Tariffs, Unemployment, and the Current Account: An Intertemporal Equilibrium Model*

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Abstract

This paper integrates labor market search into a dynamic general equilibrium model to analyze the macroeconomic effects of a tariff. The search friction creates a wedge between the marginal product of labor and the product wage. With perfectly flexible prices and wages, the model captures the intuitive effect that a permanent increase in the tariff improves the country's terms of trade, which tends to reduce the product wage and stimulates labor demand. However, the tariff also increases the price of the consumption goods bundle and reduces the marginal utility of wealth measured by imports. This consumption bundle effect raises the reservation wage and the product wage. When the consumption smoothing motive is realistically strong, the consumption bundle effect of the tariff dominates the direct product wage effect, leading to lower employment in both the short run and long run. Thus, even with persistent unemployment, raising tariffs is not the means in which a government in a small open economy can succeed in increasing employment, short run or long run. The welfare effect of the tariff is also analyzed.

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1. Introduction

Tariffs protect jobs, at least in the short run. This idea was sometimes proposed as a policy prescription to reduce unemployment (Cripps and Godley (1978)) and has been rationalized by models with sticky nominal prices or wages. By creating a gap between the real wage and the marginal product of labor, the nominal rigidities generate unemployment and leave output to be determined by aggregate demand. If nominal prices are sticky, as in the celebrated framework of Mundell (1968) and Fleming (1962), tariffs shift demand from imports to domestic goods, which is absorbed by increased labor demand and output. This mechanism continues to work in modern versions of sticky price models, including the intertemporal model by Obstfeld and Rogoff (1995). If nominal wages are sticky, the shift in demand toward domestic goods improves the country's terms of trade and so reduces the wage measured in domestic goods – the product wage. Again, labor demand and output increase.

The assumption of nominal rigidities is controversial and the positive employment effect of tariffs may be rejected on this ground alone.[1] However, this rejection is a weak one, because central to the argument for a positive employment effect of tariffs is not the nominal rigidity *per se* but the existence of unemployment created by the deviation of the real wage from the marginal product of labor. The nominal rigidity is only a convenient way to generate such a deviation. To have a stronger rejection to the positive employment effect of tariffs, one must then show that tariffs reduce employment even when persistent unemployment exist for reasons other than nominal rigidities.

This paper does exactly that. To achieve this purpose, I must first construct a model that has two necessary ingredients. One is a real friction in the labor market that generates unemployment in the long run without nominal rigidities, for reasons discussed above. The second is a dynamic general equilibrium framework that permits the distinction between short-run and long-run effects

[1] For example, van Wijnbergen (1987) shows that, if the nominal wage is instead fully indexed to a consumer price index and hence responds to a tariff in the same proportion as does the price index, it increases by more than do the terms of trade when the tariff increases. Thus, the product wage, which is measured in the domestic good only, rises and employment falls. Other counter-arguments to the positive employment effect of tariffs include Mundell (1961), who argues that an improvement in the terms of trade induced by tariffs reduces aggregate demand and output via the Laursen-Metzler effect (see Eichengreen (1981) for more discussions) and Sen and Turnovsky (1989), who argue that tariffs induce a substitution toward leisure and hence reduce employment.

of tariffs. This is necessary because optimal wealth accumulation places restrictions on the long-run product wage and has direct implications on the effects of tariffs. For example, with time-additive preferences, optimal wealth accumulation implies that the long-run capital-labor ratio is exogenously determined by the equality between the long-run marginal product of capital and the subjective discount rate. If the labor markt is frictionless, then the long-run product wage must also be exogenous, irrespective of the tariff.[2]

A model that has the above ingredients has recently been developed in macroeconomics (Merz (1995), Andolfatto (1996), and Shi and Wen (1997, forthcoming)). It is the intertemporal version of the search unemployment theory by Mortensen (1982) and Pissarides (1990). In this theory unemployment persists because firms must maintain costly vacancies in order to hire workers and unemployed workers must search in order to find a job. The marginal product of labor is strictly higher than the product wage so as to compensate for the firm's hiring (vacancy) cost. This gap allows tariffs to affect employment without wage rigidity: Permanent increases in tariffs can permanently affect the product wage and hence employment without affecting the marginal product of labor.

The product wage, determined by Nash bargaining between firms and workers, is a weighted sum of the marginal product of labor and a reservation wage.[3] With an intertemporal setting, the reservation wage equals the marginal rate of substitution between consumption and leisure. There are two ways in which a tariff affects the reservation wage in the current model, depicted in Figure 1.1. The first is the *direct product wage effect*: A tariff increases the price of the domestic good and so reduces the product wage. The second is the *consumption bundle effect*: A tariff increases the price of the goods bundle, both directly through the import price and indirectly through the terms-of-trade improvement, which reduces the marginal value of wealth measured in the import and raises the marginal rate of substitution between leisure and consumption.

[2]If preferences are not time-additive but instead recursive in the Uzawa-Epstein fashion, a tariff will create an incentive to accumulate foreign assets to meet the long-run "target" level of consumption. Long-run output and employment will decrease by more than in a time-additive model. See Obstfeld (1982) and Shi (1994) for applications of the Uzawa-Epstein preference in open economies.

[3]The exogenous matching function and Nash bargaining for wage determination in the Mortensen-Pissarides model and in this paper are simple but not necessary elements for the existence of unemployment. Shi and Wen (forthcoming) examine endogenous matching functions and alternative wage determination schemes.

The two effects of the tariff are opposite to each other and their relative strength depends on the elasticity of intertemporal substitution. The consumption bundle effect dominates the direct product wage effect when the elasticity of intertemporal substitution is small, in which case consumption varies very little in response to the higher price, leaving the marginal value of wealth to fall significantly and the product wage to rise. With realistic values of the elasticity of intertemporal substitution, the overall effect of the tariff is to raise the product wage and reduce employment in both the long run and short run. Thus, the presence of the search friction and unemployment is insufficient for generating a predominant, positive employment effect for the tariff.

The tariff may also change utility. In the special case where the country has no influence on the terms of trade, the tariff reduces steady state consumption, increases leisure and increases steady state utility. But the steady state utility gain is completely wiped out by the cost of transition and so the tariff has no effect on intertemporal utility. When the country has some limited influence on the terms of trade, the welfare effect of the tariff is likely to be ambiguous.

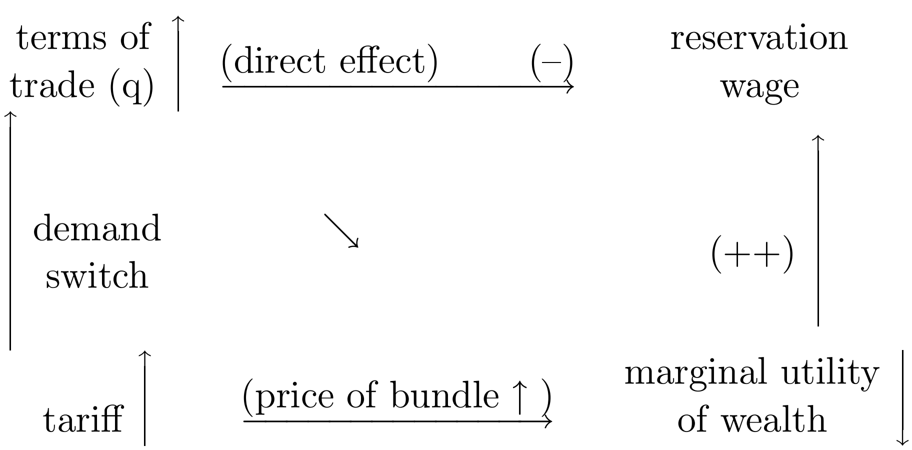

Figure 1.1

The model constructed in this paper enhances the dynamic general equilibrium framework that has been extensively used in international economics since the late 1970s. The latter has often been criticized for lacking some realistic features such as unemployment. By allowing for persistent unemployment and yet maintaining flexible prices and wages, the current framework provides a viable alternative to models with nominal rigidities for addressing international policy issues related to unemployment. The search approach to unemployment is chosen here because it matches the statistical definition of unemployment and is tractable in a dynamic optimization environment that involves a long horizon.

To emphasize the importance of the search friction, a small open economy is adopted so that the product wage effect of the tariff would be absent in the long run if the friction were eliminated as in Sen and Turnovsky (1989). The intertemporal analysis is related to the voluminous literature on the Laursen-Metzler effect (see Obstfeld (1982) and Persson and Svensson (1985)). The presence of unemployment links the analysis to some trade models, such as Matusz (1986), Fernandez (1992), Brecher (1992), and Neary (1982), but the current paper differs in two aspects. First, the current paper focuses on the macroeconomic effects of tariffs rather than the sectorial effects in the trade models. Second, the current paper employs an intertemporal structure, while those trade models are typically either static or very restrictive on agents' intertemporal decisions.

The remainder of this paper is organized as follows. Section 2 constructs an intertemporal maximization model with labor market search. Section 3 isolates the consumption bundle effect by assuming that the economy faces exogenous terms of trade. Section 4 examines the case of endogenous terms of trade. Section 5 discusses the welfare effect of the tariff. Section 6 concludes the paper and the appendices provide necessary proofs.

2. Labor Market Search in a Small Open Economy

2.1. Goods and Assets

Consider a small open country that imports a good whose price in the world market is normalized to one. The country imposes a tariff rate τ on imports and the country's residents face an import price $(1+\tau)$. The country produces a single good, called the *domestic good*, which can be consumed or exported. The relative price of the domestic good to the import in the world market is q, which is the country's terms of trade. The assumption of complete specialization in production places the focus of this paper on the aggregate effect of the tariff rather than its sectorial allocative effects. As in Sen and Turnovsky (1989), the country may be able to influence the terms of trade and so tariffs can affect employment through the terms of trade. This influence is captured by an export function, $x(q)$, which satisfies:

$$x'(q) \leq -x(q)/q \leq 0. \tag{2.1}$$

These properties ensure that the foreign demand for the country's good is a decreasing function of the good's price and has an elasticity greater than unity. The latter reflects the fact that a small country's influence on the terms of trade is limited. A special case is $x'(q) = -\infty$, in which case the country faces exogenous terms of trade.

The country consists of many identical households whose size is normalized to one. Households have an unrestricted access to the world good and asset markets. In particular, capital is perfectly mobile across countries and so households can borrow and lend at a constant world interest rate $\rho > 0$. A household's portfolio consists of domestic capital K, measured in terms of domestic goods, and foreign assets F, measured in terms of the import before the tariff. The rental rate of capital is r. Because the terms of trade can vary over time, holding domestic capital yields a capital gain (or loss) $\dot{q}/q$ relative to holding foreign assets. Therefore, the arbitrage between domestic capital and foreign assets yields

$$r + \frac{\dot{q}}{q} = \rho. \tag{2.2}$$

In contrast to capital mobility, labor is immobile across countries.

