the policies.

¹⁰ Burdett et al. (1984) indicate a different estimate of $\theta = 0.15$. We have done the simulation also with this estimate but found results similar to the ones reported here.

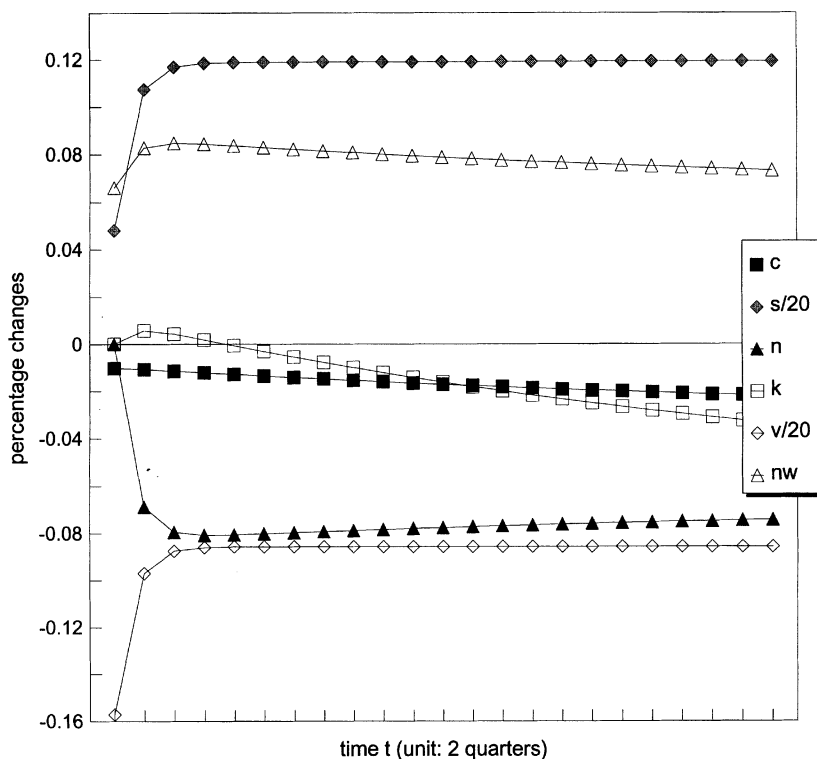


Fig. 2. Dynamic responses to an unemployment subsidy.

labor market ($x_A < 0$, not drawn). The main difference between τ_u and τ_w is their effects on the after-tax wage rate and unemployment. An unemployment subsidy raises the after-tax wage rate but a labor income tax reduces it. By raising the after-tax wage rate, an unemployment subsidy encourages more agents to search ($s_A > 0$) and hence increases the labor force participation along the entire dynamic path ($n_A + s_A > 0$). A labor income tax does the opposite. It lowers the net income from employment and discourages agents from participating in the labor force. The unemployment level immediately falls after the labor-income tax. Along the entire dynamic path, a labor-income tax reduces employment by a larger magnitude than an unemployment subsidy does. To increase employment, a policymaker may find it attractive at the margin to cut the unemployment subsidy to finance a cut in the labor income tax.

4.3. Comparison between τ_k and τ_w

A capital income tax τ_k and a labor income tax τ_w share some long-run features, as depicted in Figs. 3 and 4. They both reduce consumption, the capital

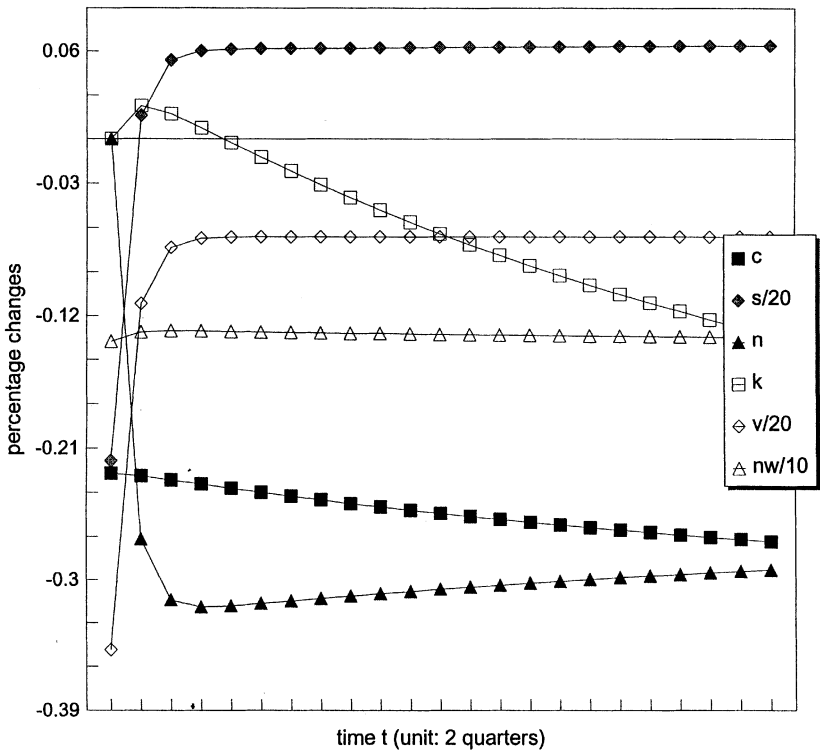


Fig. 3. Dynamic responses to a labor income tax.

stock and the after-tax wage rate in the long-run. However, the two taxes differ in their long run effects on employment. While a labor income tax reduces long-run employment, a capital income tax increases long-run employment under the current parameter values. As explained in Section 3, this difference arises because a labor income tax raises the reservation wage but a capital income tax reduces it.

The two taxes also differ in their immediate effects on investment, consumption and the after-tax wage rate. A capital income tax immediately reduces investment, increases the after-tax wage rate and increases consumption, but a labor income tax does the opposite. The positive short-run responses of the after-tax wage rate and consumption to a capital income tax are opposite to their long-run responses. To explain, note that a capital income tax reduces investment immediately by reducing the after-tax rate of return to capital. Since a capital income tax also induces firms to cut job vacancies, the generated resources within each firm must be absorbed by a temporarily higher wage rate. The higher wage rate and the lower capital accumulation provide resources for

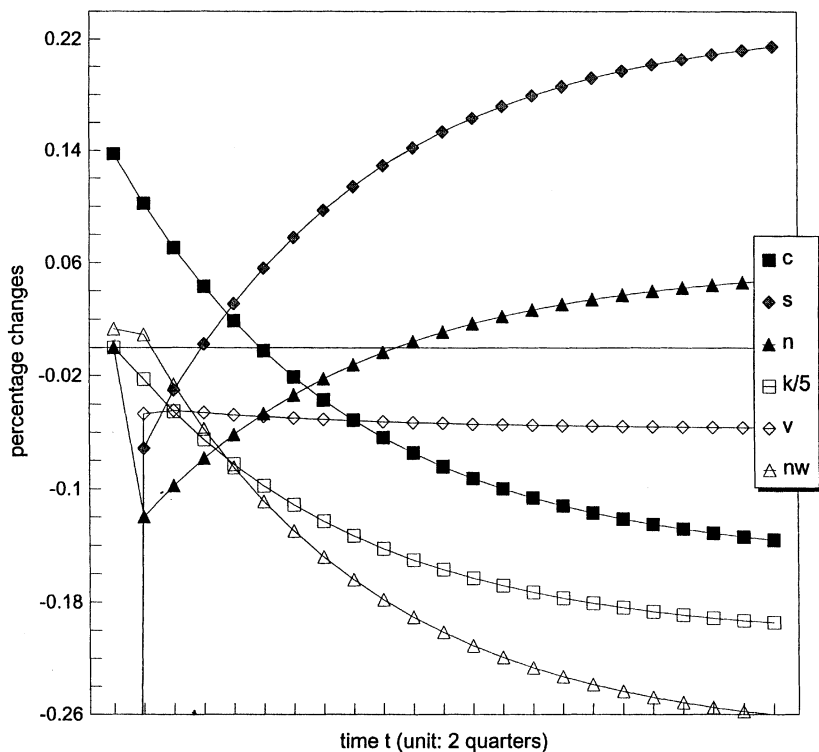


Fig. 4. Dynamic responses to a capital income tax.

households to increase consumption temporarily. In contrast, a labor income tax immediately reduces the after-tax wage rate, which immediately reduces consumption. Firms immediately increase investment to absorb the resources generated by the lower wage and fewer vacancies.

