Bargains, Barter, and Money

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We examine a search money model in which there is a symmetric coincidence of wants in all barter matches. However, when bargaining outcomes are asymmetric across matches, the barter economy is inefficient. Then a robust monetary equilibrium exists provided that money holders enjoy adequate bargaining terms. Fiat money may be welfare improving. In contrast to the literature, it is the asymmetry in bargains across matches rather than asymmetry in demands that generates these results. Journal of Economic Literature Classification Numbers: C78, E40.

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1. INTRODUCTION

Ever since Jevons (1875) and Menger (1892), the absence of the double coincidence of wants has been the key feature used to motivate a role for money as a medium of exchange. Recent contributions by Jones (1976),
Kiyotaki and Wright (1989, 1991, 1993) among many others, provide general equilibrium search models that prove how an absence of the coincidence of wants gives rise to valued money. In other money models, like that of Williamson and Wright (1994), informational frictions generate an effective absence of the double coincidence of wants. In all these models, money fails either to be robustly valued or to improve welfare if there is a frictionless double coincidence of wants in matches. It is tempting to presume that this is a general result.

This paper examines a search money model in which there is no impediment to barter from Jevons’ “want of a coincidence of wants” in matches. The particular model we develop is a variant of Shi (1995) and Trejos and Wright (1995) where goods are divisible and there is diminishing marginal utility of consumption. We modify the model so that there is a symmetric double coincidence of wants in all barter matches: both agents in any match receive the same utility from consuming equal quantities of the other’s good.

The model is used to determine under what circumstances, if any, robust monetary equilibria would exist with frictionless exchange in matches. We find equilibria in which money holders strictly prefer to hold money in search, when barter generates asymmetric bargains across heterogeneous matches and money holders enjoy sufficiently good bargaining terms of trade.

In describing the patterns of exchanges consistent with our results, we take an agnostic view toward particular bargaining solutions. Rather than restricting our analysis to a particular mechanism (like split-the-surplus or take-it-or-leave-it solutions), we explore all individually rational bargains. Bargains on a match’s surplus frontier are referred to as efficient bargains.

To get a more specific characterization we restrict the analysis to efficient bargains arising from the generalized Nash bargaining solution. Asymmetric bargains result whenever the Nash bargaining weights vary across matches according to the mix of items traded. This captures the idea that some traders may get the better of others in particular matches. We examine the convention where each agent receives a superior bargaining weight $\Omega > 1/2$ in as many barter matches as it has an inferior weight, $1 - \Omega$. With random matching, all agents are in a similar position ex ante.

With asymmetric bargains and diminishing marginal utility of consumption, barter generates an inefficiency. The benefits from receiving better bargains in half of the matches is not compensated by the loss of receiving poor terms in the remaining matches. On average agents are worse off relative to when the bargains are symmetric, $\Omega = 1 - \Omega = 1/2$.

When the barter convention generates an inefficiency, monetary equilibria may exist provided that money holders enjoy sufficiently good bargains.
We assume that all money holders are in the same bargaining position versus good traders and have a fixed bargaining weight, $w$. In equilibrium this gives money fixed purchasing power. We characterize combinations of bargaining weights $(w, \Omega)$ for which monetary equilibria exist and find equilibria for $w < 1/2$. Money’s value comes partly from self-fulfilling expectations that it is valuable. Money is also valued because using it reduces the volatility of consumption relative to barter. In particular, money holders avoid the possibility of receiving poor terms in the next barter match.

Can money be welfare improving when there is a symmetric coincidence of wants and all individual bargains are efficient? The answer is yes, if barter generates an inefficiency and the crowding-out effect on goods production is not too strong. The crowding-out effect arises from the standard inventory assumption that agents cannot hold money and goods. We extend the model to allow money holders to produce goods and illustrate a wide range of monetary equilibria where money improves welfare. Money improves welfare by facilitating the transfer of utility between agents by reducing the volatility of consumption relative to barter.

Our analysis is most closely related to Engineer and Shi (1998). Unlike the current paper, our earlier paper has asymmetric demands and symmetric bargaining weights. Another major difference is that the current model incorporates divisible goods with diminishing marginal utility, whereas our earlier paper used indivisible goods with divisible sidepayments. The current paper establishes that barter general equilibrium inefficiency can arise from the exchange of asymmetric quantities. This is perhaps more satisfying than requiring sidepayments transfer utility imperfectly. We are also able to do the analysis with general functions rather than linear ones.

The paper proceeds as follows. Section 2 lays out the model without money. Section 3 describes the set of barter bargains and the barter equilibria. Money bargains and monetary equilibria are analyzed in Section 4. Section 5 uses the generalized Nash bargaining solution to generate bargains. Section 6 examines welfare. Section 7 concludes.

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2 Monetary equilibria exist where all agents in all matches have equal bargaining weights. Money is valued because of the general equilibrium effect which yields money holders’ better terms of trade.
2. THE MODEL WITHOUT MONEY

The basic model is similar to Shi (1995) and Trejos and Wright (1995) except that all agents in our model like each others’ goods equally—there is a universal symmetric coincidence of wants in all good matches. We modify the model in this way to stress that the results are generated by bargains.

Time is continuous. There is a continuum of infinitely lived agents indexed by \( i \) on the unit circle who meet bilaterally according to an anonymous matching process with Poisson arrival rate \( \beta \). Agent \( i \) does not consume his own good but enjoys utility \( u(q_i) \) from consuming \( q_i \) units of any other good \( j \neq i \). This captures the symmetric coincidence of wants between agents. We assume that \( u(q) \) is strictly concave, \( u'(q) > 0 \) and \( u''(q) < 0 \), and that \( u(0) = 0, u'(0) = \infty \) and \( u'(\infty) = 0 \). Denote \( \bar{q} \) such that \( u(q) = q^\delta \), where \( 0 < \delta < 1 \).