For the demand for goods, denote a representative household's consumption of the domestic good by d and the consumption of the import by f. To simplify analysis, assume that d and f enter the household's utility function through a linearly homogeneous aggregator $H(d, f)$ that is increasing and concave in each argument.[4] In this case, a household's optimal consumption can be chosen in two stages. First, for any given $c > 0$, it is optimal to choose the bundle (d, f) to solve:

$$c \cdot p(q, \tau) \equiv \min_{(d,f)} \{qd + (1 + \tau)f : H(d, f) \geq c\}.$$

The function $p(q, \tau)$ defined above is the unit cost (or expenditure) function dual to H, which will be referred to as the *price index* of the consumption bundle. In the second stage of the consumption

[4] Linear homogeneity of H implies that the two goods are complementary in the sense $H_{12} > 0$. This implication is plausible and used by Sen and Turnovsky (1989).

choice, c is chosen to maximize intertemporal utility, as described in the next subsection. I will refer to c as consumption of the goods bundle.

It is well known that $p(q,\tau)$ is increasing and concave in each argument and is linearly homogeneous in $(q, 1+\tau)$. For given c, the demand functions for goods are:

$$d = c \cdot p_1(q,\tau), \quad f = c \cdot p_2(q,\tau). \tag{2.3}$$

It is reasonable to require that the share of consumption on the domestic good, qp_1/p, be a non-decreasing function of the tariff and a non-increasing function of the terms of trade. That is, $pp_{12} \geq p_1 p_2$ and $pp_1 \leq q(p_1^2 - pp_{11})$. These requirements can be easily satisfied if, for example, H is a Cobb-Douglas aggregator.

2.2. Households

Each household consists of many infinitely-lived agents, each endowed with a fixed flow of time, T. At any given point in time, an agent can choose only one of the following activities: working for wages, searching for a job or enjoying leisure. Agents who are searching for jobs are called unemployed agents. Unemployed agents are randomly matched with job vacancies according to a matching function described later. Since the timing of a match is random, agents face idiosyncratic risks in income and leisure. This randomness can complicate the analysis by generating distributions of wealth and consumption across agents. To focus on the aggregate behavior, I will assume that each household consists of a continuum of agents with measure T and that all members care only about the household's utility. In this case, individual risks in consumption and leisure are completely smoothed within each household. A similar approach is adopted in the literature on indivisible labor, where employment lotteries are used to smooth the risk across states of employment (see Hansen (1985) and Rogerson (1988)).[5]

The utility function of a household is

$$U = \int_0^\infty \{u(c) - \beta[n + l(s)]\} e^{-\rho t} dt, \quad \beta > 0, \tag{2.4}$$

[5] The approach is also common in other well-known macroeconomic models. For example, in a monetary model, Lucas (1990) assumes that household members go to different markets and pool the receipts.

where $c = H(d, f)$ is the household's consumption of the bundle, n the size of household members in work, and s the size of unemployed members. The fraction of members in work is n/T and s/T the fraction of members unemployed. The labor force participation rate is $(n+s)/T$ and the unemployment rate is $s/(n+s)$. The function $l(s)$ measures the efficiency units of time in search relative to working. Note that the utility function is linear in the hours of work, as implied by the above cited literature on indivisible labor with employment lotteries. Also, the rate of time preference equals the international interest rate, which is necessary for consumption to converge to a steady state in a small open economy with a constant rate of time preference.

The function $u(\cdot)$ is assumed to be increasing and concave, with an intertemporal elasticity of substitution $\sigma \equiv -u'(c)/[cu''(c)]$. Hall (1988) and Epstein and Zin (1991) have found that the intertemporal elasticity of substitution is empirically small and below unity. I thus assume $\sigma \leq 1$. I also assume that the search effort is inelastically supplied and so s is fixed at a level $s_0 = 1$. This assumption is made for analytical tractability: without it the model cannot be analytically solved. It should be interpreted as an extreme approximation for the reality that the search effort is much less elastic than vacancy (Layard et al. (1991)). Accordingly, the qualitative results obtained herein should hold more generally for such economies. Note that fixing s fixes the level of unemployment but leaves the rate of unemployment to be determined endogenously.[6]

As in other search models, employment in the current model is predetermined at each given time; it changes only gradually as workers quit or unemployed agents find jobs:

$$\dot{n} = ms_0 - \theta n. \tag{2.5}$$

The constant θ is the rate of job separation and m the rate at which each unemployed agent finds a job. As discussed later, m depends on the ratio of aggregate vacancy to unemployment. However, an individual household takes m as given.

A representative household's maximization problem is

$$(PH) \qquad \max_{(c,\dot{F})} U$$

[6]In a search model without tariffs (an earlier version of Shi and Wen (1997)), it is shown that, if the search effort is much more elastic than job vacancy, a permanent productivity increase generates the counter-factual result that the ratio of job vacancy to the number of unemployed agents immediately falls.

subject to:

$$\dot{F} = \rho F + q(wn + \pi) - pc + L; \tag{2.6}$$

$$F(0) = F_0 \text{ given.}$$

Here π is the dividend to capital (defined later), measured in terms of the domestic good, w is the wage measured in terms of the domestic good (i.e., the product wage), and L is the lump-sum rebate of the tariff revenue. The household takes (m, w, π, q, p, L) as given in the maximization. Note that n is not in the list of the household's choice variables. This is because, with an inelastic search effort, employment dynamics described by (2.5) are exogenous to the household. Employment is demand-driven, determined by the firm's hiring decision and a wage equation described later.

Let ϕ be the current-value shadow price of wealth measured in terms of import before tariffs. Standard dynamic optimization techniques generate:

$$\dot{\phi} = 0 \tag{2.7}$$

$$u'(c) = p\phi. \tag{2.8}$$

Since the world interest rate always equals the rate of time preference, the shadow value of wealth must be constant over time along any continuous transition path, as is typical in a small open economy. That is, any changes in ϕ must be once-and-for-all and occur immediately after shocks hit the economy. Eq. (2.8) states the familiar relation that the marginal utility of consumption of the goods bundle is equal to the marginal value of wealth, evaluated with the price index p. Once c is determined, the demand for each good is given by (2.3).

2.3. Firms

There are many identical firms in the economy. The production function is $G(K, n)$ which is increasing and concave in each argument, and linearly homogeneous. Each firm maintains vacancies in order to hire workers. The cost of maintaining a number v of job vacancies is $B(v)$ in terms of the domestic good. This cost function is increasing and convex, with a *vacancy elasticity* $\epsilon = B'(v)/[vB''(v)]$. Let μ be the rate at which a vacancy finds a match. Like m, the rate μ depends on aggregate vacancy and unemployment, but an individual firm takes μ as given. A firm's employment evolves as follows:

$$\dot{n} = \mu v - \theta n. \tag{2.9}$$

Adjustments in physical capital are also costly, as in Hayashi (1982). That is, to increase physical capital by an amount i, the firm must invest a total amount $Q(i)$. The function Q has the following properties:

$$Q'(i) > 0,\ Q''(i) > 0,\ Q(0) = 0,\ Q'(0) = 1.$$

An individual firm takes as given the wage rate w offered by other firms. The firm also takes (μ, q, r) as given and maximizes the present value:

$$(PF)\quad \max_{(v,i,n,K)} \quad \int_0^\infty \pi(t) e^{-\int_0^\tau r(\tau)d\tau}\, dt$$

subject to (2.9) and the following constraints:

$$\pi = G(K, n) - wn - B(v) - Q(i); \tag{2.10}$$

$$\dot{K} = i;$$

$$n(0) = n_0,\ K(0) = K_0 \text{ given.}$$

Let Ψ be the current-value shadow price of an additional worker to the firm and - the marginal value of capital. The optimal conditions for (PF) are

$$\Psi = B'(v)/\mu; \tag{2.11}$$

$$\dot{\Psi} = (\theta + r)\Psi - (G_2 - w); \tag{2.12}$$

$$- = Q'(i); \tag{2.13}$$

$$\dot{-} = r- - G_1. \tag{2.14}$$

Eq. (2.11) characterizes the firm's optimal decision for vacancy – the investment in employment. It requires the marginal cost of a vacancy, $B'(v)$, to be equal to the marginal benefit, $\mu\Psi$. Eq. (2.12) requires the "return" to employment, $(\theta + r)\Psi$, to be equal to the sum of the "dividend" from hiring, $(G_2 - w)$, and the capital gain, $\dot{\Psi}$. Eq. (2.13) and (2.14) are similar conditions for the investment in physical capital.

I will refer to the difference $(G_2 - w)$ as the firm's surplus from hiring. In contrast to a typical neoclassical model, the marginal product of labor here must exceed the wage rate in order to give firms a positive surplus from hiring that compensates for the hiring cost. If $G_2 = w$, the shadow price of an additional worker to the firm would be zero in the steady state and so vacancies and employment would be zero in the steady state (see (2.12)).

2.4. Matching and Wage Determination

The matching for each vacancy and unemployed agent is random but the aggregate number of job matches is deterministic and given by a matching function. Let $\bar{v}$ and $\bar{s}_0$ $(= 1)$ be the aggregate number of vacancies and unemployed agents, respectively. The flow of job matches is:

$$M(\bar{v}, \bar{s}) = M_0 \bar{v}^{\alpha} \bar{s}_0^{1-\alpha}, \qquad \alpha \in (0, 1), \tag{2.15}$$

where M_0 is a positive constant. The matching technology exhibits constant returns-to-scale, as is empirically supported (see Blanchard and Diamond (1989)). The Cobb-Douglas form is adopted for analytical simplicity. With the normalization $\bar{s}_0 = 1$, we have:

$$m(\bar{v}) \equiv M/\bar{s}_0 = M_0 \bar{v}^{\alpha}, \qquad \mu(\bar{v}) \equiv M/\bar{v} = m(\bar{v})/\bar{v}. \tag{2.16}$$

Note that the matching rate for vacancy, μ, is a decreasing function of $\bar{v}$. Also, $\mu\bar{v} = m$ and so the two laws of motion for n, (2.5) and (2.9), coincide in any symmetric equilibrium. I will suppress the bar in $\bar{v}$ and $\bar{s}_0$.