4.4. Comparisons between (τ_v, τ_u) and between (τ_v, τ_I)

The effects of a vacancy subsidy τ_v are opposite to those of an unemployment subsidy on all variables except the wage rate (see Fig. 5). It reduces unemployment and increases consumption, vacancies and employment. This is hardly surprising, since a vacancy subsidy creates incentives (or disincentives) to firms and workers that are just opposite to an unemployment subsidy.

A vacancy subsidy is a subsidy to ‘investment’ in employment, as discussed in Section 2.2. Since labor and capital are complementary in the production function, a subsidy to the investment in employment is shared by capital

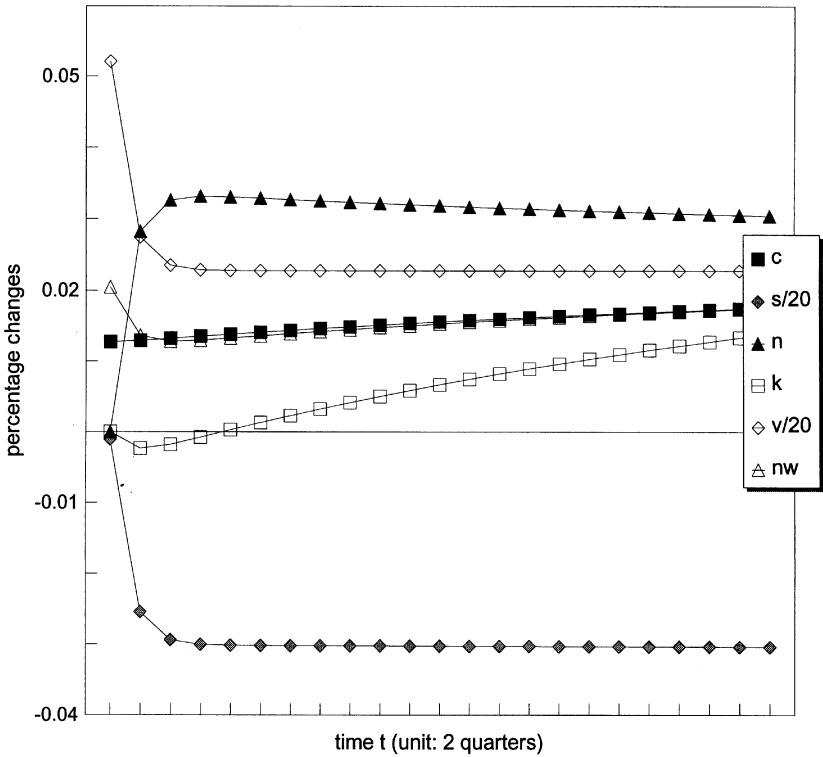


Fig. 5. Dynamic responses to a vacancy subsidy.

through equilibrium factor prices, and vice versa. Not surprisingly, a vacancy subsidy and an investment tax credit have many similar effects. Comparing Figs. 5 and 6 shows that both policies (a) immediately increase vacancy, relax the tightness of the labor market and increase employment, (b) increase the capital stock and reduce the search effort in the long run; (c) increase long-run consumption and after-tax wage rate.

The two policies differ in two aspects. First, their long-run effects on employment differ, although their short-run effects are the same. In contrast to the positive effect of a vacancy subsidy, an investment tax credit reduces long-run employment. This is because an investment tax credit increases the average product of labor and long-run consumption by such a large proportion that steady state reservation wage rises by a large proportion (see Section 3 for a discussion). This is supported by Fig. 6, where percentage increases in steady state consumption and wage rate are much larger than those in Fig. 5.

Second, although a vacancy subsidy and an investment tax credit have the same long run effects on investment, their immediate effects on investment differ.

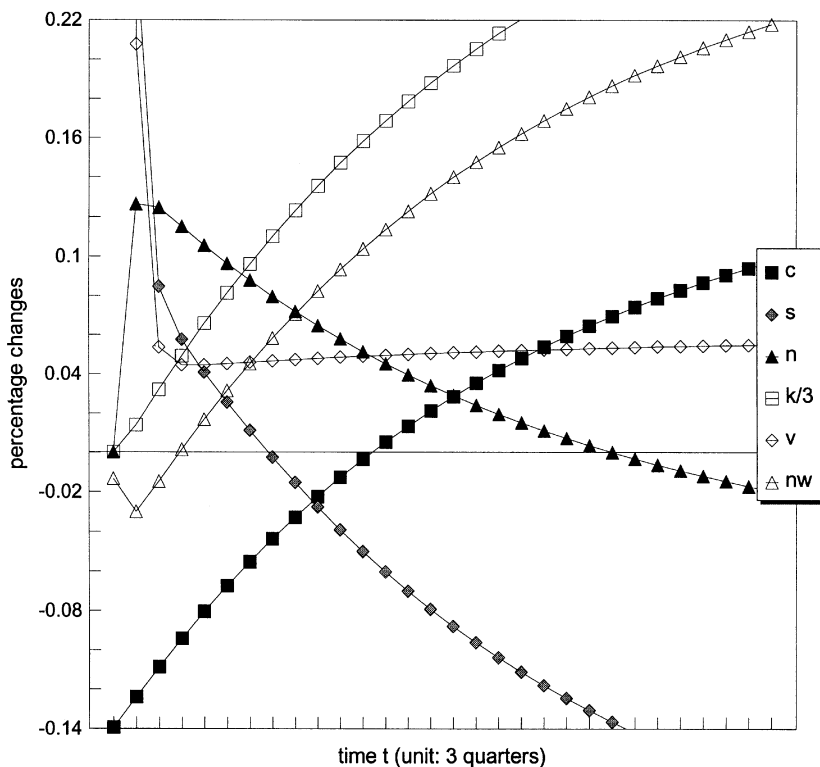


Fig. 6. Dynamic responses to an investment tax credit.

An investment tax credit immediately increases investment but a vacancy subsidy immediately reduces investment. Also, a vacancy subsidy immediately increases the net wage and consumption, but an investment tax credit does the opposite immediately, although the two policies agree on the long-run effects on consumption and wage rate. The immediate fall in investment in the case of a vacancy subsidy is necessary for creating resources for hiring, given the positive responses of consumption and wage rate. In contrast, when the investment tax credit increases, it is not optimal for firms to reduce investment and so the increased resources in hiring come from reduced wages and consumption.

To summarize this section, we describe two common features of the dynamic responses to the policy changes. First, the initial responses take place primarily in vacancy and the search effort. The percentage changes of these two variables are so much larger than those of other variables that they are often re-scaled to fit into Figs. 2–6. In contrast, employment responds smoothly to the policy changes, a feature explored by Andolfatto (1996) and Shi and Wen (1997).