Goods are nonstorable and hence are produced for immediate delivery. An agent endowed with a production opportunity can instantaneously produce \( q \) units of its own good at disutility \( c(q) \). As the form of the cost and utility functions are the same across agents, all agents have symmetric demand schedules for other agents’ goods in terms of their own. We normalize \( c(q) = q \) without loss of generality. Denote \( q^* \) such that \( u'(q^*) = c'(q^*) = 1 \).

Let \( V_i \) denote agent \( i \)'s value to searching while holding a production opportunity. Once matched, the surplus for \( i \) is his payoff from trading \( q_i \) less its reservation utility, \( S_i = U_i - \bar{U}_i \). The reservation utility is the value of leaving the match without trading, \( \bar{U}_i = V_i \). After trading each agent instantly consumes, immediately acquires a new production opportunity, and then resumes searching. Hence, in a barter match \( S_i = u(q_i) - q_i + V_i - \bar{U}_i = u(q_i) - q_i \). Similarly, \( S_j = u(q_j) - q_j \). Notice that the average surplus in a barter match, \( \Sigma = (S_i + S_j) / 2 \), is maximized if both agents produce \( q^* \), yielding \( \Sigma^* = u(q^*) - q^* \).

3. BARGAINS AND BARTER

In this section we explore the effects of all individually rational bargains on the barter economy. Asymmetric bargains are particularly interesting because, in our random matching setup, they yield asymmetric quantities exchanged across matches. Consider an extreme example. Suppose the good \( i \) trader makes a take-it-or-leave-it offer to the good \( j \) trader, \( j \) makes a take-it-or-leave-it offer to trader \( k \), and \( k \) makes a take-it-or-
leave-it offer to \( i \). With random matching among the three traders, each trader has a 50% chance of entering into a match where it makes the offer or receives the offer.\(^3\) Asymmetric bargains within matches lead to asymmetric quantities exchanged across matches.\(^4\)

Below we first justify looking at all bargains. Then we detail a structure similar to the example where ex ante all agents are in a similar position but ex post match type determines the bargaining outcome.

3.1. Bargains and Convention

In this paper we take an agnostic approach to bargains and analyze all bargains that can be supported by convention. We do this partly for generality but also because barter and money arose under historical circumstances where identities, backgrounds, and culture undoubtedly played a huge role.\(^5\) When ordinary barter was established it seems (according to Einzig, 1966, p. 341–342) it often displayed fixed ratios influenced by tradition. We take bargains as determined by convention, perhaps having originated with factors which may or may not be currently present.

A convention is a regularity in behavior that is customary, expected, and self-enforcing (see Lewis, 1969). Convention involves “sociological preconditions” that determine a bargaining solution. In regard to the two-person pure bargaining game, Shubik (1984, p. 283–284) remarks: “…game theory teaches us to expect an indeterminate outcome in the absence of sociological preconditions.” We model convention by having agents condi-

\(^3\)The example is reminiscent of the childhood game “paper-rock-scissors” where paper dominates rock, rock dominates scissors, and scissors dominates paper. Of course, the game is different than ours because the players pick either paper, rock, or scissors as a strategy. However, the games are similar if agents must choose their trader type prior to being matched. The games look very similar when agents choose different goods so that each good on the unit circle corresponds to an agent, as in our model.

\(^4\)To see why, consider a counterexample. Suppose that the good \( i \) trader is decisive against both trader \( j \) and \( k \). Then both \( j \) and \( k \) receive the minimum \( q \) in exchange with \( i \). Now consider the match between traders \( j \) and \( k \). In this match at least one trader generally receives more than the minimum \( q \). Thus either \( j \) or \( k \) experiences asymmetric bargaining outcomes across matches. Finally, if \( i \) is not decisive versus \( k \), then \( i \) receives different quantities across matches.

\(^5\)In a “Hobbesian” brute and nasty state of society, coordinating on even very asymmetric outcomes may be acceptable given the alternatives. There is some evidence that early barter was often fractious and failed in tribal societies and that “silent barter” preceded ordinary barter; see Einzig (1966) and Quiggin (1979). Silent barter is often associated with the parties historically intensely disliking each other and involves bartering without face to face meetings where one agent leaves goods and the other literally takes them (in exchange) or leaves them.
tion their bargaining behavior on the mix of goods being traded (equivalent to identities in barter matches).

Our approach is probably closest to that of Rosenthal and Landau (1979) who also examine symmetric bargaining in a random matching game (also see Kreps, 1990, p. 412–413). They endogenously derive when customs emerge. Agents with (uncertain) reputations for not yielding receive the better bargaining terms. Our model can be viewed as one where each trader knows with certainty their relative reputations (and hence bargaining position) vis-à-vis other groups of traders.

There have been prominent attempts to model bargaining without appealing to sociological preconditions. Abstract assumptions like symmetry and efficiency are often imposed to construct a deductive equilibrium selection theory. However, the efficacy of such assumptions are put seriously in doubt by Van Huyck et al. (1995, 1997) who provide compelling experimental evidence that unequal-division conventions can systematically emerge in symmetric bilateral bargaining games.6

Sequential games are also used to motivate bargaining solutions. These games yield efficient bargains that are inherently asymmetric. In Rubinstein’s (1982) game with alternating offers, bargains favor the first agent to make an offer except in the limit as the time between offers goes to zero. In a more general game where nature probabilistically decides who moves first in each round of bargaining (Binmore, 1987), symmetry only occurs in the limit when the probability is one-half. By varying the probability it is possible to achieve all bargains on the efficiency frontier (see Section 5). The point is that the structure of sequential bargaining is an exogenous protocol or sociological precondition. By changing the protocol a wide range of solutions are possible. Furthermore, Kreps (1990, p. 563) remarks, “The solution bounces around an incredible amount for what seem to be insignificant differences in the bargaining protocol.” It is for the reason that both theory and evidence point to convention-influenced outcomes that we study the general structure below.

