The Variability of Velocity of Money in a Search Model*

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Abstract

We construct a dynamic search model to examine the behavior of velocity. The prominent feature of the model is costly search in both goods and labor markets. Incorporating money growth shocks and productivity shocks, we calibrate the model to the US time series data. Even though there is no substitution between money and credit or other assets, our model generates volatile velocity and a negative correlation between velocity and consumption growth. Both features are noticeably lacking in some other equilibrium monetary models. We also examine the propagation mechanism for the shocks and the correlations between endogenous variables.

Keywords: Velocity; Search; Inflation; Inventory; Propagation.

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1. Introduction

In this paper we analyze an economy where there is costly search is both the goods market and the labor market. Calibrating the economy to the US data, we examine whether this model can account for the variability of velocity of money, with shocks to money growth and productivity. We also analyze the propagation mechanism of the shocks in such a search economy.

Figures 1.1 and 1.2 here.

Velocity of money varies considerably over time in the US data. Figure 1.1 shows the level of consumption velocity of money in the period 59:I – 98:III, with $M1$ and $M2$ as monetary aggregates, and Figure 1.2 shows the variation in velocity of $M2$ around the mean. Changes in velocity are important in monetary economics, because they are indicative of how macroeconomic shocks affect aggregate real activities.[1] However, the large variability of velocity seems inconsistent with equilibrium monetary models. In a model where all consumption is purchased with cash in advance, velocity is unity and so it does not vary at all. The variability of velocity increases only slightly after credit goods and different information structures are incorporated into the cash-in-advance model, as Hodrick et al. (1991) find with calibration. To generate non-negligible volatility of velocity, their models require unrealistically high real interest rates. Even with a quarterly real interest rate as high as 2.6%, the coefficient of variation in velocity, defined as the standard deviation of velocity in percentage of the mean, is at most 1.66 for the period 59:II – 88:I in their models (Table 6 therein). The sample value is 4.48.[2]

We examine whether a search model might help explaining the variability of velocity. In this model, there is search in both the goods market and the labor market. The goods market functions like that in Kiyotaki and Wright (1991, 1993), where fiat money serves as a medium of exchange. The important exceptions are that money and goods are divisible and that search intensities are endogenous. In particular, buyers choose their search intensity, sellers choose inventory, and the

[1] See Friedman (1956) and Brunner and Meltzer (1963) for the discussions on the importance of velocity.

[2] Christiano (1991) finds that such a low variability of velocity extends to economies that exhibit the liquidity effect modelled by Lucas (1990).

aggregate number of trades is an increasing function of these two intensities. The labor market functions like that in Mortensen (1982) and Pissarides (1990), where the aggregate number of matches is an increasing function of the numbers of vacancies and unemployed workers. In both markets, bargaining determines the terms of trade between the two agents in a match.

Consumption velocity of money in our model is equal to the number of trades per buyer in the goods market. So, velocity is determined by a novel element, the "extensive margin" of trade, as opposed to the intensive margin which reflects the quantity of goods traded in each match. Under our specification of the matching function, velocity is an increasing function of buyers' search intensity and sellers' inventory. The extensive margin of trade provides a novel channel through which shocks make velocity vary. A positive shock to money growth, for example, reduces the real value of money and induces buyers to spend money quickly. To do so successfully in a non-Walrasian, buyers must search more intensively. Such high buyers' search intensity increases the number of trades per buyer, and hence increases velocity. A positive productivity shock, in contrast, reduces buyers' search intensity by increasing the supply of goods. Since inventory is pre-determined, the number of trades falls, and so does velocity. In future periods, however, velocity rises because the higher productivity increases inventory which leads to more trades.

To assess the model's quantitative predictions, we calibrate the model to the US time series data, with stochastic money growth and productivity. Our model generates mean values of consumption close to the sample value, without the introduction of credit goods. Velocity is also more volatile in our model than in Hodrick et al. (1991). For the quarterly sample 59:II – 88:I, the coefficient of variation in velocity is 1.91, with the relative risk aversion being 4. Although this coefficient is still smaller than the sample value (4.48), it is significantly higher than in Hodrick et al. (1991), considering that our model matches the quarterly real interest rate with the sample value. This higher volatility of velocity comes from the effects of shocks on the number of trades through inventory and buyers' search intensity, as discussed above.

The model also generates interesting correlations between endogenous variables. First, velocity and consumption growth are negatively correlated, as in the data, while such a correlation is either positive or indistinguishable from zero in Hodrick et al. (1991). This negative correlation arises in our model because consumption is the product of the number of trades and the quantity of goods traded in each match, while velocity depends exclusively on the former. When a shock affects these two margins of trade in opposite directions, it can change consumption growth and velocity in opposite directions. Second, the real interest rate and inflation are negatively correlated, where the real interest rate is defined as the intertemporal price of goods. This negative correlation indicates an interesting propagation mechanism. In particular, a positive money growth shock reduces future consumption by more than current consumption, while a positive productivity shock increases future consumption by more than current consumption.