Second, the capital stock and employment typically follow non-monotonic dynamic paths. For example, when the labor income tax increases, the capital stock first rises and then falls. In contrast, in a standard model the capital stock falls monotonically after a labor income tax.

5. Marginal welfare costs of taxes and subsidies

5.1. Marginal deadweight loss

We now compute the marginal welfare costs of taxes and subsidies. Let U_Δ be the change in intertemporal utility caused by a policy change. Then

$$U_\Delta = u_1 \int_0^\infty c_\Delta(t) e^{-\rho t} dt - u_2 \int_0^\infty [n_\Delta(t) + s_\Delta(t)] e^{-\rho t} dt.$$

U_Δ can be computed by substituting $(c_\Delta, n_\Delta, s_\Delta)$ from Eq. (4.2). This deadweight loss can be expressed as a loss in the present value of real income (consumption) of the magnitude U_Δ/u_1 . Since different taxes raise different levels of revenues, we measure this deadweight loss relative to the revenue. The present value of the revenue raised by a tax increase is

$$R \equiv \int_0^\infty L_\Delta(t) e^{-\rho t} dt.$$

The function $L_\Delta(t)$ can be derived by differentiating Eq. (2.15) and substituting Eq. (4.2).

The *marginal deadweight loss* (MDL) proposed by Judd (1987) takes the following form:¹¹

$$\text{MDL} = -\frac{U_\Delta}{u_1 R}. \quad (5.1)$$

For example, the marginal deadweight loss of capital taxation is computed by setting $\tau_{k1} = 1$ and $\tau_{w1} = \tau_{u1} = \tau_{v1} = \tau_{I1} = 0$. Denote MDL_k as the marginal deadweight loss of capital taxation and denote MDL_w , MDL_u , MDL_v and MDL_I similarly. In a normal case, MDL is positive for a tax and negative for a subsidy. Table 2 reports marginal deadweight losses for different values of ϵ, σ and τ_{u0} when $\tau_{k0} = \tau_{w0} = 0.3$ and $\tau_{v0} = \tau_{I0} = 0$. The numbers confirm the intuition that marginal deadweight losses from capital income taxation and

¹¹ Judd defines MDL as $\rho U_\Delta/(u_1 R)$. The numerical results reported in Judd (1987) are actually $U_\Delta/(u_1 R)$. Our definition corrects this typographical error. Also, we added a negative sign to his definition in order to report welfare losses of taxes as positive numbers.

Table 2
MDL of permanent changes in τ

ε	σ	$\tau_{r0} = 0.45$					$\tau_{r0} = 0.55$				
		(1)	(2)	(3)**	(4)*	(5)*	(1)	(2)	(3)**	(4)*	(5)*
0.1	-0.1	0.108	0.338	-0.585	-8.269	0.965	0.232	0.350	-0.742	-1.815	0.981
	-0.5	0.105	0.307	-0.582	-9.313	0.843	0.226	0.318	-0.740	-1.835	0.859
	-2.0	0.096	0.239	-0.572	-15.047	0.606	0.211	0.248	-0.732	-1.899	0.619
	-5.0	0.085	0.181	-0.560	-103.191	0.429	0.192	0.188	-0.723	-1.997	0.440
0.4	-0.1	0.197	0.410	-0.551	-7.810	1.299	0.328	0.422	-0.723	-1.861	1.310
	-0.5	0.178	0.361	-0.547	-10.364	1.066	0.302	0.372	-0.719	-1.910	1.080
	-2.0	0.137	0.271	-0.538	-68.058	0.713	0.246	0.281	-0.710	-2.048	0.728
	-5.0	0.102	0.211	-0.529	14.207	0.517	0.200	0.220	-0.701	-2.210	0.531
0.6	-0.1	0.240	0.444	-0.535	-7.621	1.486	0.374	0.457	-0.714	-1.882	1.494
	-0.5	0.209	0.384	-0.532	-10.810	1.176	0.334	0.396	-0.710	-1.942	1.188
	-2.0	0.149	0.284	-0.526	714.512	0.757	0.256	0.294	-0.702	-2.101	0.772
	-5.0	0.106	0.222	-0.522	11.138	0.549	0.202	0.231	-0.695	-2.271	0.564

Notes: Parameters have base values. Column (1) is for MDL_{us}, (2) for MDL_{de}, (3) for MDL_{jp}, (4) for MDL_{fr}, and (5) for MDL_{it}. **: $U_j < 0$; *: $U_j > 0$.

labor income taxation increase with the labor supply elasticity ε and consumption demand elasticity $1/(1 - \sigma)$.

5.2. Comparison between different policies

Additional results can be drawn from Table 2. First, the marginal deadweight loss of capital income taxation is much lower than in Judd (1987). In Table 2, no figure of MDL_k exceeds 50 cents. In contrast, Judd (1987) has found that MDL_k often exceeds one dollar. The capital income tax has a smaller welfare cost here because it increases consumption temporarily and increases long-run employment by reducing the reservation wage (see Sections 4.2 and 4.3). In contrast, consumption falls immediately and long-run employment falls in a standard model. Since the steady state capital-labor ratio falls by the same amount in the present model as in the standard model, the increase in steady state employment implies that steady state capital stock and output fall by a smaller amount in the present model.

Second, the relative welfare cost of the capital income tax to the labor income tax is lower in the current model than in Judd (1987). For $\tau_{u0} = 0.45$ in Table 2, the capital income tax is about 20 cents more costly than the labor income tax. For $\tau_{u0} = 0.55$, the welfare cost of the labor income tax is very close to that of the capital income tax. The welfare ranking of the two taxes can even be reversed when $\tau_{k0} = 0.5$ and $\tau_{w0} = 0.4$, as shown later in Table 3. In comparison, Judd (1987) has never found such a reversal. Instead, his results indicate that MDL_k is three to four times as large as MDL_w . The contrast in results arises because a labor income tax here directly reduces employment by raising the reservation wage (see Section 3) but only indirectly so through the equilibrium effect of the wage rate in a standard model.

The third result from Table 2 is that MDL_u is negative. This is because the subsidy reduces government revenue ($R < 0$) and reduces welfare ($U_A < 0$). It is not surprising that the unemployment subsidy reduces the governmental revenue. The subsidy also reduces intertemporal utility because it reduces consumption and increases the labor force participation (see Section 4.2). Since both consumption and leisure are lower, utility falls. The absolute value of MDL_u is above 50 cents and increases with τ_{u0} . There is a large marginal welfare gain from cutting the unemployment subsidy to finance a cut in the labor income tax. This welfare gain, measured by $MDL_w - MDL_u$, can easily exceed 50 cents for $\tau_u = 0.45$ and can exceed a dollar for $\tau_u = 0.55$. The negativity of MDL_u is quite robust with respect to changes in ε, σ and τ_{u0} , as Table 2 indicates.

The fourth result from Table 2 is that a vacancy subsidy is quite efficient. It increases intertemporal utility in all combinations of parameters in Table 2 and increases the governmental revenue in all but a few cases. The government revenue increases because capital income and labor income are increased sufficiently to generate additional tax revenue that outweighs the subsidy. The

magnitude of MDL_v is large and exceeds 1.8 dollars. Cutting the unemployment subsidy to subsidize hiring has a marginal welfare gain of at least 2.5 dollars. Compared with this magnitude, the marginal gains from switching between other taxes are pale.

Like a vacancy subsidy, an investment tax credit also increases welfare. However, it typically reduces the government revenue and so MDL_I is positive. That is, with the parameter values used in Table 2 an investment tax cannot be self-financed.¹² To explain why an investment tax credit reduces tax revenue, recall that an investment subsidy reduces long-run employment. The fall in steady state employment reduces the labor income tax revenue and the total tax revenue.