3.2. Barter Bargains

At the simplest level, our approach simply allows for some traders to get the better of others. To explore the possibilities, we examine a simple convention which allows for a wide range of patterns of outcomes across

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6 See Binmore et al. (1993) for a discussion of the various focal points for the Nash bargaining problem and the ease of getting subjects to change focal points. Of focal points, Kreps (1990, p. 415) says, “Context, the identity of players, their backgrounds, and other such fuzzy factors play a huge role in this.” Schelling (1980) argues against the psychological salience of symmetry.
matches according to the items exchanged. We assume that each good (or equivalently each agent) receives superior barter terms in as many matches as it has inferior terms. This has the virtue that with random matching, all agents are in a similar position ex ante.

Consider the convention where the combination of goods traded determines the bargain. Suppose two good traders, agents $i$ and $j$, are matched. Let $d(i, j)$ be the clockwise distance from agent $i$ to agent $j$ along the unit circle. If $d(i, j) \leq 1/2$, convention determines the bargain $(q_i, q_j) = (A, B)$; whereas, if $d(i, j) > 1/2$, the quantities are reversed so that $(q_i, q_j) = (B, A)$. We consider all pairs $(A, B)$ consistent with nonnegative surpluses: $\{(A, B) \mid u(A) - B \geq 0, u(B) - A \geq 0\}$. All bargains satisfy the individual rationality constraints and are Nash equilibria when the alternative is to separate.

As agents are randomly matched, the ex ante surplus is then just the average of the two surpluses, $\tilde{\Sigma} = [u(A) - B + u(B) - A]/2$. Bargains $(q_i, q_j) = (q^*, q^*)$ maximize the ex ante surplus, attaining a maximum $\Sigma^* = u(q^*) - q^*$. Thus, $\tilde{\Sigma} \leq \Sigma^*$ for all $(A, B)$ and $\tilde{\Sigma} < \Sigma^*$ for $(A, B) \neq (q^*, q^*)$.

### 3.3. Barter Economy

The value function for a good trader in the nonmonetary economy is $V^n_{e} = \Sigma / R$, where $R = r/\beta$ and $r > 0$ is the discount rate. For the solution $\Sigma$, the value of search with a good is just $\tilde{V}^n_{e} = \tilde{\Sigma} / R$. Let $V^* = \Sigma^* / R$ denote the maximum expected utility.

**Proposition 3.1.** When $(A, B) = (q^*, q^*)$ expected utility is maximized, $\tilde{V}^n_{e} = V^*$. When $(A, B) \neq (q^*, q^*)$ the barter economy is inefficient, $\tilde{V}^n_{e} < V^*$.

Barter can generate an inefficiency from two sources. Obviously, quantities $(A, B)$ that do not lie on the surplus frontier generate an inefficiency. However, even if the quantities are on the surplus frontier, they generate inferior expected utility wherever $A \neq B$. Then quantities are exchanged which do not equate marginal benefits and costs between traders: utility is transferred imperfectly at the margin (see Section 5.1).

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7Alternatively, when agents are matched they are randomly assigned with 50% probability to be a type 1 agent or a type 2 agent. Type 1 agents get $(A, B)$ and type 2 agents get $(B, A)$. This distinguishing of agent type and random assignment is used, for example, by Binmore et al. (1993).
4. FIAT MONEY

As in Shi (1995) and Trejos and Wright (1995), we model fiat money as an indivisible storage item that no one produces or consumes. Initially, a fraction $M < 1$ of the agents is randomly chosen and endowed with a unit of money. Those endowed with money may either keep it or dispose of it. Those without money are endowed with a production opportunity. As is standard, agents can only hold in inventory either one unit of money or one production opportunity (relaxed in Section 6). This implies that every trade is either a trade of goods for goods or a trade in which an agent with one unit of money trades it for goods. Hence, if nobody disposes of money there will always be $M$ agents with money called *money holders* and $1 - M$ agents called *good traders*. When nobody believes money is valuable, it will be discarded and the nonmonetary economy will be described by the pure barter equilibrium.

4.1. *Money Matches*

We model money as purchasing a fixed quantity of goods in all matches. This is the simplest formulation and puts all money holders in the same bargaining position versus good traders as bargains are conditional on the mix of items in inventory. Let $q_g$ be the convention-determined quantity of services produced by the good trader for the money holder in a money match. After trading $q_g$ for money, the good trader immediately becomes a money holder. The money holder consumes $q_g$ immediately and becomes a good trader. This trade yields a surplus $S_m$ to the money holder and a surplus $S_g$ to the good trader:

$$S_m = u(q_g) - Y \quad \text{and} \quad S_g = -q_g + Y,$$

where $Y = V_m - V_g$. Both $V_g$ and $V_m$ and hence $Y$ are taken as exogenous by agents.

4.2. *The Monetary Economy*

**Definition 4.1.** A monetary economy is a value of $q_g > 0$ that yields nonnegative surpluses in all money matches, $S_m \geq 0$ and $S_g \geq 0$.

When surpluses are nonnegative, the value functions are:

$$RV_g = (1 - M) \Sigma + M S_g,$$

$$RV_m = (1 - M) S_m.$$  \hspace{1cm} (4.2)

Subtracting (4.2) from (4.3) yields

$$Y = \frac{(1 - M)[u(q_g) - \Sigma] + M q_g}{1 + R}.$$  \hspace{1cm} (4.4)
Note that $S_g \geq 0$ implies $Y > 0$. Thus, if there is a monetary economy, it is robust in the sense that money holders strictly prefer to hold money to a production opportunity.