Both the goods market search and labor market search are important for the propagation mechanism. In the case of a positive monetary growth shock, buyers' search intensity responds by the most at the time of the shock and then declines. The initial response in buyers' search intensity mitigates a large part of the negative response in the intensive margin of trade, thus preventing consumption from falling by a large quantity immediately. This mitigating force weakens as buyers' search intensity declines next period, and so consumption continues to fall. In the case of a positive productivity shock, costly search in the labor market prevents employment and output from increasing in large magnitudes immediately, thus creating the gradual rise in output and consumption.

Another interesting result is that productivity shocks affect the volatility of velocity by much more than money growth shocks do. In fact, money growth shocks explain only a small fraction of the variability of velocity. Because productivity shocks are highly persistent, they have persistent effects on inventory and the number of trades, and hence on velocity. The dominance of productivity shocks also yields the result that output has a larger variance than sales.

A large body of the literature on velocity focuses on the statistical relationship between velocity and other aggregate variables, which often relates to the estimation of the money-demand equation. For example, Brunner and Meltzer (1963) and Lucas (1988) find that velocity is a stable, increasing function of the nominal interest rate. Like Hodrick et al. (1991), we compute an equilibrium model and examine how shocks affect endogenous variables, including velocity and interest rates, simultaneously. This examination reveals the propagation mechanism, which is useful for economic analyses but difficult to be obtained from the money-demand estimation. In this sense our analysis is complementary to such estimation exercises. Moreover, we abstract from assets other than money and inventory. This is not because other assets are unimportant for the behavior of velocity, but rather because we want to focus on the role of search in explaining velocity, which is largely unknown.[3]

The labor market structure in our framework resembles a standard search model of unemployment, e.g. Mortensen (1982) and Pissarides (1990). The goods market structure extends a standard search money models, such as Shi (1995) and Trejos and Wright (1995), to allow for divisible money and inventory. These extensions are already done in Shi (1998). Relative to Shi (1998), the main contribution here is that we introduce stochastic productivity and money growth, in order to study the stochastic properties of velocity. We also adopt a more reasonable equilibrium concept.

2. The Description of the Economy

2.1. The Household and Matches

Time is discrete. There are a continuum of households with measure one and a continuum of types of goods, both being represented by points along a circle H. A household $h \in H$ is specialized in producing good h but consumes a subset of goods produced by other households. Agents get zero utility from consuming their own output. In each period agents meet with each other randomly according to a matching function described later. In this decentralized environment, fiat money can be valuable in facilitating exchanges and, in fact, every exchange involves one agent giving up money for the partner's goods.[4]

In contrast with a standard search model, but in common with Shi (1998), our model allows

[3]Gordon and Leeper (2000) and Gordon et al. (1998) compute general equilibrium models where there is a transaction technology in the goods market and agents choose the labor input in transaction versus production. Their focus is to examine how agents' expectations of monetary and fiscal policies affect the trend and cyclical features of velocity.

[4]See Kiyotaki and Wright (1991, 1993), Shi (1995) and Trejos and Wright (1995).

goods to be stored by (and only by) their producers, and such inventory depreciates at rate $\delta_i \in (0, 1)$. This modelling device makes it possible for shocks to be propagated through inventory. To investigate this particular propagation mechanism, we abstract from other types of investment by assuming that the stock of productive capital is fixed.[5]

Also in contrast with a standard search model, we allow the aggregate stock of money to grow or shrink over time and permit agents to exchange any feasible quantities of both money and goods in a match. This extension is necessary for examining money growth, but it can induce non-degenerate distributions of money holdings, consumption, and inventory across agents. To focus on the aggregate behavior and to maintain tractability, we abstract from such distributions using the assumption of large households introduced by Shi (1997, 1999). That is, each household consists of a continuum of agents who share consumption and regard the household's utility as the common objective. In this case, individual matching risks are smoothed out within each household. To the extent that idiosyncratic matching risks may increase consumption volatility, the risk-smoothing assumption may under-estimate the variability of velocity.[6]

With this modelling device, we can examine a symmetric equilibrium in which all households' decisions are the same, except for the types of goods they consume and produce. Let us pick an arbitrary household, of type h, as a "representative" household and describe its decisions in detail. Use lower-case letters to denote this household's decisions. Add a hat to other households' decisions or aggregate variables, which the representative household takes as given.

A household consists of five types of agents. One group of members enjoy leisure, while the other four types are active in the market. The active market participants are entrepreneurs (a fraction a_p of the household members), workers (a fraction $a_p \hat{n}_t$), unemployed agents (a fraction u), and buyers (a fraction a_b). An entrepreneur performs the dual tasks of a producer and a seller: he hires workers from other households to produce and sells output to other households. An unemployed agent searches for a job. A worker inelastically supplies one unit of labor per

[5]Later in the quantitative exercise we account for fixed investment by assuming that it is a fixed fraction of aggregate output. For the accumulation of productive capital in a search monetary model, see Shi (1999, 2001).