5.3. Importance of the labor market friction

We now illustrate the degree to which our results are robust to changes in the labor market friction. Table 2 has already contained such an illustration by presenting the welfare costs under two unemployment subsidy rates, 0.45 and 0.55. In the sense that an unemployment subsidy reduces agent's incentive to work, an increase in the subsidy exacerbates the labor market friction. Table 2 shows that such an increase in the friction increases the relative cost of labor income taxation and can even make labor income taxation more costly than capital income taxation.

Consider three other parameters λ, α , and b . Table 3 lists the results when (λ, α) take values in $\{0.2, 0.4, 0.6, 0.8\}$ and (τ_{k0}, τ_{w0}) take two sets of values. For fixed α , increasing the worker's bargaining weight λ makes both capital income taxation and labor income taxation more inefficient. The relative cost of labor income taxation to capital income taxation increases. This is because firms' profitability from hiring is already low and the labor income tax reduces it further. Similarly, for fixed λ , increasing α reduces the gap between the welfare costs of the two factor income taxes, although both taxes become more inefficient. When α is sufficiently large, labor income taxation become more costly than capital income taxation.

The marginal costs of the unemployment subsidy and the vacancy subsidy exhibit non-monotonic patterns when (λ, α) increase. We explain only the pattern for the unemployment subsidy, while a similar explanation applies to the vacancy subsidy. Starting from very low values of (λ, α) (e.g. $\lambda = 0.2$ and $\alpha = 0.2$), an increase in λ or α reduces the marginal welfare cost of the unemployment subsidy. As λ or α increases further, the pattern is reversed. To explain this non-monotonic pattern, recall that the condition $\lambda = 1 - \alpha$ is required for the search equilibrium to internalize the labor market externalities when there are no distortionary taxes (see Section 3). When unemployment is subsidized

¹²To economize on space, the numbers for U_A and R are not listed in the tables.

($\tau_{u0} \neq 0$) but vacancy is not ($\tau_{v0} = 0$) as in the current numerical exercise, the labor market is distorted even when $\lambda = 1 - \alpha$. If λ is not so much lower than $1 - \alpha$ (e.g., $\lambda \geq 0.4$ and $\alpha \geq 0.4$), unemployed agents are compensated too much for their search effort but firms are compensated too little for vacancies, relative to their contributions to the match creation. In this case, efficiency can be improved if the worker's bargaining weight is reduced (for fixed α) or if the elasticity of vacancies in job matches is reduced (for given λ). That is, MDL_u increases with (λ, α) . However, if λ is much lower than $1 - \alpha$, the nature of inefficiency in the labor market is reversed. For example, when $\alpha = 0.2$ and $\lambda = 0.2$, the contribution of the worker's search effort to the job match creation, measured by $1 - \alpha = 0.8$, exceeds the worker's share of the match surplus by a wide margin. In this case, the search effort is the one which is compensated too little. An increase in the unemployment subsidy improves efficiency in this case.

The above dependence of the welfare costs on (λ, α) is robust with respect to changes in the base tax rates (τ_{k0}, τ_{w0}). Table 3 shows the same pattern of dependence when τ_{k0} changes from 0.3 to 0.5 and τ_{w0} from 0.3 to 0.4. As in Judd (1987), the increase in the base tax rates significantly raises the marginal welfare costs of the capital and income taxes. With these increased base tax rates, a labor income tax is often more costly than a capital income tax when $\alpha \geq 0.4$.

We now turn to the marginal cost of vacancy, b . Since b is identified by matching the steady state unemployment rate $s^*/(n^* + s^*)$ to a particular number un^* (see Section 4), increasing un^* amounts to an increase in b . Table 4 lists the marginal welfare costs of capital and labor income taxes when un^* takes different values. When un^* increases from 0.01 to 0.12, both labor income taxation and capital income taxation become more costly, with the cost of labor income taxation increasing faster than that of capital income taxation. To explain, note that the calibration exercise fixes steady state labor force participation $n^* + s^*$ at a value 0.68. With given labor force participation, an increase in un^* amounts to higher steady state unemployment. The existence of more unemployed agents widens the difference between a labor income tax and a capital income in their effects on employment. Thus, as our model deviates further along the dimension of unemployment from a Walrasian labor market, labor income taxation becomes more costly than capital income taxation.

6. Alternative wage determination and matching schemes

In the sensitivity analysis in the last section we have kept the Nash bargaining scheme and the matching function. Since these two elements are obvious deviations from a conventional model, we now analyze how the welfare results change when they are removed.

Table 3
Dependence of MDL on (α, λ) and (τ_{k0}, τ_{w0})

λ	(τ_{k0}, τ_{w0})	(0.3, 0.3)					(0.5, 0.4)				
		0.2	0.4	0.6	0.8		0.2	0.4	0.6	0.8	
0.2	(1)	-0.072	0.036	0.164	0.320	0.047	0.473	1.134	4.090		
	(2)	0.216	0.236	0.255	0.275	0.641	0.673	0.707	0.743		
	(3)**	2.377	0.348	-0.386	-0.766	0.476	-0.470	-0.764	-0.908		
	(4)*	-0.736	-0.187	1.641	-22.334	-0.448	-2.105	-1.561	-1.452		
	(5)*	0.560	0.616	0.678	0.744	1.542	1.658	1.788	1.933		
0.4	(1)	0.054	0.142	0.245	0.368	0.260	0.714	1.640	4.583		
	(2)	0.249	0.265	0.281	0.298	0.672	0.702	0.734	0.768		
	(3)**	0.271	-0.404	-0.710	-0.884	-0.392	-0.752	-0.883	-0.951		
	(4)*	-0.341	-3.959	-2.048	-1.792	-1.358	-1.210	-1.185	-1.175		
	(5)*	0.627	0.676	0.728	0.784	1.597	1.709	1.832	1.970		
0.6	(1)	0.099	0.180	0.274	0.384	0.337	0.801	1.749	4.762		
	(2)	0.262	0.276	0.291	0.307	0.683	0.712	0.744	0.776		
	(3)**	-0.217	-0.617	-0.809	-0.923	0.618	-0.838	-0.922	-0.966		
	(4)*	-2.545	-1.536	-1.440	-1.404	-1.133	-1.113	-1.108	-1.106		
	(5)*	0.652	0.697	0.746	0.798	1.616	1.726	1.848	1.983		
0.8	(1)	0.122	0.199	0.288	0.392	0.376	0.846	1.805	4.855		
	(2)	0.268	0.282	0.296	0.312	0.688	0.718	0.749	0.781		
	(3)**	-0.434	-0.718	-0.858	-0.942	0.722	-0.879	-0.940	-0.973		
	(4)*	-1.359	-1.295	-1.278	-1.271	-1.080	-1.077	-1.075	-1.075		
	(5)*	0.665	0.708	0.755	0.805	1.626	1.735	1.855	1.989		

Notes: $\tau_{w0} = 0.55$, base values for other parameters. Row (1) is for MDL_{vs}, (2) for MDL_k, (3) for MDL_m, (4) for MDL_v and (5) for MDL_r. **: $U_d < 0$; *: $U_d > 0$.

Table 4
Dependence of MDL on un

	$un^* = s^*/(s^* + n^*)$				
	0.01	0.03	0.06	0.10	0.12
MDL_w	0.1098	0.1588	0.2455	0.3955	0.4918
MDL_k	0.2752	0.2778	0.2814	0.2858	0.2878

Notes: $\tau_{u0} = 0.55$, base values for other parameters.