**Proposition 4.2.** For $R$ sufficiently small, there exists a monetary economy if and only if $q_g > \min\{A, B\}$ and barter generates an inefficiency, $(A, B) \neq (q^*, q^*)$. No monetary economy exists when barter does not generate an inefficiency.

**Proof.** Substituting (4.4) into (4.1) yields

$$
(1 + R)S_g = (1 - M)\left[u(q_g) - q_g - \bar{\Sigma} \right] - Rq_g. 
$$

Recall that $u(q^*) - q^* > \bar{\Sigma}$ if and only if $(A, B) \neq (q^*, q^*)$. Hence, for $R$ sufficiently small and $(A, B) \neq (q^*, q^*)$, there exists a $q_g > \min\{A, B\}$ sufficiently close to $q^*$ such that $S_g \geq 0$. If $q_g \leq \min\{A, B\}$, $u(q_g) - q_g - \bar{\Sigma} < 0$ and $S_g < 0$.

Substituting (4.4) into (4.1) yields

$$
(1 + R)S_m = M\left[u(q_g) - q_g \right] + (1 - M)\bar{\Sigma} + Ru(q_g). 
$$

Hence, for any $q_g \leq \bar{q}$, $S_m > 0$. As $q^* < \bar{q}$, there exists a $q_g > \min\{A, B\}$ that yields $S_m \geq 0$ and $S_g \geq 0$.

Monetary exchange must command at least as much as the worst outcome in barter, $q_g > \min\{A, B\}$. However, money may be valued if it receives less than the average of the barter quantities, $q_g < (A + B)/2$. In this case, money is valued because it smooths consumption relative to barter.

5. THE ECONOMY WITH GENERALIZED NASH BARGAINING

To get a more specific characterization, we concentrate on efficient bargains using the generalized Nash bargaining solution. This mechanism is the natural choice as agents are matched bilaterally and it can generate all efficient bargains on a match’s surplus frontier.\(^8\)

\(^8\)It is well known that this mechanism can be generated from a sequential offers game where the bargaining weight for an agent corresponds to the probabilities with which that agent makes an offer in each round. For analyses of noncooperative bargaining solutions see Binmore (1987), Kreps (1990), and Wolinsky (1987).
5.1. Barter

The Nash solution maximizes the weighted product of the surpluses from exchange.

\[
\max_{q_i, q_j} S_i^{\omega(i,j)} S_j^{1-\omega(i,j)} \quad \text{s.t. } S_i \geq 0, S_j \geq 0, q_i \geq 0, q_j \geq 0,
\]

where \( \omega(i,j), 0 \leq \omega(i,j) \leq 1 \), is the weight on agent \( i \)'s surplus.\(^9\)

To generate asymmetric bargains we allow the weight \( \omega(i,j) \) to differ depending on the match type \((i,j)\). As before, \( d(i,j) \) is the clockwise distance from agent \( i \) to agent \( j \) along the unit circle. If \( d(i,j) \leq 1/2 \), set \( \omega(i,j) = \Omega \), and if \( d(i,j) > 1/2 \), set \( \omega(i,j) = 1 - \Omega \). Take-it-or-leave-it offers correspond to \( \Omega = 0, 1 \) so that \( \Omega(1 - \Omega) = 0 \). In contrast, if \( \Omega = 1/2 \) (so that \( \Omega(1 - \Omega) = 1/4 \)), the bargaining weight does not depend on the partner. Notice that bargaining weights are more symmetric the greater is \( \Omega(1 - \Omega) \).

Suppose agent \( i \) has been matched to an agent \( j \) where \( d(i,j) \leq 1/2 \), so that \( \omega(i,j) = \Omega \). The first-order conditions from the maximization problem imply:

\[
q_j = Q(q_i) = u^{-1} \left( \frac{1}{u'(q_i)} \right),
\]

\[
\Omega [u(q_i) - Q(q_i)] - (1 - \Omega) u'(q_i) [u(Q(q_i)) - q_i] = 0. \tag{5.2}
\]

Note that \( Q'(q) < 0 \) and that \( q > Q(q) \) if and only if \( q > q^* \). Denote the solution to (5.2) by \( \hat{q}_i \) and denote \( \hat{q}_j = Q(\hat{q}_i) \).

**Lemma 5.1.** Solution \( \hat{q}_i \) exists and is unique, with the following properties:

(i) \( d\hat{q}_i/d\Omega < 0 \) and \( d\hat{q}_j/d\Omega > 0 \); and (ii) \( \hat{q}_i > q^* \) (\( \hat{q}_j < q^* \)) if and only if \( \Omega < 1/2 \).

**Proof.** The LHS of (5.2) is an increasing function of \( q_i \), having a value \( -\infty \) when \( q_i = 0 \) and a positive value when \( q = \bar{q} \) (note \( u'(\bar{q}) < 1 \) implies \( Q(\bar{q}) < q^* < \bar{q} \)). Thus, there is a unique solution to (5.2) lying in \((0, \bar{q})\).

\(^9\)There exists a sequential game where the surplus weight \( \omega(i,j) \) corresponds to the probability of agent \( i \) moving first in each round of bargaining. The reservation utility depends on agents encountering other agents between bargaining rounds and leaving the existing match if they encounter a desirable new trading partner. Trejos and Wright (1995) refer to this as search while bargaining. The above Nash solution corresponds to the limit as the period of delay between bargaining rounds becomes small. See Engineer and Shi (1996) for a lengthy derivation of the generalized Nash bargaining solutions used in a similar monetary economy. In this earlier work we did not look at models with asymmetric information to avoid the complications of multiple equilibria.
The property (i) can be verified directly. For (ii), note that \( \hat{q}_i > q^* \) if and only if the LHS of (5.2) is negative when \( q_i = q^* \) and this condition is equivalent to \( \Omega < 1/2 \). 