Once an unemployed agent is matched with a vacancy, the agent and the firm negotiate the agent's current and future wage rates. The outcome is determined by Nash bargaining which maximizes the weighted surpluses of the household and firm. To be precise, let t_0 be the time when a match is created. Denote by $\{\hat{w}(t)\}_{t \geq t_0}$ the path of wage rates to be determined for the new worker, conditional on the continuation of the agent's employment. Wage rates are measured in terms of the domestic good. Hiring an additional worker of size dn with the wages increases the firm's current-valued surplus at each time $t \geq t_0$ by $[G_2(t) - \hat{w}(t)]dn$. Having an additional member working at the wages increases the household's income (in terms of imports) at each time $t \geq t_0$ by $\hat{w}(t)q(t)dn$. The value of such increased income is $\phi(t)\hat{w}(t)q(t)dn$, since $\phi(t)$ is the marginal utility of time-t income (wealth).[7] The associated leisure cost is βdn and so the household's surplus is $[\hat{w}(t)q(t)\phi(t) - \beta]dn$ at each time $t \geq t_0$. With normalization, the Nash bargaining solution solves

$$\max_{\hat{w}(t)}[G_2(t) - \hat{w}(t)]^{1-\lambda}\left[\hat{w}(t) - \frac{\beta}{q(t)\phi(t)}\right]^{\lambda}, \quad \text{for } t \geq t_0.$$

The parameter $\lambda \in (0,1)$ can be interpreted as the worker's bargaining power.[8] Solving this bargaining problem yields

$$\hat{w}(t) = \lambda G_2(t) + (1-\lambda)\frac{\beta}{q(t)\phi(t)}. \tag{2.17}$$

Since all firms are identical, they must offer the same wage in any symmetric equilibrium. Since the wage formula is independent of when the match is formed (i.e., independent of t_0), two workers who are hired by the same firm at different times must be paid the same wage at any given time. Thus, $\hat{w}(t) = w(t)$ for all t and the hat is suppressed.

The product wage rate is a weighted sum of the marginal product of labor, G_2, and the reservation wage, $\beta/(q\phi)$, with the weights being the bargaining powers of the worker and the firm. Since $G_2 > w$, as argued before, $G_2 > \beta/(q\phi)$. The product wage lies between the marginal rate of substitution $\beta/(q\phi)$ and the marginal product of labor G_2, in contrast to a standard neoclassical model where $w = \beta/(q\phi) = G_2$. Even if the marginal product of labor is constant, a tariff can still affect the product wage through the terms of trade and the marginal value of wealth. These induced responses of q and ϕ will be the two channels through which a tariff affects employment, as analyzed later.

[7]Equivalently, the increased income $\hat{w}(t)q(t)dn$ can be used to purchase $\hat{w}(t)q(t)dn/p(t)$ units of the consumption bundle and so yields utility $u'(c(t))\hat{w}(t)q(t)dn/p(t)$. Since $u'(c)/p = \phi$, this utility is $\hat{w}(t)q(t)\phi(t)dn$.

[8]In a stationary environment, the solution to the Nash bargaining problem coincides with the solution to some non-cooperative sequential bargaining games (Wolinsky (1987)). Coles and Wright (1998) discuss the relationship between the two solutions in a nonstationary environment.

2.5. Equilibrium Definition

For a finitely elastic export function $x(q)$, the terms of trade are determined by the market clearing condition for the domestic good:

$$d + x(q) + B(v) + Q(i) = G. \tag{2.18}$$

An equilibrium can be defined as follows:

Definition 2.1. *A search equilibrium is a converging sequence of* $\{c(t), d(t), f(t), n(t), F(t), K(t), v(t), i(t)\}_{t\geq 0}$, *good prices* $\{q(t)\}_{t\geq 0}$, *factor returns* $\{r(t), w(t)\}_{t\geq 0}$, *dividends* $\{\pi(t)\}_{t\geq 0}$, *matching rates* $\{m(t), \mu(t)\}_{t\geq 0}$ *and rebates* $\{L(t)\}_{t\geq 0}$ *such that*

(i) given $\{q, r, w, \pi, m, \mu, L\}$, $\{c, F\}$ *solve* (PH) *and* $\{d, f\}$ *satisfy (2.3);*

(ii) given $\{q, r, w, m, \mu, L\}$, $\{n, K, v, i\}$ *solve the firm's problem* (PF)*;*

(iii) $\{r, \pi, w\}$ *satisfy (2.2), (2.10), and (2.17);*

(iv) $\{m, \mu\}$ *are given by (2.16);*

(v) $L = \tau f$ *and* q *satisfies (2.18).*

Central to this equilibrium is the feature that employment is driven by the firm's decision on vacancy. The dynamics of vacancy can be obtained from (2.11) and (2.12) by eliminating Ψ and substituting the wage equation (2.17):

$$\dot{v} = \gamma \left[(\theta + r)v - \frac{(1-\lambda)m(v)}{B'(v)} \left(G_2 - \frac{\beta}{q\phi} \right) \right], \tag{2.19}$$

where $\gamma \equiv \epsilon/[1+(1-\alpha)\epsilon] > 0$. Vacancy increases if and only if the return to vacancy, $(\theta+r)vB'/m$, exceeds the firm's surplus from hiring. In the steady state, the two are equal and so steady state vacancy, denoted v^*, is given by

$$(\theta + \rho)\frac{v^* B'(v^*)}{m(v^*)} = (1-\lambda) \left(G_2 - \frac{\beta}{q^*\phi} \right). \tag{2.20}$$

I have used the fact $r = \rho$ in the steady state.

The marginal product of labor in the steady state is exogenous, as the steady state capital-labor ratio is pinned down exogenously by $G_1 = \rho$. Therefore, a tariff can affect steady state job vacancy and employment only through the reservation wage $\beta/(q^*\phi)$. This effect can be channeled either through a change in the terms of trade – the *direct product wage effect* of tariffs, or through a change in the marginal value of wealth – the *consumption bundle effect* of tariffs.

Eq. (2.20) implicitly characterizes the long-run supply of the goods market. It gives a positive relation between steady state vacancy (and hence output) and the marginal utility of wealth (ϕ), depicted by the upward sloping curve VV in Figure 3.1. The VV curve will be called the long-run "aggregate supply curve", with the marginal utility of wealth being the "price". A high marginal utility of wealth lowers the reservation wage, increases the firm's surplus from hiring and so stimulates hiring (and output).

3. The Case of Exogenous Terms of Trade

In this section I isolate the consumption bundle effect of the tariff. This is achieved in a special case where the country faces an infinitely elastic foreign demand for its goods, i.e., $x'(q) = -\infty$. In this case the terms of trade are constant, eliminating the direct product wage effect of the tariff. With constant terms of trade, the rental rate of capital must be equal to the world interest rate at each point of time, i.e., $r(t) = \rho$ for all $t \geq 0$. To ease exposition in this section, I also assume that the marginal adjustment cost in investment is flat, i.e., $Q'' = 0$. In this case the marginal value of capital is unity and $G_1 = \rho$ for all $t \geq 0$. Therefore, the capital-labor ratio is constant, denoted $\kappa = K/n$. The variable K can be replaced by κn and $\dot{K}$ by $\kappa \dot{n}$.

3.1. The Dynamic System and the Solution

The dynamic system for this special case consists of differential equations for (v, n, F). The dynamic equation for v is given by (2.19) with $r = \rho$. The dynamic equation for n is given by (2.5). To obtain the dynamic equation for F, substitute π from (2.10) and $L = \tau f$ into the dynamic equation for F in (2.6) to obtain:

$$\dot{F} = \rho F + q[G - B - \kappa(m - \theta n)] - (p - \tau p_2)c. \tag{3.1}$$

The initial conditions for the dynamic system of (v, n, F) are $n(0) = n_0$ and $F(0) = F_0$.

Vacancy is constant along the transition path in this special case. To see this, notice that (q, r, G_2, ϕ) are all constant along the transition path and so the equation (2.19) is an autonomous equation for vacancy. Since the right-hand side of (2.19) is an increasing function of vacancy, the return to vacancy exceeds the firm's surplus from hiring if and only if vacancy exceeds its steady state level v^* defined by (2.20). Thus, vacancy increases over time if and only if vacancy exceeds the steady state level. The steady state level can be reached only when $v(t) = v^*$ for all t. More precisely, when responding to disturbances like the tariff, vacancy jumps immediately to the steady state level and stays there afterward.

With constant vacancy, the dynamic equations for n and F are linear differential equations that can be solved to generate the following proposition (see Appendix A).

Proposition 3.1. *When the terms of trade are constant, the stable paths of (n, F) are characterized as follows for any given (n_0, F_0):*

$$n(t) = \frac{m(v^*)}{\theta} + \left[n_0 - \frac{m(v^*)}{\theta}\right] e^{-\theta t}, \tag{3.2}$$

$$F(t) = F^* - \frac{q^*}{\theta + \rho}\left(\frac{G}{n} + \theta\kappa\right)\left[n(t) - \frac{m(v^*)}{\theta}\right], \tag{3.3}$$

where F^* is the steady-state value of F and is given by

$$F^* = \frac{1}{\rho}\left\{(p - \tau p_2)c - q^*[G - B(v^*)]\right\}. \tag{3.4}$$

Proposition 3.1 states that claims on foreign assets are negatively related to employment and hence to output along the stable path. This is because an increase in employment raises the marginal product of capital, which in turn induces agents to switch investment from foreign assets to domestic capital. As a result, the current account, $\dot{F}$, is negatively related to changes in employment.

Proposition 3.1 also implies that the steady state depends on the initial conditions (n_0, F_0), as is typical in a small open economy model with a constant rate of time preference. Eq. (3.3) at $t = 0$ helps to determine the marginal utility of wealth, ϕ. Substituting (3.4) into (3.3), setting $t = 0$ and noticing $c = u'^{-1}(p\phi)$ yields:

$$\frac{G_2}{\theta + \rho}m(v^*) - B(v^*) + \rho(\kappa + \frac{G_2}{\theta + \rho})n_0 = \frac{1}{q^*}\left[(p - \tau p_2)u'^{-1}(p\phi) - \rho F_0\right]. \tag{3.5}$$

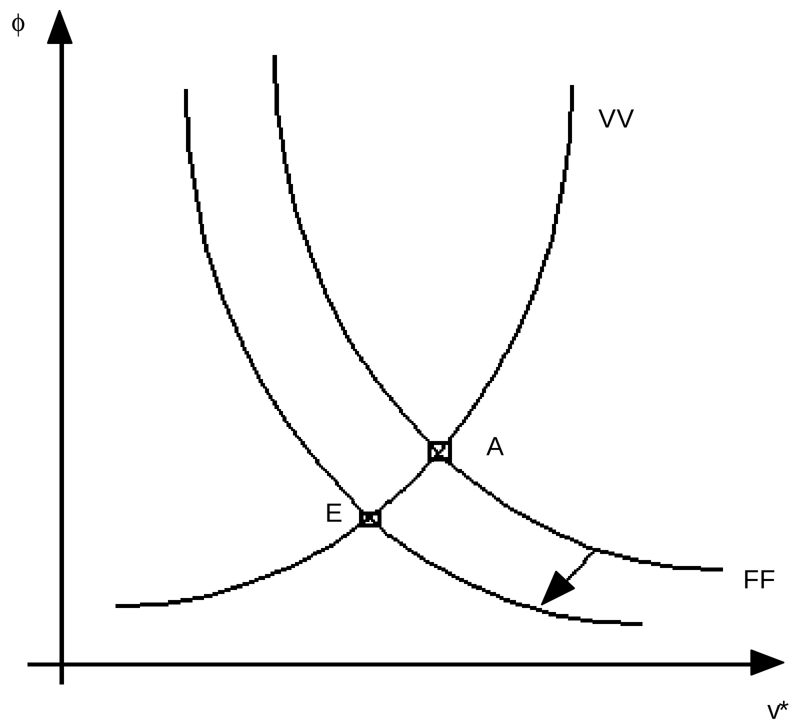

Figure 3.1:

This steady state equation gives a negative relationship between steady state vacancy and the marginal utility of wealth, depicted by the curve FF in Figure 3.1. The left-hand side of (3.5) is an increasing function of v^*, measuring the amount of goods available for consumption and export.[9] The right-hand side of the equation is a decreasing function of ϕ, measuring the expenditure on goods and foreign debt service. The FF curve will be called the long-run "aggregate demand curve".