6.1. Wage bargaining with insiders and outsiders

We first change the wage determination scheme but maintain the matching function. To begin, note that the Nash bargaining solution (2.14) is restrictive because the weights are fixed, not because it expresses the wage as a weighted sum of the reservation wage and the marginal product of labor. Any wage determination scheme that gives both sides of the match a non-negative surplus can be expressed in the form of Eq. (2.14) with the weights being suitably modified. There are many ways to make the bargaining weights respond to the labor market conditions. Here we adapt the insider-outsider sequential bargaining approach in Shaked and Sutton (1984), which encompasses both the competitive solution and the Nash bargaining solution as special cases.

The Shaked-Sutton model considers sequential bargaining between a firm and a matched worker (an insider), where the two sides alternate in making wage offers. The key ingredient is that the firm has the opportunity to switch to a new worker (an outsider) after T periods of negotiation with the insider. We keep this ingredient. To facilitate the comparison with previous sections, we generalize the Shaked-Sutton model by letting nature choose which side to make the offer in each round of bargaining. The firm is chosen with probability $p \in (0,1)$ and the worker with probability $1 - p$. Appendix A contains a detailed description of the bargaining sequence and a proof for the following proposition.

Proposition 6.1. *The bargaining game has a unique subgame perfect equilibrium. The first proposal is accepted and there is no delay nor switching in the equilibrium. Moreover, when offers are made infinitely quickly, the worker's share of surplus approaches the following limit:*

$$\lambda_A = \frac{T(1-p)^2}{p + T(1-p)}.$$

When the firm's switching opportunity arises infinitely quickly ($T \rightarrow 0$), the firm extracts all the match surplus ($\lambda_A \rightarrow 0$); if the firm's switching opportunity

arises infinitely slowly ($T \rightarrow \infty$), the worker obtains the Nash bargaining share $(1 - p)$. In equilibrium, T depends on the labor market tightness $x = \bar{v}/\bar{s}$. Since each vacancy is matched with a worker at the rate μ , $T = 1/\mu = x/m(x)$ and the worker's share of the match surplus is

$$\lambda_A(x) = \frac{x(1-p)^2}{pm(x) + x(1-p)}. \quad (6.1)$$

The worker gets a larger share of the surplus when the labor market is less tight (x larger). The wage rate is given by Eq. (2.14) with λ being replaced by $\lambda_A(x)$.

One can also examine the opposite case where the worker but not the firm can switch to a new firm after T rounds of negotiation. Since the rate at which a worker finds a match is $m(x)$, it is reasonable to set $T = 1/m(x)$ in this case. The worker's share of the match surplus when offers are made infinitely quickly can be calculated as

$$\lambda_B(x) = \frac{(1-p)[p + m(x)]}{p + (1-p)m(x)}. \quad (6.2)$$

Again, the worker's share is higher when the labor market is less tight (x larger). $\lambda_B \rightarrow 1$ when $x \rightarrow \infty$ and $\lambda_B \rightarrow 1 - p$ when $x \rightarrow 0$. Not surprisingly, $\lambda_B(x) > \lambda_A(x)$ for any $x \geq 0$.

In reality both the firm and the worker can switch to a different bargaining partner and so the worker's share is likely to be between Eqs. (6.1) and (6.2). We compute the welfare costs for both cases. Since p is the firm's Nash bargaining share, it is chosen to be 0.6, the value used for $1 - \lambda$ before. The welfare results are reported in Table 5.

The results can be compared with those under Nash bargaining (the cell in Table 2 with $\varepsilon = 0.4$ and $\tau_{u0} = 0.55$). First, changing the wage determination scheme does not change much the welfare cost of the capital income tax and the welfare benefit of the investment tax credit. There is a small decrease in both in the two cases. Second, when only the firm has the opportunity to switch bargaining partner (the top half of Table 5), the vacancy subsidy can no longer be self-financed, although it continues to increase welfare. As a result, MDL_v becomes positive numbers. The welfare costs of the labor income tax and the unemployment subsidy are much smaller than before. MDL_w is only about two-fifth and MDL_u is only about one-ninth of the corresponding number in Table 2. This is because the wage scheme (6.1) gives the worker a share of the match surplus that is much lower than the worker's contribution to the match formation ($\lambda_A(x^*) = 0.145 < 1 - \alpha$). Thus, subsidizing unemployment becomes less inefficient and subsidizing hiring becomes less efficient.

Similarly, when only the worker has the opportunity to switch bargaining partner (the bottom half of Table 5), the worker is given a relatively high share of the match surplus which exceeds the worker's contribution to the match

Table 5
MDL with insider–outsider bargaining

	σ	MDL _w	MDL _k	**MDL _u	*MDL _v	*MDL _l
Only firms	– 0.1	0.148	0.352	– 0.075	0.331	1.121
can switch	– 0.5	0.132	0.311	– 0.081	0.299	0.929
	– 2.0	0.097	0.235	– 0.096	0.233	0.631
	– 5.0	0.067	0.183	– 0.110	0.182	0.462
Only workers	– 0.1	0.290	0.420	– 0.751	– 1.511	1.265
can switch	– 0.5	0.266	0.271	– 0.748	– 1.533	1.047
	– 2.0	0.215	0.281	– 0.739	– 1.592	0.710
	– 5.0	0.172	0.220	– 0.730	– 1.657	0.519

Notes: (ε, τ_0) : base values; $\tau_{u0} = 0.55$; $p = 0.6$; *: $U_A > 0$; **: $R < 0$ and $U_A < 0$.

formation ($\lambda_B(x^*) = 0.606 > 1 - \alpha$). As a result, the unemployment subsidy is slightly more inefficient than before and the vacancy subsidy raises higher revenue than before (which produces a smaller absolute value for MDL_v). The labor income tax is slightly less inefficient than before as the tax raises a higher revenue.

6.2. Wage posting

The exercise in the last subsection is limited in the sense that a firm cannot use its wage rate to directly affect how quickly its vacancies are filled. In this subsection we examine the case where firms post wages intending to attract workers. The analysis relies on neither the exogenous matching function nor the Nash bargaining solution. The specific wage-posting framework used here is the one in Peters (1991), which is tractable because there is no wage dispersion among homogeneous jobs and workers. Since the framework closely follows Peters', we only briefly describe the environment and state the results.

There are a large number of vacancies and a large number of unemployed workers. The relative number of the two equals the labor market tightness, x . Firms post wages simultaneously. After observing all posted wages, unemployed agents decide which job to apply to. If a job gets only one applicant, the worker gets the job at the posted wage. If a job gets more than one applicant, only one applicant is chosen randomly (with equal probability) to get the job at the posted wage. Let q be the expected number (the queue length) of applicants for a job. The probability that an applicant gets the job to which he applies is $m(q)$ and the probability that the firm fills the vacancy is $\mu(q)$. When the numbers of vacancies and unemployed agents are sufficiently large, these matching rates

approach:

$$m(q) = (1 - e^{-q})/q, \quad \mu(q) = 1 - e^{-q}.$$

Since firms post wages before workers apply, a firm anticipates that its posted wage will affect the number of applicants. To see this, note that an applicant must be indifferent between applying for the job with (w, q) and any other job with (\tilde{w}, \tilde{q}) :

$$m(q) \cdot \left[w - \frac{u_2}{(1 - \tau_w)u_1} \right] = \text{ESW} \equiv m(\tilde{q}) \cdot \left[\tilde{w} - \frac{u_2}{(1 - \tau_w)u_1} \right],$$

where ESW is the applicant's expected surplus from applying for any other job. When there are many vacancies and applicants in the market, each firm's influence on ESW is negligible. For given ESW, the above worker's indifferent relation gives $q = q(w)$. It is easy to verify $q'(w) > 0$ and so a firm can attract more applicants by posting a higher wage. The matching rate for a vacancy, $\mu(q(w))$, is an increasing function of the posted wage.