This lemma shows that asymmetric weights yield asymmetric bargains and that the greater an agent’s weight, the fewer goods he/she gives (and the more he/she receives) in barter.

Before knowing his/her relative position in the barter trade, an agent’s expected surplus from a barter match is \((\hat{S}_i + \hat{S}_j)/2\), which can be computed as:

\[
\Sigma(\hat{q}_i) = \left[ u(\hat{q}_i) - \hat{q}_i + u(Q(\hat{q}_i)) - Q(\hat{q}_i) \right]/2.
\]  

(5.3)

For brevity, we sometimes denote this surplus simply as \( \hat{S} \). The following lemma shows how the bargaining weights affect the expected surplus.

**Lemma 5.2.** As the bargaining weights become more symmetric, the expected surplus of a barter match increases, i.e., \( \partial \Sigma/\partial \Omega(1 - \Omega) > 0 \). When the bargaining weights are equal, \( \Omega(1 - \Omega) = 1/4 \), expected surplus reaches a maximum, \( \Sigma^* \). At the other extreme, where one agent has all the bargaining power, \( \Omega(1 - \Omega) = 0 \), expected surplus is at the minimum.

**Proof.** Since \( d\Sigma/d\Omega = \Sigma'(\hat{q}_i)d\hat{q}_i/d\Omega \) and \( d\hat{q}_i/d\Omega < 0 \) by Lemma 5.1, \( d\Sigma/d\Omega > 0 \) iff \( \Sigma'(\hat{q}_i) < 0 \). Differentiating (5.3), we have \( \Sigma'(\hat{q}_i) < 0 \) iff \( \hat{q}_i > q^* \) and, by Lemma 5.1, iff \( \Omega < 1/2 \). That is, \( \Sigma \) increases in \( \Omega \) for \( \Omega < 1/2 \) and decreases in \( \Omega \) for \( \Omega > 1/2 \). Since the function \( \Omega(1 - \Omega) \) depends on \( \Omega \) in the same way, \( \hat{S} \) increases when \( \Omega(1 - \Omega) \) increases and attains the maximum when \( \Omega(1 - \Omega) \) does so. The maximum of \( \Omega(1 - \Omega) \) is \( 1/4 \), attained at \( \Omega = 1/2 \). Since \( \hat{q}_i = q^* \) when \( \Omega = 1/2 \), the maximum of \( \hat{S} \) is \( \Sigma^* \). Similarly, \( \hat{S} \) attains the minimum at \( \Omega = 0 \) or \( \Omega = 1 \), where one agent receives zero surplus and the other the maximum surplus.

**Proposition 5.3.** A unique barter equilibrium exists in which all agents discard their money. Expected utility, \( V^n_s = \hat{S}/R \), increases the more symmetric the bargaining weights, \( \partial V^n_s/\partial \Omega(1 - \Omega) > 0 \). The equilibrium is inefficient, \( V^n_s < V^*, \) when the weights are asymmetric, \( \Omega(1 - \Omega) < 1/4 \), and the equilibrium is efficient when the weights are equal.

Barter trades the socially efficient quantities of goods \((q^*, q^*)\) if and only if the bargaining weights are equal between the two agents. Unequal weights \( \Omega < 1 - \Omega \) yield inefficient bargains where \( \hat{q}_i > q^* > \hat{q}_j \). Then, trader \( j \)’s marginal utility is less than the marginal cost to trader \( i \), \( u'(\hat{q}_j) < 1 \). Hence, utility is transferred imperfectly at the marginal. For trader \( i \) marginal utility is greater than marginal cost, \( u'(\hat{q}_i) > 1 \). Thus, expected utility could be increased by marginally decreasing \( \hat{q}_i \) and increasing \( \hat{q}_j \). The opposite is true for \( \Omega > 1 - \Omega \).
5.2. Money

As before, money holders are in the same bargaining position versus good traders. This implies a fixed bargaining weight for money holders and in equilibrium fixed purchasing power of \( q_g \) for money.

The generalized Nash solution with bargaining weight \( w \) for money holders is

\[
\max_{q_g} \left\{ S_m^w S_g^{1-w} : S_g \geq 0, S_m \geq 0, q_g \geq 0 \right\}.
\]

The solution satisfies

\[
Y = \frac{(1 - w)u(q_g) + wu'(q_g)q_g}{(1 - w) + wu'(q_g)}.
\]

Equating (5.4) and (4.4) yields an equation in \( q_g \):

\[
\left[ 1 - M - \frac{(1 + R)(1 - w)}{1 - w + wu'(q_g)} \right] [u(q_g) - q_g] = (1 - M)\hat{\Sigma} + Rq_g. 
\]

Denote the left-hand side as \( L(q_g, w, R, M) \). The analysis is restricted to symmetric stationary pure strategy equilibria.

**Definition 5.4.** A monetary equilibrium is a solution to (5.5) for \( q_g > 0 \).

**Proposition 5.5.** When bargaining weights are asymmetric in barter matches, \( \Omega(1 - \Omega) < 1/4 \), there exists a \( w_0 \in (0, 1) \) for \( R \) sufficiently small such that for \( w > w_0 \) a monetary equilibrium exists. When the weights are equal, \( \Omega(1 - \Omega) = 1/4 \), no monetary equilibrium exists. Thus, a robust monetary equilibrium exists only if the barter equilibrium is inefficient.