The intersection between the two curves VV and FF in Figure 3.1 determines steady state vacancy and the marginal utility of wealth. Once (v^*, ϕ) are determined, other steady state values (n^*, K^*, F^*, c) can be recovered accordingly.

3.2. A Permanent Increase in the Tariff

Suppose that the economy is in a steady state at time 0, with $\tau = 0$ and $(n(0), K(0), F(0)) = (n_0, \kappa n_0, F_0)$. Then the tariff rate has a once-and-for-all, unexpected increase to a new level $d\tau > 0$ which is sufficiently small.[10] Since the terms of trade are fixed, the tariff affects the product wage only through its effect on the marginal value of wealth, ϕ. This consumption bundle effect arises because the tariff makes the consumption bundle more expensive, i.e., increases p. The marginal value of wealth, $\phi = u'(c)/p$, falls and the reservation wage rises. The product wage rises, which reduces the firm's surplus from hiring and reduces vacancy. Depicted in Figure 3.1, the long-run aggregate demand curve FF shifts to the left, as consumers now can only afford to buy a smaller quantity of the consumption bundle than before for any given ϕ. The VV curve does not shift and so job vacancy is lower in the new steady state (point E) than in the original steady state (point A). Consequently, steady state employment and capital stock are lower in the new steady state.

The employment response to the tariff clearly relies on the reservation wage being endogenous. It also depends critically on the non-Walrasian feature of the labor market. In particular, the bargaining power of the firm in the wage determination $(1 - \lambda)$ plays a key role. If the firm has

[9]The condition required for the left-hand side of (3.5) to be increasing in v^* is $\alpha/(1-\lambda) > 1 - \beta/(q\phi G_2)$, which is satisfied if the firm's bargaining power in the wage determination $(1-\lambda)$ does not exceed its contribution to the match formation (measured by α) by too large a margin. Such a condition is maintained here (see Hosios (1990) for more discussions on the difference between $1-\lambda$ and α).

[10]Throughout this paper, I will examine only permanent changes in tariff. Transitory changes can also be examined but omitted here.

a very low bargaining power, for example, changes in the product wage induced by the tariff will have only a small effect on the firm's surplus of hiring, in which case the responses of vacancy and employment to the tariff will be small. In terms of Figure 3.1, a lower bargaining power of the firm corresponds to a steeper long-run aggregate supply curve VV, in which case the shift in the FF curve generates a large change in ϕ but only a small change in v.

The importance of the labor market friction sets the current analysis apart from the Sen-Turnovsky (1989) model, where the labor market is Walrasian. In a Walrasian labor market, the marginal product of labor equals the marginal rate of substitution between consumption and leisure. In this case the VV curve is horizontal and an increase in the tariff generates the *largest* (negative) consumption bundle effect. Thus, the search friction in the labor market attenuates the consumption bundle effect which a tariff has on employment.

The transitional dynamics after the tariff increase can be analyzed as follows. Since $(n^*, K^*) < (n_0, K_0)$, (3.2) implies that employment and the capital stock monotonically decrease along the stable path. Thus, raising the tariff reduces employment and output, both in the long-run and in the short-run — there is no trade-off between the short run and the long run effects of a tariff in this special case. The tariff also raises the long-run level of claims on foreign assets, which can be verified from (3.4). The country experiences current account surpluses along the entire transition path (see (3.3)), as investors switch investment from domestic capital to foreign assets.

The dynamic adjustments in the labor market can be expressed in Figure 3.2 in the subspace of the vacancy rate $vv \equiv \frac{v}{n+1}$ and the unemployment rate $ss \equiv \frac{1}{n+1}$, where $n+1$ is the size of the labor force. The long-run relationship between these two variables is given by $\dot{n} = 0$, i.e., by $m(v^*) = \theta n^*$, which is depicted by the downward sloping Beveridge curve, BEV. The increase in the tariff moves the economy from one steady state (point A) to another (point E). The transition of (vv, ss) traces a stylized counter-clockwise trajectory around the Beveridge curve (see Layard et al. (1991)), as depicted by the path ABE. At the instant when the tariff increases, the unemployment rate ss does not change, since n is pre-determined. In contrast, job vacancy immediately falls to the new long-run level, inducing an over-adjustment in the vacancy rate vv relative to its long-run adjustment.

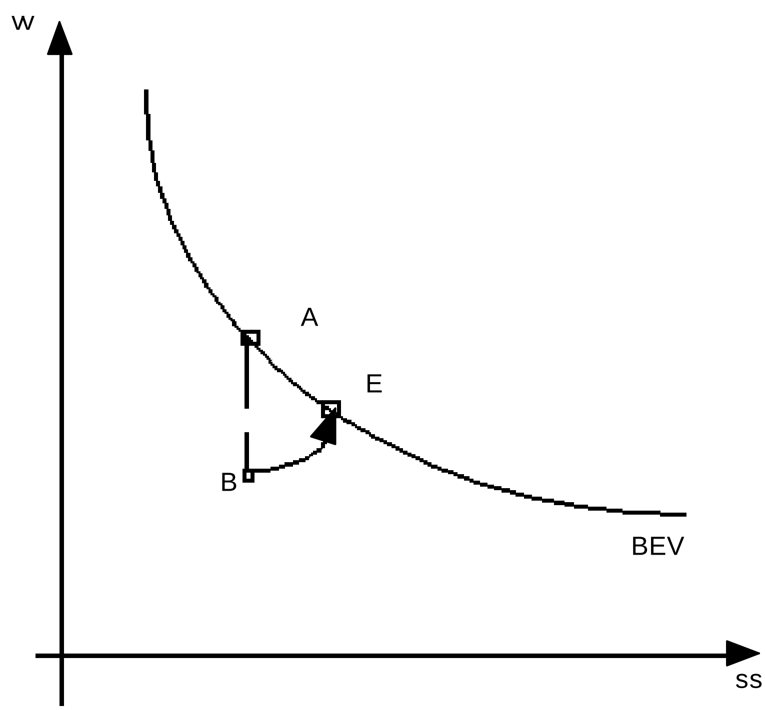

Figure 3.2:

This is the discontinuous drop from point A to point B in Figure 3.2. After this instantaneous change, the vacancy rate and the unemployment rate both rise gradually to reach the new steady state (point E) as employment falls to the new long-run level.

The results of this subsection can be summarized as follows:

Proposition 3.2. *When the country faces exogenous terms of trade, the tariff reduces employment, capital and output in both the long run and short run. The transition features a current account surplus and an over-adjustment (a fall) in the vacancy rate followed by increases in both the vacancy rate and the unemployment rate.*

3.3. A Permanent Improvement in the Terms of Trade

In this subsection I examine the effect of an exogenous improvement in the terms of trade, which generates an exogenous direct product wage effect. The purpose of the exercise is to highlight the conflict between the consumption bundle effect illustrated in the last subsection and the direct product wage effect.

Suppose that the economy is in a steady state at time 0, with $q = q_0$ and $(n(0), K(0), F(0)) = (n_0, \kappa n_0, F_0)$. The terms of trade then have an unanticipated, once-and-for-all (marginal) increase to q^*. Like the tariff in the last section, the terms-of-trade improvement makes the consumption bundle more expensive. This generates the consumption bundle effect that increases the product wage for any given q. The terms-of-trade improvement also directly reduces the product wage for any given ϕ. This direct product wage effect increases vacancy. Overall, the product wage falls if and only if the direct product wage effect outweighs the consumption bundle effect.

The conflict between the two effects can be illustrated with Figure 3.1 (where the corresponding shifts of the curves for the current case are not drawn). The direct product wage effect shifts the aggregate supply curve VV down to the right. That is, for any given marginal value of wealth ϕ, a higher value of q increases the firm's surplus from hiring and increases vacancy. The consumption bundle effect corresponds to a downward shift of the aggregate demand curve FF to the left. That is, for any given ϕ, an increase in q increases the price of the goods bundle and reduces the amount agents can consume; to maintain the equilibrium, job vacancy must fall to reduce the supply accordingly.[11] Overall, the marginal value of wealth is unambiguously lower in the new steady state than in the old one, but vacancy can be either higher or lower in the new steady state. Vacancy increases only if the aggregate supply curve VV shifts downward by more than the aggregate demand curve FF does.

Whether the direct product wage effect dominates the consumption bundle effect depends on the elasticity of intertemporal substitution, σ. The larger the elasticity of intertemporal substitution, the weaker the consumption bundle effect and the more likely that the direct product wage effect dominates the consumption bundle effect. The explanation is as follows. When the elasticity of intertemporal substitution is large, the consumption smoothing motive is weak. In this case consumption on the goods bundle falls a lot in response to the increase in the goods price. The resulted increase in the marginal utility of consumption mitigates the rise in price and so the marginal utility of wealth $\phi = u'(c)/p$ falls very little, leading to a small consumption bundle effect. In contrast, when the elasticity of intertemporal substitution is small, consumption of the goods bundle falls very little in response to the price increase, leaving the marginal utility of wealth to fall significantly.

The above explanation can be supported by showing that the terms-of-trade improvement raises job vacancy if and only if

$$\sigma > qx/f. \tag{3.6}$$

Whether this condition is satisfied clearly depends on the nature of the economy. There are realistic economies that satisfy (3.6). For example, if the value of the export is 70% of the import, (3.6) would require the elasticity of intertemporal substitution to exceed 0.7, which is possible with some of the estimates in Epstein and Zin (1991). Despite this possibility, we will show in the next section that the terms-of-trade improvement induced by a tariff is not sufficient to produce a dominant, direct product wage effect.