Anticipating such dependence of the matching rate on the posted wage, each firm chooses w in addition to (v, k^d, n^d) to maximize the present value of profit given in (PF). The optimality conditions for (v, k^d, n^d) have the same forms as before, with μ being replaced by $\mu(q(w))$. The optimality condition for w is

$$n = v\Omega_F \cdot \mu'(q) \cdot q'(w).$$

In a symmetric equilibrium the wage rate is the same across vacancies and hence each vacancy attracts the same expected number of applicants, $q = 1/x$. Slightly abusing the notation we can rewrite equilibrium matching rates for unemployed workers and vacancies as

$$m(x) = x(1 - e^{-1/x}), \quad \mu(x) = 1 - e^{-1/x}. \quad (6.3)$$

Using $1/x$ to replace q and substituting $(\mu'(q), q'(w), \Omega_F)$ in the optimality condition for w yields the following wage equation:

$$w = \frac{u_2}{(1 - \tau_w)u_1} + b(1 - \tau_v) \frac{v}{n} \left[\frac{1}{\alpha(x)} - 1 \right], \quad (6.4)$$

where $\alpha(x) \equiv xm'(x)/m(x)$. The endogenous matching function implicit in Eq. (6.3) have some interesting features. First, the function exhibits constant returns to scales – the matching rates depend only on the relative number of vacancies to unemployed agents and satisfy $m(x) = x\mu(x)$. Second, the wage rate lies between the reservation wage and the marginal product of labor around the

steady state. To see this, notice that the steady state satisfies $v^*/n^* = \theta/\mu(x^*)$ and

$$b(1 - \tau_v) = \mu(x^*)\Omega_F^* = \frac{\mu(x^*)}{\theta + \rho}(F_2 - w^*).$$

With these relations the steady state version of Eq. (6.4) becomes

$$w^* = \lambda(x^*)F_2 + [1 - \lambda(x^*)]\frac{u_2}{(1 - \tau_w)u_1},$$

where $\lambda(x^*)$ is the worker's share of the match surplus in the steady state:

$$\lambda(x^*) = \frac{\theta}{\theta + \rho\alpha(x^*)} \cdot [1 - \alpha(x^*)]. \quad (6.5)$$

Since $\lambda(x^*) \in (0,1)$, the wage rate is indeed between the reservation wage and the marginal product of labor around the steady state. Also, $\lambda(x) < 1 - \alpha(x)$ around the steady state.¹³

With the new wage formula (6.4) and the new forms of $(m(x), \alpha(x))$, the dynamic equation for x is now replaced by the following:

$$\dot{x} = \frac{1}{1 - \alpha(x)} \left\{ x \left[\theta + (1 - \tau_k) \frac{F_1 - \delta(1 - \tau_I)}{1 - \tau_I(1 - \tau_k)} \right] + \frac{vm(x)}{n} \left[\frac{1}{\alpha(x)} - 1 \right] - \frac{m(x)}{b(1 - \tau_v)} \left[F_2 - \frac{u_2}{(1 - \tau_w)u_1} \right] \right\}.$$

One can then identify b with the steady state version of this equation and revise the identification of β accordingly. The welfare costs of taxes and subsidies are reported in Table 6.

Endogenizing the matching function and the wage determination scheme does not change much the inefficiency of the capital income tax and the efficiency of the investment tax credit. The welfare costs (benefits) of these two policies slightly fall in comparison with those in Table 2 (the cell with $\varepsilon = 0.4$ and $\tau_{w0} = 0.55$). However, the welfare cost of the wage income tax is only about two-fifths of that before; the welfare cost of the unemployment subsidy is only about one-third of that before; and the efficiency of a vacancy subsidy increases by about 80%. These changes occur because wage-posting generates both a much higher worker's share of the match surplus, $\lambda(x^*) = 0.732$, and a much

¹³ Because ρ is small, $\lambda(x^*)$ in Eq. (6.5) is close to $1 - \alpha(x^*)$. The difference between the two is created by the firm's concern that existing workers are paid the posted wage as well. If the firm ignored this effect, the wage rate would be determined by $\max(F_2 - w)\mu(q(w))$, which would generate a wage equation with precisely $\lambda(x) = 1 - \alpha(x)$. The welfare costs of policies with this alternative wage formula are very similar to the ones reported here.

Table 6
MDL with wage posting

σ	MDL _w	MDL _k	**MDL _u	*MDL _v	*MDL _I
– 0.1	0.140	0.389	– 0.248	– 2.918	1.117
– 0.5	0.125	0.346	– 0.250	– 3.102	0.939
– 2.0	0.092	0.265	– 0.254	– 3.705	0.655
– 5.0	0.064	0.209	– 0.257	– 4.624	0.488

Notes: (ε, τ_0) : base values; $\tau_{u0} = 0.55$; *: $U_A > 0$; **: $R < 0$ and $U_A < 0$.

lower contribution of vacancy to the match formation, $\alpha(x^*) = 0.233$. The high worker's share implies that a wage income tax generates a large revenue and hence a low welfare cost. The higher wage also implies that the unemployment subsidy requires a larger revenue, which leads to a smaller measure of MDL_u. Finally, since the labor market is quite tight, a vacancy subsidy generates a large welfare gain by increasing the firm's incentive to hire.

In summary, the welfare ranking of different taxes and subsidies is robust to changes in the wage determination scheme and the match formation, although the magnitudes change with the framework. In particular, a vacancy subsidy increases welfare but an unemployment subsidy reduces the government revenue and welfare. Interestingly, the welfare costs of the capital income taxation and the investment tax credit do not change much with these changes in the labor market arrangements. Moreover, there is a sense in which the welfare cost gap between capital income taxation and labor income taxation widens as the labor market becomes more competitive (Table 6 and the top half of Table 5).

7. Conclusion

In an intertemporal model with unemployment, we have examined the dynamic effects and welfare costs of a labor income tax, a capital income tax, an unemployment subsidy, a vacancy subsidy, and an investment tax credit. When compared with a standard model such as Judd (1987), the current model reduces the relative cost of capital income taxation to labor income taxation. In some cases a labor income tax can even be more costly than a capital income tax. Moreover, an unemployment subsidy is inefficient: It reduces government revenue and reduces intertemporal utility. A vacancy subsidy is similar to an investment tax credit and is very efficient: It increases government revenue and increases intertemporal utility. The welfare ranking between policies is robust to changes in the labor market structure that determines matches and wages.

The key differences between our results and Judd's (1987) can be attributed to the non-Walrasian labor market. Since it is costly for firms to hire workers, employment becomes a stock that must be maintained through 'investment'. A labor income tax, by increasing the reservation wage and reducing the firm's profitability from hiring, reduces the firm's incentive to invest in employment. A capital income tax, on the other hand, reduces the reservation wage and may have a positive effect on the profitability from hiring. This difference increases the relative cost of a labor income tax to a capital income tax. Similarly, an unemployment subsidy is inefficient because it pushes up the reservation wage and reduces firms' profitability from hiring. A vacancy subsidy is efficient because it is a subsidy to investment in employment.