**Proof.** First, when \( \Omega(1 - \Omega) = 1/4 \), \( q_g^* = q^* \) and \( \hat{\Sigma} = \Sigma^* \); the LHS of (5.5) is strictly less than the RHS for all \( q_g \), so there is no monetary equilibrium in this case. Similarly, there is no monetary equilibrium when \( w = 0 \). Let us assume \( \Omega(1 - \Omega) \neq 1/4 \) and \( w > 0 \). Also, the total surplus in a monetary trade, \( u(q_g) - q_g \), must be nonnegative and so we restrict \( q_g \in (0, \bar{q}] \) where \( u(\bar{q}) = \bar{q} \). Next, consider the case \( w = 1 \). At the two ends \( q = 0 \) and \( q = \bar{q} \), the function \( L(q_g, w = 1, R, M) \) equals 0 and is less than the RHS. Also, for \( R \) sufficiently close to zero,

\[
L(q_g = q^*, w = 1, R, M) = (1 - M)\Sigma^* > \text{RHS}(5.5)|_{q_g = q^*}.
\]
Thus, there are at least two solutions for \( q \) to (5.5), both in \((0, \bar{q})\). These solutions satisfy \( Y > 0 \) (see (5.4)). Since the LHS of (5.5) is a decreasing function of \( w \), continuity implies that robust monetary equilibria exist when \( w > w_0 \) for some \( w_0 \in (0, 1) \).

5.3. An Example

Suppose \( u(q) = q^\delta \), where \( 0 < \delta < 1 \). Let \( \delta = 0.1 \), \( M = 0.05 \), and \( R = 0.01 \). Figure 1 illustrates the minimum bargaining weight for a money holder, \( w_0 \), that supports a robust monetary equilibrium. For any barter bargaining weight, \( \Omega \), a robust monetary equilibrium exists if and only if \( w > w_0(\Omega) \). In fact, \( w > w_0 \), two monetary equilibria exist where agents strictly prefer to hold money. As \( \Omega \) increases toward \( 1/2 \), the minimum weight \( w_0 \) approaches 1. The graph \( w_0(\Omega) \) is symmetric with respect to \( \Omega = 1/2 \) and so, for \( \Omega > 1/2 \), \( w_0(\Omega) \) decreases in \( \Omega \).

At an extreme value of the barter bargaining weight, \( \Omega = 0.01 \), robust monetary equilibria exist for any \( w > 0.324 \). This example corresponds to when the asymmetry in barter weights is very large. This extreme example is interesting because money is valued even when it yields the holder a bargaining weight less than the weighted average of the barter weights, \( w < [\Omega + 1 - \Omega]/2 = 1/2 \).

![FIG. 1. Minimum bargaining weight for a money holder that supports a monetary equilibrium, \( w_0 \), as a function of the bargaining weight of one of the barter traders, \( \Omega \).](image-url)
In this and other examples we found \( w_0 > \min[\Omega, 1 - \Omega] \) but were not able to prove it generally. This may be because monetary bargains depend not only on the weights but also on the equilibrium value of \( Y \). When money is more highly valued it receives better terms of trade for a given bargaining weight. Agents also hold money because it reduces the volatility of consumption relative to barter. In particular, money holders avoid the possibility of receiving poor terms in the next barter match. With diminishing marginal utility, agents would like to smooth consumption. They are able to do this using money, as it has fixed purchasing power.

6. WELFARE-IMPROVING MONETARY EQUILIBRIA

In the above model, an increase in the money stock increases the number of money holders by crowding out an equal number of good traders. This strong crowding-out effect restricts the range of equilibria and the welfare-improving role of money. It arises from our assumption that money holders cannot produce.

A natural extension of the model is one in which each money holder can produce with probability \( \theta \in [0, 1] \) upon being matched.\(^{10}\) By increasing \( \theta \), we can reduce the crowding-out effect and increase the chance for money to improve welfare. In this section we examine the case \( \theta = 1 \) and show that money can indeed improve welfare. By continuity, money improves welfare for \( \theta \) sufficiently close to 1.\(^{11}\)

As before, there is a unit upper bound on each agent's money holdings and when two agents without money are matched, they trade in the same way as described before. There are two new elements. One is that two money holders can trade. In such a match the two agents are symmetric in their money holdings and so we assume their bargaining weights are determined by their relative position along the circle. Then the quantities of goods traded in such a match are the same as those in a barter match, i.e., the solutions \( (\hat{q}_i, \hat{q}_j) \) to (5.1) and (5.2). The expected surplus for each of the two agents is \( \Sigma(\hat{g}_i) \) defined in (5.3).

The second new element is that, in a match between a money holder and an agent without money, the money holder can supply goods in addition to money. Let \( q_m \) be the quantity of goods supplied by the money holder.

\(^{10}\) We would like to thank Neil Wallace for suggesting this extension.

\(^{11}\) We focus on the case \( \theta = 1 \) to reduce algebraic complexity. When \( \theta \) is strictly between 0 and 1, there are two types of matches between a money holder and an agent without money. In one the money holder can produce and, in the other, the money holder cannot produce. The quantities of goods traded are different in these two matches and the algebra is more involved than in the case \( \theta = 1 \).
holder and $q_g$ be the quantity supplied by the agent without money. Then the surplus from trade for the money holder ($S_m$) and the surplus for the agent without money ($S_g$) are, respectively,

$$S_m = u(q_g) - q_m - Y; \quad S_g = u(q_m) - q_g + Y,$$

where $Y \equiv V_m - V_g$ as before.

The generalized Nash solution with bargaining weight $w$ for the money holder is

$$\max_{q_g, q_m} \left\{ S_m S_g^{1-w}: S_g \geq 0, S_m \geq 0, q_g \geq 0, q_m \geq 0 \right\}.$$

For $q_g, q_m < \bar{q}$ the solution satisfies

$$Y = \left( 1 - w \right) \frac{u(q_g) - Q(q_g)}{1 - w} - \frac{w u'(q_g) \left( u(Q(q_g)) - q_g \right)}{w u'(q_g)}, \quad (6.1)$$

where $q_m = Q(q_g)$ and the function $Q(\cdot)$ is defined in (5.1). The average surplus for each agent in this match is $(S_m + S_g)/2 = \Sigma(q_g)$, where $\Sigma(\cdot)$ is defined in (5.3).