The dynamic responses of (n, F) to the terms-of-trade improvement can be analyzed using Figure 3.3. The lines STP and STP' depict the stable path, given by (3.3), before and after

[11] Precisely, the FF curve shifts downward to the left if and only if $G^* - B(v^*) > (1-\sigma)d^*$, or equivalently, $x + \sigma d > 0$, which is easily satisfied if the country exports a positive quantity of goods.

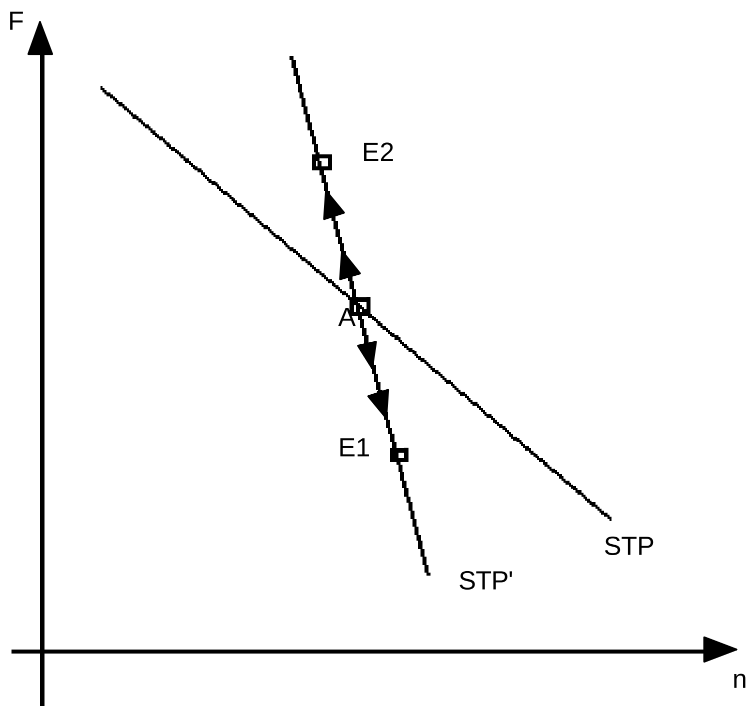

Figure 3.3:

the terms-of-trade improvement. Since the slope of the stable path depends positively on q^*, the terms-of-trade improvement increases the slope of the stable path. The initial steady state is at point A. The dynamics depend on whether (3.6) is satisfied. If (3.6) is satisfied, the new steady state is at point $E1$, in which case employment increases and the current account is in a deficit along the adjustment path. If (3.6) is violated, the new steady state is at point $E2$, in which case employment falls and the current account is in a surplus along the adjustment path.

4. The Case of Endogenous Terms of Trade

I now examine the dynamic effects of a tariff when the terms of trade are endogenous. A global dynamic analysis like the one in the last section is no longer possible and hence only local dynamics are considered. As in the last section, let the tariff change be a marginal, permanent and unanticipated increase from the initial value 0. The economy is in a steady state prior to the tariff change, with $(n(0), K(0), F(0)) = (n_0, \kappa n_0, F_0)$.

4.1. Characterization of the Stable Dynamic Path

With endogenous terms of trade, the dynamic system consists of seven variables, $(r, -, q, v, n, K, F)$. Solving for equilibrium dynamics is analytically possible only when $\alpha = 1-\lambda$, which will be assumed thereafter as in Merz (1995) and Andolfatto (1996). This condition requires that the firm's power in wage bargaining, $1-\lambda$, exactly compensates for the contribution of the vacancy to the match formation, measured by α (see Hosios (1990)). Without losing the essence of the analysis, I will also restrict the marginal adjustment cost of investment to be sufficiently flat around the steady state, i.e., $Q''(0) \approx 0$ and so $Q(i) \approx i$. In this case the marginal value of capital, - , is close to 1 and so the rental rate of capital is close to the marginal product of capital. The dynamics of the other five variables (q, v, n, K, F) can be approximated by the following system:[12]

$$(E) \quad \begin{cases} \dot{q} = q(\rho - G_1) \\ \dot{v} = \gamma \left[(\theta + G_1)v - \frac{(1-\lambda)m(v)}{B'(v)}(G_2 - \frac{\beta}{q\phi}) \right] \\ \dot{n} = m(v) - \theta n \\ \dot{K} = G - [d + x(q)] - B(v) \\ \dot{F} = \rho F + qx(q) - f. \end{cases}$$

The initial conditions are $(n(0), K(0), F(0)) = (n_0, \kappa n_0, F_0)$, where κ is the steady state capital-labor ratio, given by $G_1(\kappa) = \rho$. The conditions for $\dot{q}$ and $\dot{v}$ are derived from (2.2) and (2.19) by replacing r with its proxy G_1. The conditions for $\dot{n}$ is a copy of (2.5). The condition for $\dot{K}$ comes from the goods market clearing condition (2.18) using the approximation $Q(i) \approx i$. The condition for F is derived by substituting $(L, \pi, \dot{K})$ into (2.6). Since the variables (d, f) are functions of (q, τ, ϕ) (see (2.3)), the system (E) is a complete dynamic system of the five variables (q, v, n, K, F) once ϕ is determined. According to (2.7), ϕ is constant along the equilibrium dynamic path. Its value is determined through a stability requirement described later.

Denote world-wide consumption of the country's good by $D(q, \phi, \tau) \equiv d + x(q)$, where $d = p_1 u'^{-1}(p\phi)$. The earlier assumptions on x and p imply $D_1 < 0$. Appendix B shows that the dynamic system is saddle-path stable if the demand for the domestic good is sufficiently elastic (i.e., if D_1 is sufficiently negative). This result is not surprising since the dynamic system is stable when $D_1 = -\infty$, as demonstrated in the previous section. In particular, the dynamic system has two real, negative roots $\omega_2 < \omega_1 < 0$. Denote $Y = (q, v, n, K)^T$ and Y^* the steady state value of Y. The stable path is characterized as follows (see Appendix B).

Proposition 4.1. *The stable path of (E) is:*

$$Y(t) - Y^* = (Z_1, Z_2) \begin{pmatrix} b_1 e^{\omega_1 t} \\ b_2 e^{\omega_2 t} \end{pmatrix}, \tag{4.1}$$

$$F(t) - F^* = (n_0 - n^*)(\Gamma_2 e^{\omega_2 t} - \Gamma_1 e^{\omega_1 t}), \tag{4.2}$$

[12]To show that the dynamics of (E) approach the true dynamics when $Q''(0) \approx 0$, one can start with $Q'' > 0$, linearize the dynamics of $(r, - , q, v, n, K, F)$ and then take the limit $Q'' \to 0$ to show that the locally stable path of (q, v, n, K, F) in this dynamic system approaches that of system (E).

where Z_1 and Z_2 are 4×1 vectors and (b, Γ) are constants, both given in Appendix B, with $\Gamma_1 > \Gamma_2 > 0$ and $\omega_1 \Gamma_1 < \omega_2 \Gamma_2$.

4.2. Long-Run Effects of the Tariff

Let us first determine the steady state. Since the capital labor ratio is κ in both steady states before and after the tariff, steady state capital stock and employment always respond to the tariff in the same direction:

$$K^* - K_0 = \kappa(n^* - n_0). \tag{4.3}$$

Steady state employment is $n^* = m(v^*)/\theta$, which depends on steady state vacancy. Steady state vacancy in turn depends on the terms of trade and the marginal value of wealth. In particular, (2.20) holds in the steady state, which can be used to solve v^* as an increasing function of (ϕ, q^*). Denote this function as $v(\phi, q^*)$. Steady state terms of trade and the marginal utility of consumption are determined by the market clearing conditions for the domestic good and the condition for the country's balance of payments.

The domestic good market clearing condition is given by the $\dot{K}$ equation in (E). Setting $\dot{K} = 0$ and substituting the function $v(\phi, q^*)$ yields the following equation for (ϕ, q^*):

$$\frac{G}{n} \cdot \frac{m(v(\phi, q^*))}{\theta} - B(v(\phi, q^*)) - p_1 u'^{-1}(p\phi) - x(q^*) = 0. \tag{4.4}$$

The left-hand side of this equation is the excess supply of the domestic good. Note that G/n is an exogenous constant in the steady state. Eq. (4.4) gives a negative relation between steady state terms of trade and the marginal utility of wealth, depicted by the HH curve in Figure 4.1. A higher marginal utility ϕ decreases the reservation wage, increases vacancy and the supply of the domestic good. To clear the market for the domestic good, the price of the domestic good must fall.

The condition for the country's balance of payments is given by the $\dot{F}$ equation in the system (E). Setting $\dot{F} = 0$ and substituting F^* from the version of (4.2) at $t = 0$ gives the following equation for (ϕ, q^*):

$$p_2 u'^{-1}(p\phi) - q^* x(q^*) - \rho \left\{ F_0 - \delta \left[\frac{1}{\theta} m(v(\phi, q^*)) - n_0 \right] \right\} = 0, \tag{4.5}$$

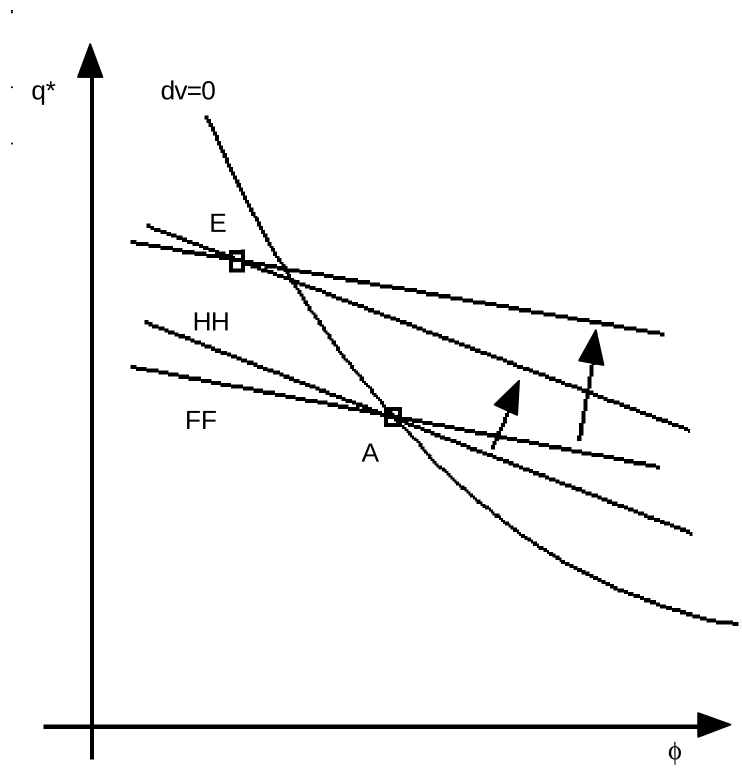

Figure 4.1:

where $\delta \equiv \Gamma_1 - \Gamma_2 > 0$. The left-hand side of this equation is the country's current account deficit in the steady state. The net of the first two terms is the net import. The last term is the interest receipts from foreign asset claims, where the change in the asset position is accounted for by the term $\delta[\cdot]$.