We view the exercises in this paper as a benchmark for future examinations. As stated earlier, the framework can be extended to study large tax changes (as in Lucas, 1990a), anticipated temporary changes (as in Judd, 1987) or other policies such as tariffs (Shi, 1995). One can also use the model to examine optimal taxation. Standard models without unemployment such as Chamley (1986) have shown that optimal capital income tax converges to zero when the economy converges to the steady state, but optimal labor income tax does not converge to zero. In the present model, employment has similar features as capital; both are stocks that must be costly accumulated. The similarity may imply the need for tax smoothing between capital and labor incomes even in the steady state.

The question of optimal unemployment subsidy is also interesting. The dynamic framework of this paper provides a novel avenue along which this old question can be examined (see Topel and Welch, 1980, for a survey of the old arguments). The results in this paper indicate that for given tax rates on capital and labor, the unemployment subsidy is inefficient. It is interesting to determine the optimal subsidy when factor taxes are chosen optimally. Similarly, the optimal vacancy subsidy may be different in the current framework of intertemporal maximization.

Appendix A. Proof of Proposition 6.1

Let us first describe in detail the sequential bargaining framework with insiders and outsiders (depicted in Fig. 7). The total surplus to be split between the firm and the worker is $F_2 - u_2 / [(1 - \tau_w)u_1]$. We determine the firm's share and the worker's share of this surplus. Once the shares are determined, the worker's surplus and the firm's surplus can be recovered easily.

The firm is denoted by F , and a generic insider worker is denoted by W (there cannot be two insiders at any point of the game). Each round of negotiation takes a length of time, Δ . Since the firm and the worker have the same discount rate ρ in the steady state, to simplify analysis we assume that they have the same

discount rate ρ in the bargaining. The discount factor between two adjacent periods is thus $D \equiv e^{-\rho\Delta}$.

As in Shaked and Sutton (1984), the firm is free to switch its partner after T periods of negotiations with the current insider. In any period where the firm and the current insider have not bargained for T or more periods, the firm cannot switch its partner. In such a period nature first chooses one player, the firm with probability p and the insider with probability $1 - p$, to be the proposer. The proposer makes an offer, the responding player may either accept the offer (Y) which ends the game or reject the offer (N) to prolong the game to the next period.

In any period where the firm is free to switch, nature first chooses randomly the proposer in the current period according to the same distribution ($p, 1 - p$). If the firm is chosen, the firm can either propose to the current insider or switch the game to an outsider. If the firm proposes to the current insider, the current insider may either accept the offer which ends the game or reject the offer to prolong the game to the next period. If the firm switches to an outsider, the current insider returns to the unemployment pool and the outsider becomes the new insider. A new game between the firm and the new insider starts in which a new switching opportunity arises for the firm after T periods of bargaining with the new insider. If the current insider is chosen to be the proposer in the current period, the insider makes an offer and the firm can either accept to end the game, or reject to prolong the game to the next period, or switch to a new insider to start a new game.

Denote the game where the firm just starts the bargaining with an insider as G , and the game where the firm is free to switch as G^0 , respectively. To prove Proposition 6.1, we establish the following two lemmas.

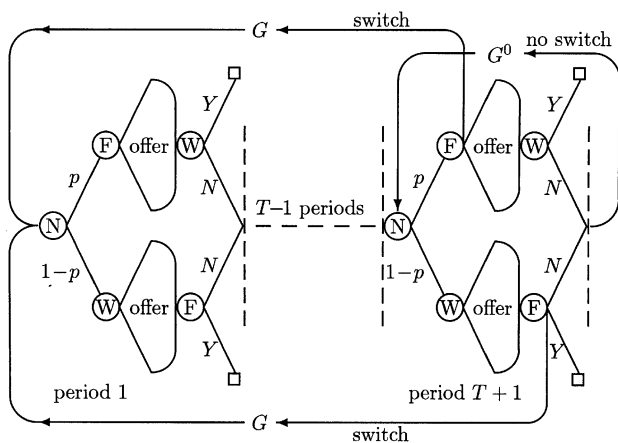


Fig. 7. Sequential bargaining with insider–outsider workers.

Lemma A.1. Let M , M^1 and M^0 be the suprema (infima) of firm's equilibrium shares in G , one period before G^0 and in G^0 , respectively. Then

$$M = p(1 - D^{T-1}) + D^{T-1}M^1, \quad (\text{A.1})$$

$$M^1 = pM_f + (1 - p)DM^0, \quad (\text{A.2})$$

$$M^0 = p\max\{M, M_f\} + (1 - p)\max\{M, DM^0\}, \quad (\text{A.3})$$

where

$$1 - M_f = D[pM_w + (1 - p)(1 - \max\{M, DM^0\})], \quad (\text{A.4})$$

$$M_w = 0 \text{ if } M_f \leq M; 1 - M_f \text{ if } M_f \geq M. \quad (\text{A.5})$$

Proof. Consider the first $T - 1$ periods where the firm cannot switch in the current period and the next period. Denote firm's highest possible share in period t as x_f when the firm proposes and x_w when the worker proposes. Similarly denote the firm's shares in period $t + 1$ as x'_f and x'_w for $t < T$. Since the firm proposes with probability p and the insider worker proposes with probability $1 - p$ in both t and $t + 1$, the firm's highest possible expected shares in period $t + 1$ is $m' = px'_f + (1 - p)x'_w$. From subgame perfection, we have $x_w = Dm'$ and

$$1 - x_f = D[p(1 - x'_f) + (1 - p)(1 - x'_w)] = D(1 - m').$$

Therefore, the firm's highest possible expected share in period t is

$$\begin{aligned} m &= px_f + (1 - p)x_w = p[1 - D(1 - m')] + (1 - p)Dm' \\ &= p(1 - D) + Dm'. \end{aligned} \quad (\text{A.6})$$

Given the firm's highest expected share M^1 in period T , applying Eq. (A.6) ($T - 1$) times yields Eq. (A.1), which gives the firm's highest possible share M in the first period of G .

In the game that starts one period before G^0 (period T), the firm cannot switch in the current period but can switch in the next period. If the worker proposes, the worker will not offer the firm more than DM^0 by subgame perfection. If the firm proposes, it will be the best for the firm to offer the worker the lowest expected share in next period. In the next period the firm is free to switch as well. If the worker proposes, the worker's share should not be less than M_w which could be either 0 or positive as defined by Eq. (A.5). If the worker proposes in the next period, its share will not be less than $1 - \max\{M, DM^0\}$. Therefore, (A.4) gives firm's highest share if the firm proposes in period T . Then Eq. (A.2) gives the firm's highest expected share in period T .

In a period where the firm is free to switch (period $T + 1$), with probability $1 - p$ the worker proposes. In this case, the firm can either switch or not switch. If the firm switches, its share will not be more than M by definition. If the firm

does not switch, the worker will not offer the firm more than DM^0 . So when the worker proposes, the firm's highest possible share is $\max\{M, DM^0\}$.

With probability p , the firm proposes in period $T + 1$. The firm can choose to switch rather than making an offer, in which case the firm's maximum share is M by definition. If the firm does not switch but proposes to the current insider, the firm has the same situation as in period T where the firm's share is not more than M_f . So when the firm proposes, its maximum share is $\max\{M, M_f\}$. Eq. (A.2) summarizes these cases to give the firm's highest possible expected share in period $T + 1$. \square

Lemma A.2. *The bargaining game has a unique subgame perfect equilibrium where*

$$M = \frac{p(1 - pD) - p(1 - p)D^{T+1}}{1 - pD - (1 - p)D^T}. \quad (\text{A.7})$$

The first offer is accepted and there is no delay or switching in equilibrium.