The value functions satisfy the following equations:

$$RV_m' = (1 - M) S_m + M \Sigma(\hat{q}_i);$$

$$RV_g' = (1 - M) \Sigma(\hat{q}_i) + MS_g.$$

The expected surplus $\Sigma(\hat{q}_i)$ appears in the value function for a money holder because of the trade with another money holder. Subtracting these two equations we have:

$$(1 + R)Y = (2M - 1) \Sigma(\hat{q}_i) + (1 - M) \left[ u(q_g) - Q(q_g) \right] - M \left[ u(Q(q_g)) - q_g \right]. \quad (6.2)$$

Equations (6.1) and (6.2) jointly determine $(Y, q_g)$. Then we can recover other variables.

With the definition $Z = MV_m + (1 - M)V_g$, ex ante welfare obeys

$$RZ = \left[ M^2 + (1 - M)^2 \right] \Sigma(\hat{q}_i) + 2M(1 - M)\Sigma(q_g).$$
Clearly, when either \( M \to 0 \) or \( M \to 1 \), the welfare level approaches the one in a barter economy. The following proposition shows that the optimal money stock can be positive:

**Proposition 6.1.** Assume \( \Omega(1 - \Omega) < 1/4 \) and that money holders can produce, \( \theta = 1 \). When \( w \) is sufficiently close to \( 1/2 \) and \( R \) is sufficiently close to \( 0 \), there is a robust monetary equilibrium \( (i.e., \ Y > 0) \) and the optimal money stock is in the interior \( M \in (0, 1) \).

**Proof.** We prove the proposition for \( w = 1/2 \) and, by continuity, the optimal money stock is positive when \( w \) is sufficiently close to \( 1/2 \). Also, we prove the proposition for the case \( \Omega < 1/2 \). (For the case \( \Omega > 1/2 \), replace \( \hat{q}_i \) by \( \hat{q}_i = Q(\hat{q}_i) \) and note that \( \Sigma(Q(\hat{q}_i)) = \Sigma(\hat{q}_i) \); then the same proof applies.) When \( \Omega < 1/2 \), Lemma 5.1 implies \( \hat{q}_i > q^* \).

First, when \( w = 1/2 \), we show that a robust monetary equilibrium exists only if \( q_s > q^* \). Clearly, with \( w = 1/2 \), Eq. (6.1) yields \( Y(q^*) = 0 \). Define \( q_s \) by \( u(Q(q_s)) = q_s \). Then \( q^* < q_s < \hat{q} \). For \( q > q_s \), \( u(Q(q)) - q < 0 \) and \( u(q) - Q(q) > u(q^*) - Q(q^*) > 0 \), implying \( Y > 0 \). For \( q \leq q_s \), the numerator of \( Y(q_s) \) is an increasing function of \( q_s \). Since \( Y(q^*) = 0 \), then \( Y < 0 \) for \( q < q^* \) and \( Y > 0 \) for \( q \in (q^*, q_s] \). Therefore \( Y(q) > 0 \) iff \( q > q^* \).

Now restrict attention to \( q_s > q^* \) to ensure \( Y > 0 \). Consider the case \( M < 1/2 \). We show that there is a unique, robust monetary equilibrium. Substituting \( Y(q_s) \) into (6.2) and evaluating at \( w = 1/2 \) we have:

\[
2 \left( 1 - M \right) u'(q_s) - \frac{M}{1 + u'(q_s)} \Sigma(q_s) - (1 - 2M) \Sigma(\hat{q}_i) = RY. \tag{6.3}
\]

When \( R \) is sufficiently close to \( 0 \), the solution \( q_s \) is close to the solution to \( LHS(6.3) = 0 \). Then, \( q_s < \hat{q}_i \); otherwise \( LHS(6.3) < 0 \). Similarly, \( (1 - M)u'(q_s) > M \). Under this condition, \( LHS(6.3) \) is a decreasing function of \( q_s \). Since the \( LHS \) of (6.3) is positive when \( q_s = q^* \) and negative when \( q_s = \hat{q}_i \), there is a unique solution for \( q_s \) and the solution lies in \((q^*, \hat{q}_i)\). Moreover, since the \( LHS \) of (6.3) is a decreasing function of \( M \) in the region \((q^*, \hat{q}_i)\), the solution for \( q_s \) satisfies \( \frac{d q_s}{d M} < 0 \), with \( q_s \downarrow q^* \) when \( M \uparrow 1/2 \).

Next, differentiating \( Z \) we have \( d Z/d M > 0 \) if and only if

\[
(1 - 2M) \left[ \Sigma(q_s) - \Sigma(\hat{q}_i) \right] + M(1 - M) \Sigma'(q_s) \frac{d q_s}{d M} > 0.
\]

Since \( \Sigma'(q) < 0 \) for \( q > q^* \) and since \( \hat{q}_i > q_s > q^* \), the first term above is positive when \( M < 1/2 \). So is the second term, since \( d q_s/d M < 0 \) when
$M < 1/2$. Thus, welfare increases in $M$ for $M < 1/2$. Since at the two ends $M = 0$ and $M = 1$ the welfare level is equal to that in a barter economy, there is an $M \in (0, 1)$ that maximizes welfare.