Eq. (4.5) gives an ambiguous relationship between steady state terms of trade and the marginal utility of consumption. To see the ambiguity, note first that a higher ϕ increases the supply of the country's export through increased vacancy and output, which must be absorbed by a fall in the relative price of the country's goods – the terms of trade. However, since the demand for the import is $f = p_2 u'^{-1}(p\phi)$, a higher ϕ also reduces the demand for the foreign good and its relative price $1/q^*$. When the intertemporal elasticity of substitution σ is small, the second effect is small and dominated by the first effect and so (4.5) gives a positive relationship between q^* and ϕ. Otherwise the relationship is negative, as depicted in Figure 4.1 by the FF curve.

Regardless of the nature of the slope of the FF curve, there is a unique solution for (ϕ, q^*). In particular, when the FF curve is negatively sloped, Appendix C shows that the HH curve is steeper than the FF curve, as depicted by Figure 4.1. Since the analysis with a positively sloped FF curve is similar to that with a negatively sloped FF curve, I will analyze only the case of a negatively sloped FF curve. Figure 4.1 also draws a reference curve $dv = 0$, along which the reservation wage is fixed at the level of the original steady state. That is, $q\phi$ is constant in the steady state along the curve $dv = 0$. Steady state values of (q, ϕ) before the increase in the tariff are given by point A. Points above the $dv = 0$ curve have more vacancies and higher employment than in the initial steady state and points below the $dv = 0$ curve have fewer vacancies and lower employment.

The long-run effect of a tariff on employment is summarized below (see Appendix C for a proof):

Proposition 4.2. *Under the assumption that the foreign demand for the country's good is sufficiently elastic in the sense that (B.2) in Appendix B holds, a permanent increase in the tariff reduces steady state vacancy, employment and output.*

The negative employment effect of the tariff can be illustrated with Figure 4.1. The increase in the tariff shifts the FF curve up because, for any given ϕ, the tariff reduces the demand for the import. The resulted current account surplus must be eliminated in the steady state by a terms of trade improvement, which increases the demand for the import and reduces the export. The tariff also shifts the HH curve up because, for any given ϕ, the tariff increases the demand for the domestic good. The resulted excess demand for the domestic good must be eliminated in the steady state by a terms of trade improvement, which stimulates domestic production and curtails the country's demand for the domestic good. The new levels of (q^*, ϕ) are given by point E. Appendix C shows that the upward shift of the FF curve is more than the upward shift of the HH curve and the new steady state is below the curve $dv = 0$. Thus, although the terms of trade improve as a result of the tariff, the improvement is proportionally less than the fall in the marginal utility of wealth. The consumption bundle effect of the tariff on the product wage through the marginal utility of wealth dominates the direct product wage effect through the terms of trade. The product wage rises and so the tariff reduces steady state vacancy and employment.

An explanation for why the consumption bundle effect dominates the direct product effect is that the improvement in the terms of trade induced by the increase in the tariff is proportionally less than the increase in the tariff itself. As a result, the price index increases by more proportionally than do the terms of trade. When the consumption smoothing motive is strong (i.e., $\sigma < 1$), most of the increase in the price index must be absorbed by the reduction in the marginal value of wealth (since $\phi = u'(c)/p$) and so the latter falls by more proportionally than the improvement in the terms of trade. Therefore, $q\phi$ falls and the product wage rises.

A more elaborate explanation relies on the following corollary (the omitted proof is a direct computation using (C.2) in Appendix C):

Corollary 4.3. *For any given marginal utility of wealth ϕ, the following inequalities hold:*

$$0 < \partial(f^* - q^*x^* - \rho F^*)/\partial q^* < q^*\partial(G^* - d^* - x^* - B^*)/\partial q^*, \tag{4.6}$$

$$\frac{-\partial(f^* - q^*x^* - \rho F^*)/\partial\tau}{\partial(f^* - q^*x^* - \rho F^*)/\partial q^*} > \frac{-\partial(G^* - d^* - x^* - B^*)/\partial\tau}{\partial(G^* - d^* - x^* - B^*)/\partial q^*} > 0. \tag{4.7}$$

Eq. (4.6) states that, in response to a terms-of-trade improvement, the current account deficit increases by less than does the excess supply of the domestic good. This is because, for fixed ϕ, the excess supply of the domestic good increases in response to an improvement in the terms of trade not only through the reduction in the demand for the domestic good, as does the current account deficit, but also through an increase in the supply when hiring increases (since wage falls for fixed ϕ). Eq. (4.7) states, as a consequence of (4.6), that to eliminate the current account surplus generated by the tariff requires a larger terms-of-trade improvement than to eliminate the excess demand for the domestic good. In Figure 4.1, this means that the upward shift in the FF curve must be larger than that of the HH curve, moving the steady state below the locus $dv = 0$. The product wage thus rises and employment falls in the steady state.

4.3. Dynamic Effects of the Tariff

The dynamic effects of the tariff on employment and capital can be obtained by differentiating with respect to time the stable path in (4.1) for these variables. The dynamics are illustrated in Figure

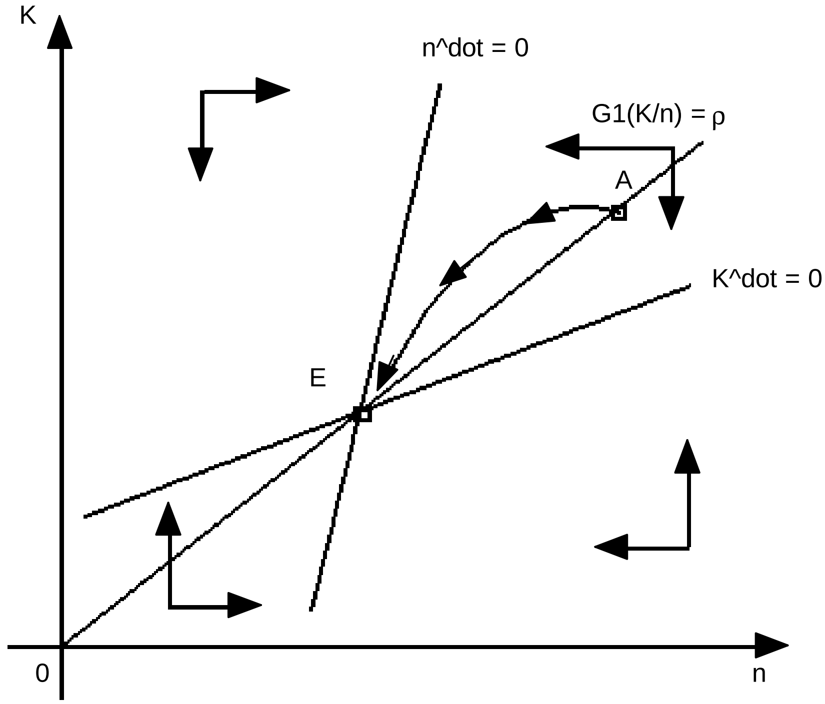

Figure 4.2:

4.2, where the long-run capital-labor ratio lies along the line $G_1 = \rho$. The explicit expressions for the $\dot{n} = 0$ and the $\dot{K} = 0$ schedules are provided in Appendix C, which establishes the following features: (i) both the $\dot{n} = 0$ schedule and the $\dot{K} = 0$ schedule are positively sloped; (ii) the $\dot{n} = 0$ schedule is steeper than the line $G_1 = \rho$ and steeper than the $\dot{K} = 0$ schedule; (iii) the $\dot{K} = 0$ schedule can be either steeper or flatter than the line $G_1 = \rho$. To avoid repetition, I discuss only the case where the $\dot{K} = 0$ schedule is flatter than the line $G_1 = \rho$.

The initial steady state is point A and the new steady state is point E, both lying on the line $G_1 = \rho$. The tariff induces the capital stock and employment to fall monotonically toward the steady state E. Output also falls monotonically. The dynamics of the job vacancy rate and the unemployment rate can be analyzed using Figure 3.2 but are omitted here. It is clear that the unemployment rate increases monotonically along the transition path and so the tariff has qualitatively the same effect on the unemployment rate both in the short run and in the long run.

Since the domestic capital stock monotonically falls, investment is re-directed toward foreign assets during the transition. That is, the current account is in surplus along the transitional path and reaches zero in the new steady state. This can be verified by differentiating (4.2) with respect to time to obtain the following expression for the current account:

$$\dot{F}(t) = (n_0 - n^*)(\Gamma_2 \omega_2 e^{\omega_2 t} - \Gamma_1 \omega_1 e^{\omega_1 t}). \tag{4.8}$$

Since $\omega_2 < \omega_1 < 0$, $\omega_1 \Gamma_1 < \omega_2 \Gamma_2 < 0$ and $n^* < n_0$, we have $\dot{F}(t) > 0$ for all t.

An interesting feature of the dynamics is that the terms of trade respond to the tariff in a non-monotonic fashion. To see this non-monotonic adjustment, notice first that $q(0) < q^*$ (see Appendix C). That is, the immediate improvement in the terms of trade after the increase in the tariff is less than in the long run. After this immediate improvement, the terms of trade continue to improve in order to maintain the arbitrage condition (2.2), since the rising capital labor ratio in the earlier stage of the transition (see Figure 4.2) pushes down the domestic interest rate. In this process the terms of trade overshoot the new long-run level. In the middle of the transition, the capital labor ratio begins to fall, which pushes up the domestic interest rate and induces the terms of trade to deteriorate toward the new long-run level. The complete adjustment of the terms of trade is characterized by an immediate jump which is followed by a continuous, hump-shaped path.

This particular adjustment path of the terms of trade implies overshooting product wage and job vacancy. Since the terms of trade rise immediately by less than in the long-run and the marginal value of wealth ϕ falls immediately to the new long-run level, the reservation wage, $\beta/(q\phi)$, must immediately overshoot its new steady state level. Since both the capital stock and employment are predetermined, the marginal product of labor is predetermined and so the product wage must immediately overshoot its new steady state level. After this overshooting, the product wage rate falls toward its new steady state level. As the product wage overshoots, job vacancy immediately falls below its long-run level.