Proof. Consider possible values of (M^0, M, M_f) . We first rule out the following three cases:

Case 1: $M \geq M_f$ and $M \geq DM^0$. In this case Eq. (A.3) implies $M^0 = M$ and so Eq. (A.4) implies $M_f > M$ for $D \in (0, 1)$, regardless of whether $M_w = 0$ or $(1 - M_f)$. This contradicts $M \geq M_f$ and so the case is impossible.

Case 2: $M \geq M_f$ and $M \leq DM^0$. In this case Eq. (A.3) yields

$$M = \frac{1 - D + pD}{p}M^0 = \left[\frac{1 - D}{p} + D \right]M^0 > DM^0,$$

which contradicts $M \leq DM^0$ for $D \in (0, 1)$. Hence, the case is impossible.

Case 3: $M \leq M_f$ and $M \leq DM^0$. In this case $M_w = 1 - M_f$ from Eq. (A.5) and so Eq. (A.4) yields

$$M_f = 1 - \frac{D(1 - p)(1 - DM^0)}{1 - pD}. \quad (\text{A.8})$$

Solving Eqs. (A.8) and (A.3) yields $M^0 = p$ and $M_f = 1 - D(1 - p)$. From Eqs. (A.2) and (A.1), we get $M^1 = M = p$, which contradicts $M \leq DM^0$ for $D \in (0, 1)$. Hence the case is impossible.

Now the only possible case left is $M \leq M_f$ and $M \geq DM^0$. In this case Eq. (A.4) yields

$$M_f = 1 - \frac{(1 - p)D}{1 - pD} + \frac{(1 - p)D}{1 - pD}M.$$

Solving this with Eqs. (A.1), (A.2) and (A.3) gives the solutions for M , M^1 , M_f , and M^0 . In particular, the solution for M is given by Eq. (A.7). One can also verify that the solutions indeed satisfy $M \leq M_f$ and $M \geq DM^0$.

Finally, since M , M^1 , M^0 and M_f are both the suprema and infima of the corresponding subgames, the equilibrium payoffs are unique. It is straightforward to recover the strategies that deliver such payoffs and verify that the first offer is accepted in equilibrium. \square

Proposition 6.1 is the limit outcome of the above lemma as $\Delta \rightarrow 0$ (i.e., $D \rightarrow 1$). The limits of M and M^0 coincide and are equal to $1 - \lambda_A$, where λ_A is given in Proposition 6.1. The limit of the result in Shaked and Sutton (1984) is a special case corresponding to $p = 1/2$. This completes the proof of Proposition 6.1.

References

- Andolfatto, D., 1996. Business cycles and labor market search. *American Economic Review* 86, 112–132.
- Auerbach, A.J., Kotlikoff, L.J., 1987. *Dynamic Fiscal Policy*. Cambridge University Press, Cambridge.
- Blanchard, O.J., Diamond, P.A., 1989. The Beveridge curve. *Brookings Papers on Economic Activity* 1, 1–60.
- Burdett, K., Kiefer, N.M., Mortensen, D.T., Neumann, G.R., 1984. Earnings, unemployment and the allocation of time over time. *Review of Economic Studies* 51, 559–578.
- Chamley, C., 1981. The welfare cost of capital income taxation in a growing economy. *Journal of Political Economy* 89, 468–496.
- Chamley, C., 1986. Optimal taxation of capital income in general equilibrium with infinite lives. *Econometrica* 54, 607–622.
- Christiano, L.J., 1988. Why does inventory investment fluctuate so much?. *Journal of Monetary Economics* 21, 247–280.
- Coles M., Wright, R., 1994. Dynamic bargaining theory. *Journal of Economic Theory*, forthcoming.
- Epstein, L., 1987. A simple dynamic general equilibrium model. *Journal of Economic Theory* 41, 68–95.
- Hansen, G., 1985. Indivisible labor and the business cycles. *Journal of Monetary Economics* 16, 309–327.
- Hansen, L.P., Singleton, K.J., 1983. Stochastic consumption, risk aversion, and the temporal behavior of asset returns. *Journal of Political Economy* 91, 249–265.
- Hosios, A., 1990. On the efficiency of matching and related models of search unemployment. *Review of Economic Studies* 57, 279–298.
- Judd, K., 1985. Short-run analysis of fiscal policy in a simple perfect foresight model. *Journal of Political Economy* 93, 289–319.
- Judd, K., 1987. The welfare cost of factor taxation in a perfect-foresight model. *Journal of Political Economy* 95, 675–709.
- Killingworth, M.R., 1983. *Labor Supply*. Cambridge University Press, Cambridge.
- King M.A., Fullerton, D., 1984. *The Taxation of Income from Capital: A Comparative Study of the United States, the United Kingdom, Sweden, and West Germany*, University of Chicago Press, Chicago.
- Layard, R., Nickell, S., Jackman, R., 1991. *Unemployment: Macroeconomic Performance and the Labour Market*. Oxford University Press, Oxford.

- Lucas, R.E. Jr., 1990a. Supply-side economics: An analytical review. *Oxford Economic Papers* 42, 293–316.
- Lucas, R.E. Jr., 1990b. Liquidity and interest rates. *Journal of Economic Theory* 50, 237–264.
- Merz, M., 1995. Search in the labor market and the real business cycle. *Journal of Monetary Economics* 36, 269–300.
- Millard, S, Mortensen, D.T., 1994. The unemployment and welfare effects of labor market policies: a case for a hiring subsidy. Manuscript, Northwestern University.
- Mortensen, D.T., 1982. Property rights and efficiency in mating, racing, and related games. *American Economic Review* 72, 968–979.
- Mortensen, D.T., 1992. Equilibrio de busqueda y ciclos economicos reales (Search equilibrium and real business cycles). *Cuadernos Economicos* 51, 151–172.
- Mortensen, D.T., Pissarides, C.A., 1993. The cyclical behavior of job creation and job destruction. In: van Ours, J.C., Pfann, G.A., Ridder, G. (Eds.), *Labor Demand and Equilibrium Wage Formation*. North-Holland, Amsterdam, pp. 201–222.
- Peters, M., 1991. Ex ante price offers in matching games: Non-steady state. *Econometrica* 59, 1425–1454.
- Pissarides, C.A., 1986. Unemployment and vacancies in Britain. *Economic Policy* 3, 499–559.
- Pissarides, C.A., 1990. *Equilibrium Unemployment Theory*. Basil Blackwell, Cambridge.
- Rios-Rull J.V., 1995. Models with heterogeneous agents. In: Cooley, T.F. (Ed.), *Frontiers of Business Cycle Research*. Princeton University Press, Princeton, New Jersey, pp. 98–125.
- Rogerson, R., 1988. Indivisible labor, lotteries and equilibrium. *Journal of Monetary Economics* 21, 3–16.
- Rogerson, R., Wright, R., 1988. Involuntary unemployment in economies with efficient risk sharing. *Journal of Monetary Economics* 22, 501–515.
- Scheinkman, J., 1976. On optimal steady states of n -sector growth models when utility is discounted. *Journal of Economic Theory* 12, 11–30.
- Shaked, A., Sutton, J., 1984. Involuntary unemployment as a perfect equilibrium in a bargaining model. *Econometrica* 52, 1351–1365.
- Shi, S., 1995. Tariffs, unemployment, and the current account: an intertemporal equilibrium model. Manuscript, Queen's University.
- Shi, S., Wen, Q., 1997. Labor market search and capital accumulation: some analytical results. *Journal of Economic Dynamics and Control* 21, 1747–1776.
- Topel, R., Welch, R., 1980. Unemployment insurance: survey and extension. *Economica* 47, 351–379.
- Wolinsky, A., 1987. Matching, search, and bargaining. *Journal of Economic Theory* 42, 311–333.