To illustrate how welfare depends on the money stock, we use the example used earlier: $u(q) = q^{0.1}$, $R = 0.01$ and $\Omega = 0.01$. First, consider a high value of the bargaining power of a money holder in a match with an agent without money, $w = 0.8$. Figs. 2.1–2.3 illustrate the equilibrium outcomes for different money stocks.\footnote{There are two solutions for $q_g$ but, as indicated by the proof for Proposition 6.1, only the one $q_g > q^*$ yields $Y > 0$ for at least some values of $M$.}

Figure 2.1 shows that as $M$ increases, an agent without money reduces the quantity of goods and the money holder increases the quantity of goods supplied to the partner. Figure 2.2 shows that as $M$ increases from initially low levels both agents with and without money benefit. This is because money facilitates the transfer of utility between agents. In particular, agents without money obtain a smoother consumption pattern over time when there are more money holders in the economy. As a result welfare increases as illustrated in Fig. 2.3. However, as $M$ becomes too high, matches in which one agent has money become rare and in most matches both agents have money. Since trades in matches between money holders resemble barter, which do not transfer utility evenly, a further increase in $M$ reduces $V_g$ and $V_m$, resulting in lower welfare. Figure 2.3 shows that the welfare-maximizing money stock lies in the interior region where a robust monetary equilibrium (i.e., $Y > 0$) exists. In Fig. 3 we consider a different bargaining weight for a money holder, $w = 0.4$. Like the previous case, there is a positive money stock that maximizes welfare. However, the optimal stock is one that pushes $Y$ to 0 so that agents are just indifferent between holding money and not holding money.

7. CONCLUSION

This paper finds that universal symmetric coincidence of wants in barter is not sufficient to preclude robustly valued money or a welfare-improving role for money. The results are consistent with our earlier paper (Engineer and Shi, 1998), where bargaining weights are symmetric across matches. There we found that asymmetry in demands was necessary for both robustly valued fiat money and welfare-improving money. Together, the analyses suggest that (for efficient bargains) asymmetry either in demands or in bargains is necessary for a welfare-improving role for money.
FIG. 2.1. Quantities of goods supplied by a money holder \( q_m \) and a non-money holder \( q_g \) in a match between the two agents as a function of the money stock.

FIG. 2.2. Value of holding money \( V_m \) and value of not holding money \( V_g \).
FIG. 2.3. Welfare ($Z$), and the difference between values of holding money and not holding money ($Y = V_m - V_p$) when $w = 0.08$.

FIG. 3. Welfare ($Z$), and the difference between values of holding money and not holding money ($Y = V_m - V_p$) when $w = 0.04$. 


In more general terms, this paper has explored the implication of various bargains supported by convention for the general equilibrium. In the analysis we concentrated on asymmetric bargains. These are efficient bargains but the general equilibrium is inefficient because the agent with the better bargain values the last unit of consumption less than the cost to the other agent—utility is transferred imperfectly at the margin. Barter results in excess quantities being exchanged. In contrast, with inefficient bargains, insufficient quantities are exchanged.

It might be argued that money is unnecessary in our framework because with symmetric bargains the barter equilibrium is efficient. Though this would be the planner's solution, it cannot be imposed in the decentralized framework. With anonymous random matching, bargains are unrestricted by social edicts. In Section 3 we argue that game theory imposes no definite restrictions that rule out inefficient barter conventions that perhaps arose from historical idiosyncracy characteristics of agents, goods, or encounters.

The inefficiency arising from the idiosyncratic nature of bargains however may be alleviated by money. Money is valued in equilibrium when it receives favorable terms of trade or reduces the variability of consumption relative to barter. The minimum bargaining weight for money holders \( w_s(\Omega) \) is a measure of how special money must be to have value. Money must impart to its holder sufficient bargaining power. This is an equilibrium, but there is something special not only about the fact that intrinsically useless tokens have value but also about money empowering the holder.\(^{13}\)

How important are bargains to barter inefficiency and the value of money?\(^{14}\) At a theoretical level we have presented a specific model with

\(^{13}\) References to the magical, sacred, and special natures of money (and peoples’ relationship with money) are replete in the anthropology, psychology, and sociology literatures (see Belk and Wallendorf, 1990; Hodges, 1988; and Zelizer, 1989). Belk and Wallendorf discuss the sacred power of money as “… power to increase oneself, to change one’s natural situation from one of smallness, helplessness, finitude to one of bigness, control, durability, importance” (p. 46), and “Money is only an extreme and specialized type of ritual … If faith is shaken, the currency is useless. So too with ritual” (p. 42). In a sense our analysis reconciles the sacred (“substantive”) and the profane (so-called economist’s “utilitarian”) views of money, by relating the bargaining power that money gives the holder to its general equilibrium value.

\(^{14}\) Interestingly, the origins of money may have been in ritualistic gift giving in sacred ceremonies (see Einzig, 1966; Hodges, 1988; and Quiggin, 1979). Our paper suggests that the bargaining power that possibly sanctified money imparts may have played a historical role in coordinating agents on better outcomes. Einzig (1966), Hodges (1988), and Quiggin (1979) all argue that the difficulties involving barter were often easily surmounted. The inefficiency of idiosyncratic bargains may have contributed to the ascendency of money. This explanation is complementary with priests and sovereigns generating seigniorage.
symmetric coincidence of wants and with fiat money. We picked this specification for simplicity and elegance. However, the general argument could be made with asymmetric demands and commodity money. At a practical level, we observe only quantities exchanged across matches. From observations it is difficult (or impossible without experiments) to empirically decompose the extent to which outcomes are due to asymmetric bargains versus asymmetric demands. The monetary literature has either ignored bargains or imposed a priori restrictions to rule out bargains as a source of money demand. In this paper, we point out that these restrictions are largely arbitrary and that by relaxing them bargains can generate a large money demand.

The search approach to monetary economics replaces the Walrasian auctioneer with decentralized exchange. Instead of endowments determining allocations through anonymous market prices, endowments and bargains determine allocations. In exploring how bargains matter to the economy, this paper demonstrates the richness of the search paradigm.

REFERENCES