5. Welfare Effects of the Tariff

Let us first examine the welfare effects of the tariff employing a commonly used welfare criterion – the steady state utility level. The tariff affects steady state utility in two ways. First, by reducing long-run employment, the tariff increases leisure and hence utility in the steady state. Second, the tariff changes steady state consumption of the goods bundle. The second effect is ambiguous, depending on the elasticity of the foreign demand for the country's export. For any finitely elastic export function, the response of steady state consumption to the tariff is:

$$c_\tau^* = \frac{\sigma cq/p}{(-DT)} \cdot \left[p_2(x + qx')E_1 - p_2 x' E_2 + cpp_{12}x\right],$$

where the subscript τ denotes the change of the variable with respect to the tariff and $DT < 0$, $E_1 > 0$ and $E_2 > 0$ are given in Appendix C. Thus, if the export function has a unit elasticity ($x + qx' \to 0$), then $c_\tau^* > 0$. In contrast, if the export function is infinitely elastic as in Section 3, then differentiating (3.5) yields:

$$c_\tau^* = \frac{q}{p}\left(\frac{G_2}{\theta + \rho} m' - B'\right) v_\tau^* = \frac{\beta\theta}{(\theta + \rho)u'} n_\tau^* < 0, \tag{5.1}$$

where the second equality comes from substituting G_2 from (2.20).

Surprisingly, in both cases the increase in leisure is strong enough to increase steady state utility. The increase in utility is obvious when the export function has a unit elasticity, since both steady state consumption and leisure increase with the tariff there. When the export function is infinitely elastic, the discounted presented value of steady state utility responds to the tariff as follows:

$$U_\tau^* = \frac{1}{\rho}(u' c_\tau^* - \beta n_\tau^*) = -\frac{\beta}{\theta + \rho} n_\tau^* > 0. \tag{5.2}$$

It is tempting to draw a similarity between this positive welfare effect and the optimal tariff literature (e.g. Johnson (1951-52)), where increasing tariffs from the level zero can improve welfare by improving the terms-of-trade. Such a similarity is superficial and misleading for two reasons. First, the tariff increases steady state welfare in the above cases primarily by increasing consumption of the non-traded good, i.e., leisure, which is not essential in the traditional optimal tariff argument. Second, unlike any static trade models, here capital can freely flow between countries and it is the intertemporal wealth accumulation decision that ultimately determines the long-run consumption level. Thus, even though the terms of trade remain unchanged in the case where the export function is infinitely elastic, the tariff reduces steady state consumption of the goods bundle.

Now let us employ the correct measure of welfare – intertemporal utility. The steady state utility is a misleading criterion of welfare because it ignores the cost of adjusting toward the steady state. In the current context, steady state utility gives the tariff a welfare effect that is biased toward the positive side. To see this, linearizing (2.4) near the steady state, we have:

$$U_\tau^* - U_\tau = \sigma c \phi p_1 \int_0^\infty [q_\tau(t) - q_\tau^*] e^{-\rho t} dt + \beta \int_0^\infty [n_\tau(t) - n_\tau^*] e^{-\rho t} dt,$$

where the relation $u'(c) = p\phi$ is used to express $c_\tau - c_\tau^*$ as a function of $q_\tau - q_\tau^*$. The right-hand side measures the dynamic welfare cost associated with the transition. To calculate the two integrals, the differences $q_\tau - q_\tau^*$ and $n_\tau - n_\tau^*$ can be calculated using (4.1) if $-x' < \infty$ and using (3.2) if $-x' = \infty$. Since employment monotonically decreases along the transition path, $n_\tau(t) > n_\tau^*$ for all $t \geq 0$ and so the second integral is positive. The first integral is also likely to be positive. This is because, as discussed above, the terms of trade overshoot the steady state level in the transition and so $q_\tau(t) > q_\tau^*$ in a large part of the transition. Thus, $U_\tau^* > U_\tau$ and so the welfare effect of the tariff is less than the steady state effect.

It is possible that the cost of transition can completely eliminate the steady state welfare gain. Consider the case $-x' = \infty$ for example. In this case the terms of trade do not change and so the first integral is zero. For changes in employment, we can use (3.2) to compute:

$$n_\tau(t) - n_\tau^* = -n_\tau^* e^{-\theta t}.$$

Then the cost associated with employment transition can be calculated as $-\beta n_\tau^*/(\theta + \rho)$, which exactly equals the steady state welfare gain, U_τ^* (see (5.2)). Thus, when the export function is infinitely elastic, there is no welfare gain from increasing the tariff from the level zero despite that the tariff increases consumption of the non-traded good. It would be nice if this type of calculation can be carried out for the general case where the country has some limited influence on the terms of trade. Unfortunately, the analytical result is not revealing and so the welfare effect of the tariff remains ambiguous in this case.

6. Conclusion

This paper integrates labor market search into a dynamic general equilibrium model to analyze the macroeconomic effects of a tariff. The search friction creates a wedge between the marginal product of labor and the product wage. With perfectly flexible prices and wages, the model captures the intuitive effect that a permanent increase in the tariff improves the country's terms of trade, which tends to reduce the product wage directly through the reservation wage and stimulates labor demand. However, the tariff also increases the price of the consumption goods bundle, reduces the marginal utility of wealth and increases the product wage through the reservation wage. With a realistically strong consumption smoothing motive, this consumption bundle effect of the tariff dominates the direct product wage effect, causing vacancy and employment to fall both in the long run and the short run. Thus, even with persistent unemployment, raising tariffs is not the means in which a government in a small open economy can succeed in increasing employment, short run or long run. International finance theorists who argue for a predominant positive employment role for tariffs must look for other labor market frictions to support their arguments.

There might be *ad hoc* rationales for a government to increase tariffs (in practice). The current paper indicates two. One is redistributional: The government might want to boost the product wage. An increase in the tariff achieves this purpose and does so in a larger scale in the short-run than in the long-run. The second reason might be current account management: An increase in the tariff produces a current account surplus along the entire transition path. However, neither rationale has a sound justification. In particular, even though a tariff increases the product wage, it is unlikely to raise workers' standard of living, because the wage measured in terms of the goods bundle is likely to fall.

For tractability, the paper has abstracted from the possible strategic responses by the rest of the world to the increase in the tariff. This omission is not as serious as it appears. Although the tariff deteriorates the terms of trade of the rest of the world, it also increases the capital flow into the rest of the world. Since it is not clear whether the rest of the world stands to lose or gain from the tariff, it is not clear whether it has the incentive to retaliate. Addressing the strategic interaction requires a two-country model and may be worth pursuing in the future research. As far as the small country is concerned, a possible short-cut to modelling the response of the rest of the world would be to assume that the export function, $x(q)$, depends on the tariff. In particular, one can view that a tariff may trigger responses that make the export function more elastic. This will exacerbate the negative effect of that tariff on the country's employment.

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Appendix

A. Proof of Proposition 3.1

Since v is constant along the transition path, $n(t)$ can be solved directly by integrating the equation for n. The result is given by (3.2). Substitute this solution into (3.1) and notice that G/n depends only on the capital-labor ratio and hence is constant along the transition path. Integrating (3.1) generates:

$$\begin{aligned} F(t) &= F^* - \frac{q}{\theta+\rho}\left(\frac{G}{n}+\theta\kappa\right)\left(n(t)-\frac{m}{\theta}\right) \\ &\quad + \left[F_0 - F^* + \frac{q}{\theta+\rho}\left(\frac{G}{n}+\theta\kappa\right)\left(n_0-\frac{m}{\theta}\right)\right]e^{\rho t}, \end{aligned}$$

For F to converge to a steady state, it is necessary and sufficient that the second term is zero for all t and this holds when

$$F_0 - F^* = -\frac{q}{\theta+\rho}\left(\frac{G}{n}+\theta\kappa\right)\left(n_0-\frac{m}{\theta}\right).$$

Under this condition, the solution for $F(t)$ given above becomes (3.3). ■

B. Proof of Proposition 4.1

Since the dynamics of $Y = (q, v, n, K)^T$ are autonomous for any given ϕ, let us examine them first. Linearizing the dynamic equations for Y in (E) yields:

$$\dot{Y} = J(Y - Y^*), \tag{B.1}$$

where Y^* is the steady state value of Y and J is the following matrix:

$$J = \begin{bmatrix} 0 & 0 & -qG_{12} & -qG_{11} \\ -\frac{\gamma A\beta}{\phi q^2} & \theta+\rho & \gamma G_{12}(v+A\kappa) & \gamma G_{11}(v+A\kappa) \\ 0 & m' & -\theta & 0 \\ -D_1 & -B' & G_2 & \rho \end{bmatrix}.$$

Here $A = \alpha m/B' > 0$ and all elements in the matrix are evaluated at the steady state with $\tau = 0$.

Denote a typical eigenvalue of matrix J by ω and let $\xi = \omega(\omega - \rho)$. The determinant of matrix J can be expressed as the following quadratic function of ξ:

$$\begin{aligned} g(\xi) \equiv & \; \xi^2 - [\theta(\theta+\rho) + qD_1G_{11} - \gamma G_{11}(v + A\kappa)(B' + \kappa m')]\xi \\ & + \theta(\theta+\rho)qD_1G_{11} - \gamma G_{11}\frac{A\beta}{q\phi}[m'(G_2 + \rho\kappa) - \theta B']. \end{aligned}$$

Denote the two solutions to the equation $g(\xi) = 0$ by ξ_1 and ξ_2. These roots are real numbers if and only if

$$\begin{aligned} 0 < & \; [\theta(\theta+\rho) + qD_1G_{11} - \gamma G_{11}(v + A\kappa)(B' + \kappa m')]^2 \\ & -4\theta(\theta+\rho)qD_1G_{11} + 4\gamma G_{11}\frac{A\beta}{q\phi}[m'(G_2 + \rho\kappa) - \theta B']. \end{aligned}$$

The right-hand side of the above inequality can be equivalently written as:

$$\begin{aligned} & [\theta(\theta+\rho) - qD_1G_{11} - \gamma G_{11}(v + A\kappa)(B' + \kappa m')]^2 \\ & -4\gamma G_{11}[qD_1G_{11}(v + A\kappa)(B' + \kappa m') - \frac{A\beta}{q\phi}(m'(G_2 + \rho\kappa) - \theta B')], \end{aligned}$$

and so a sufficient condition for the inequality to hold is

$$qD_1G_{11} > \frac{A\beta}{\phi q} \cdot \frac{m'(G_2 + \rho\kappa) - \theta B'}{(v + A\kappa)(B' + \kappa m')}. \tag{B.2}$$

This condition requires the export function to be sufficiently elastic, as $-D_1$ increases with $-x'$. The condition (B.2) is maintained throughout. Then (ξ_1, ξ_2) are positive and distinct. Let $\xi_1 < \xi_2$. It can be shown that $g(qD_1G_{11}) < 0$ and so $qD_1G_{11} \in (\xi_1, \xi_2)$.

Since ξ_1 and ξ_2 are positive, matrix J has two positive real eigenvalues and two negative real eigenvalues, calculated through the equations $\omega(\omega - \rho) = \xi_i$ $(i = 1, 2)$. The two negative real eigenvalues are $\omega_i \equiv [\rho - (\rho^2 + 4\xi_i)^{1/2}]/2$, $i = 1, 2$. Clearly, $\omega_2 < \omega_1 < 0$. The property $qD_1G_{11} \in (\xi_1, \xi_2)$ can then be written as

$$\omega_1(\omega_1 - \rho) < qD_1G_{11} < \omega_2(\omega_2 - \rho). \tag{B.3}$$

The number of negative eigenvalues of J (two) falls short of the number of pre-determined variables (n, K, F) in the system (E) by one, leaving the stable path of the equilibrium dependent on the initial conditions.

The stable manifold of Y is given by (4.1), where Z_i is the eigenvector of J corresponding to ω_i and is given as follows:

$$Z_i = \begin{pmatrix} z_{i1} \\ z_{i2} \\ z_{i3} \\ z_{i4} \end{pmatrix} = \begin{pmatrix} \frac{-qG_{11}}{\xi_i - qD_1G_{11}} \left[\frac{\beta}{q\phi} + (\kappa + \frac{B'}{m'})(\rho - \omega_i)\right] \\ (\omega_i + \theta)/m' \\ 1 \\ \frac{1}{\xi_i - qD_1G_{11}} \left[qD_1G_{12} - \frac{B'}{m'}\xi_i + \frac{\beta}{q\phi}\omega_i\right] \end{pmatrix}. \tag{B.4}$$

To determine (b_1, b_2) in (4.1), set $t = 0$ and use (4.3). We have

$$\begin{pmatrix} b_1 \\ b_2 \end{pmatrix} = \frac{(n_0 - n^*)}{z_{14} - z_{24}} \begin{pmatrix} \kappa - z_{24} \\ \\ z_{14} - \kappa \end{pmatrix}. \tag{B.5}$$

Thus, (b_1, b_2) are uniquely determined for any given ϕ if and only if $z_{14} \neq z_{24}$. Compute:

$$\begin{aligned} z_{14} - z_{24} \quad = \quad & -\frac{(\omega_1 - \omega_2)}{(\xi_1 - qD_1G_{11})(\xi_2 - qD_1G_{11})} \\ & \times \left\{ qD_1G_{11} \left[(\kappa + \frac{B'}{m'})(\rho - \omega_1 - \omega_2) + \frac{\beta}{q\phi}\right] + \omega_1\omega_2 \frac{\beta}{q\phi} \right\}. \end{aligned}$$

This is positive, because $qD_1G_{11} \in (\xi_1, \xi_2)$ and $0 > \omega_1 > \omega_2$. Therefore, the system (B.1) is stable for any given ϕ.

To find the stable path for F, linearize the $\dot{F}$ equation in (E), substitute the stable manifold (4.1), and integrate. Imposing the condition $\lim_{t\to\infty} F(t) = F^* < \infty$ yields (4.2), where

$$\begin{aligned} \Gamma_1 &= \frac{(z_{24} - \kappa)z_{11}}{(z_{14} - z_{24})(\rho - \omega_1)} \left[c(p_{12} - \frac{\sigma p_1 p_2}{p}) - (x + qx')\right], \\ \Gamma_2 &= \frac{(z_{14} - \kappa)z_{21}}{(z_{14} - z_{24})(\rho - \omega_2)} \left[c(p_{12} - \frac{\sigma p_1 p_2}{p}) - (x + qx')\right]. \end{aligned}$$

The path (4.2) at $t = 0$ also provides a condition which helps to determine ϕ.

To verify the features of $(\delta, \Gamma_1, \Gamma_2)$, note that $z_{11} < 0$, $z_{21} > 0$ and $z_{14} > \kappa > 0 > z_{24}$. Then, $\Gamma_1 > 0$ and $\Gamma_2 > 0$. Substituting $(z_{11}, z_{21}, z_{14}, z_{24})$ and using the notation $\delta = \Gamma_1 - \Gamma_2$ yields:

$$\begin{aligned} \delta \quad = \quad & \frac{[c(p_{12} - \sigma p_1 p_2/p) - (x + qx')]}{(-D_1)\left[\kappa + \frac{B'}{m'} + \frac{\beta}{q\phi}(1 + \frac{\omega_1\omega_2}{qD_1G_{11}})/(\rho - \omega_1 - \omega_2)\right]} \\ & \times \left[\frac{\beta}{q\phi} + (\kappa + \frac{B'}{m'})/(\rho - \omega_1)\right] \left[\frac{\beta}{q\phi} + (\kappa + \frac{B'}{m'})/(\rho - \omega_2)\right]. \end{aligned} \tag{B.6}$$

Since $p_{12} > p_1 p_2/p$, $\sigma \leq 1$, $x + qx' < 0$, and $D_1 < 0$, we have $\delta > 0$. Similarly one can verify $\omega_1\Gamma_1 < \omega_2\Gamma_2$. This completes the proof of Proposition 4.1. ■

C. Proofs of Proposition 4.2 and Other Statements in Section 4

In this appendix, we verify the following results used in Section 4: (i) The HH schedule is negatively sloped, while the FF schedule may be either positively or negatively sloped; (ii) The HH schedule is steeper than the FF schedule when the latter is negatively sloped; (iii) $dq^*/d\tau > 0$ and $d\phi/d\tau < 0$; (iv) Proposition 4.2: $dv^*/d\tau < 0$ and $dn^*/d\tau < 0$; (v) $q(0) < q^*$; and (vi) The dynamics of (n, K) are as described in the text and illustrated in Figure 4.2.

To show (i) – (v), differentiate (2.20) and suppress the asterisk associated with the steady state:

$$dv = \frac{\gamma v}{G_2 q\phi/\beta - 1}\left(\frac{dq}{q} + \frac{d\phi}{\phi}\right). \tag{C.1}$$

Denote

$$E_1 = \frac{v\gamma[Gm'/(n\theta) - B']}{G_2 q\phi/\beta - 1}, \quad E_2 = \frac{v\gamma\rho\delta m'/\theta}{G_2 q\phi/\beta - 1}.$$

Differentiating (4.4) and (4.5), substituting (C.1), we have:

$$\begin{bmatrix} E_1 - q[x' + c(p_{11} - \frac{\sigma p_1^2}{p})], & E_1 + \sigma c p_1 \\ E_2 + q[c(p_{12} - \frac{\sigma p_1 p_2}{p}) - (x + qx')], & E_2 - \sigma c p_2 \end{bmatrix} \begin{pmatrix} dq/q \\ d\phi/\phi \end{pmatrix} = c \begin{bmatrix} p_{12} - \frac{\sigma p_1 p_2}{p} \\ \frac{\sigma p_2^2}{p} - p_{22} \end{bmatrix} d\tau.$$

Since $p_{12} > p_1 p_2/p$, $\sigma \le 1$ and $x + qx' < 0$, it is clear that the elements of the above 2×2 coefficient matrix are positive, with the only possible exception for the element $E_2 - \sigma c p_2$. Thus the HH schedule is negatively sloped. The FF schedule is also negatively sloped if and only if $E_2 > \sigma c p_2$.

Denote the determinant of the above 2×2 coefficient matrix by DT. When the FF schedule is negatively sloped, the HH schedule is steeper than the FF schedule if and only if $DT < 0$. To verify $DT < 0$, notice $qp_1 + p_2 = p$, $qp_{11} = -p_{12}$ and $qp_{12} = -p_{22}$, all from the homogeneity of the price index p. Then we can compute

$$\begin{aligned} DT &= -\sigma c[cpp_{12} - q(px' + p_1 x)] + \frac{v\gamma}{G_2 q\phi/\beta - 1}\Delta \\ \Delta &= \rho\delta\frac{m'}{\theta}\left[c(p_{12} - \frac{\sigma p_1 p_2}{p}) - qx'\right] - \left(\frac{Gm'}{n\theta} - B'\right)\left[c(qp_{12} + \frac{\sigma p_2^2}{p}) - q(x + qx')\right]. \end{aligned}$$

A sufficient condition for $DT < 0$ is $\Delta < 0$, which can be verified using the following relations:

$$D_1 = x' - \frac{c}{q}\left(p_{12} + \frac{\sigma p_1^2}{p}q\right); \quad \frac{Gm'}{n\theta} - B' = \frac{\rho m'}{\theta}\left(\kappa + \frac{B'}{m'} + \frac{\beta}{q\phi}\rho^{-1}\right);$$

$$\delta < \frac{1}{(-D_1)}\left[c(p_{12} - \frac{\sigma p_1 p_2}{p}) - (x + qx')\right]\left[\frac{\beta}{q\phi} + (\kappa + \frac{B'}{m'})/(\rho - \omega_1)\right]. \tag{C.2}$$

With (C.2), one can also show that $dq^*/d\tau > 0$ and $d\phi/d\tau < 0$. Furthermore,

$$\frac{d(q\phi)}{d\tau} = \frac{cq^2\phi}{(-DT)}\left[\sigma p_2 x' - x\left(p_{12} - \frac{\sigma p_1 p_2}{p}\right)\right] < 0.$$

Thus, $dv^*/d\tau < 0$ and $dn^*/d\tau < 0$.

The inequality $q(0) < q^*$ can be verified directly using the equation for q in (4.1). This completes the proofs of (i) – (v).

For part (vi), differentiating the equations for (n, K) in (4.1) with respect to time yields

$$\begin{pmatrix} \dot{n} \\ \dot{K} \end{pmatrix} = \frac{1}{z_{14} - z_{24}}\begin{pmatrix} \omega_2 z_{14} - \omega_1 z_{24} & \omega_1 - \omega_2 \\ -(\omega_1 - \omega_2) z_{14} z_{24} & \omega_1 z_{14} - \omega_2 z_{24} \end{pmatrix}\begin{pmatrix} n - n^* \\ K - K^* \end{pmatrix}.$$

Notice that $z_{14} > 0$ and $z_{24} < 0$. It is then evident that the $\dot{n} = 0$ and $\dot{K} = 0$ schedules are both positively sloped. Since the coefficient matrix has two negative eigenvalues (ω_1 and ω_2), its determinant is positive and so the $\dot{n} = 0$ schedule is steeper than the $\dot{K} = 0$ schedule. The $\dot{n} = 0$ schedule is steeper than the line $G_1 = \rho$ if and only if

$$-\frac{\omega_2 z_{14} - \omega_1 z_{24}}{\omega_1 - \omega_2} > -\frac{G_{12}}{G_{11}} = \kappa,$$

which can be verified by substituting z_{14} and z_{24}. However, the $\dot{K} = 0$ schedule may or may not be steeper than the line $G_1 = \rho$. ■