[6]This risk-smoothing assumption resembles the one used in other monetary models (Lucas 1990) and in labor economics (Rogerson 1988 and Hansen 1985).

period. A buyer searches to buy the household's desired good. The fractions (a_p, u, a_b) are constant. In contrast, the number of employed workers is endogenous. Changes in employment are accompanied by opposite changes in leisure. Since workers in a household work for other households' firms, the number of employed workers in a household is determined by other households' firms. So, it is $a_p \hat{n}_t$, rather than $a_p n_t$, where n is the number of workers employed by a firm. Let $B \equiv a_b/a_p$ be the ratio of buyers to sellers in the goods market.

The matching technologies are as follows. In the labor market, the total number of matches between entrepreneurs and workers is $(a_p \hat{v}_t)^A u^{1-A}$, where $A \in (0,1)$ and $\hat{v}_t$ is the number of vacancies per firm.[7] The matching rate per vacancy is

$$\mu(\hat{v}_t) \equiv [u/(a_p \hat{v}_t)]^{(A-1)}. \tag{2.1}$$

The matching rate per unemployed worker is $(a_p \hat{v}_t/u)^A$.

The matching function in the goods market is described similarly. Let $\hat{s}_t$ be the search intensity per buyer in period t and $\hat{\imath}_t$ the inventory of goods per seller at the beginning of t. Call a match between a buyer and a seller a trade match if the seller can produce the buyer's desirable goods. The total number of trade matches is:

$$\hat{g}_t \equiv z_0 (a_b \hat{s}_t)^\alpha [a_p \psi(\hat{\imath}_t)]^{1-\alpha}, \qquad \alpha \in (0,1). \tag{2.2}$$

The function $\psi(\hat{\imath}_t)$, explained later, converts a seller's inventory into the selling intensity. The matching rate for each unit of selling intensity is

$$\hat{g}_s(\hat{s}_t, \hat{\imath}_t) \equiv \frac{\hat{g}_t}{a_p \psi(\hat{\imath}_t)} = z_0 z_t^\alpha, \;\; z_t \equiv (a_b \hat{s}_t)/[a_p \psi(\hat{\imath}_t)], \tag{2.3}$$

where z_t is the relative search intensity of buyers to sellers. Similarly, the matching rate for each unit of a buyer's search intensity is

$$\hat{g}_b(\hat{s}_t, \hat{\imath}_t) \equiv \frac{\hat{g}_t}{a_b \hat{s}_t} = z_0 z_t^{\alpha-1}. \tag{2.4}$$

[7] The Cobb-Douglas matching function is often used in the search model of unemployment, see Blanchard and Diamond (1989) for empirical evidence supporting the specification. Also, we assume that unemployed workers' search intensity is inelastic, which is not far from reality (see Layard et al. 1991).

The function $\psi(i)$ requires an explanation. Its appearance in the matching function captures the intuitive idea that the number of trade matches depends on inventory per seller, as well as the number of sellers in the market. This is similar to the idea that the number of matches in the labor market depends on the number of vacancies per firm as well as the number of firms. The function ψ can be interpreted as the number of shops or warehouses per seller that are stocked up. High inventory reduces the probability of stock-out, and hence increases the probability of trade for the seller. We capture this benefit of inventory to successful trades by assuming $\psi' > 0$, which is necessary for there to be a non-trivial level of inventory in the steady state. We also impose $\psi'' < 0$, for the benefit of inventory to successful trades diminishes at the margin.

Table 1: Statistics of Matches

	rate of matches	measure of agents	quantities supplied in each trade
entrepreneurs	$\mu(\hat{v}_t)$	a_p	w_t units of real money
unemployed agents	$a_p\hat{v}_t\mu(\hat{v}_t)/u$	$a_p\hat{v}_t\mu(\hat{v}_t)$	one unit of labor
sellers	$\hat{g}_{st}\psi(i_t)$	$a_p\hat{g}_{st}\psi(i_t)$	$\hat{q}_t$ units of goods
buyers	$\hat{g}_{bt}s_t$	$a_b\hat{g}_{bt}s_t$	x_t units of money

With the above matching technologies, we summarize a representative household's matching statistics in Table 1. In period t, each buyer finds a desirable seller at a rate $s_t\hat{g}_{bt}$ and each seller finds a desirable buyer at a rate $\hat{g}_{st}\psi(i_t)$. Thus, the measure of buyers with trade matches is $s_ta_b\hat{g}_{bt}$ in the household and the measure of sellers with trade matches is $a_p\hat{g}_{st}\psi(i_t)$. These two measures are equal to each other in all symmetric equilibria. Similarly, the number of vacancies that are matched in period t, $v_t\hat{\mu}_t$, is equal to the number of unemployed workers who are matched in period t, whenever $v = \hat{v}$.

2.2. Money Growth and Productivity

There are two types of shocks in the economy, one to the money growth rate and the other to productivity. The production function is $f(n_t) = z_{pt}n_t^{e_f}$, where $e_f \in (0, 1)$ and n_t is employment in period t. The productivity shock, z_{pt}, obeys:

$$z_{pt} = (1 - \rho_p)z_p^* + \rho_p z_{p,t-1} + \varepsilon_{pt}, \tag{2.5}$$

$$E(\varepsilon_p) = 0, \qquad E(\varepsilon_p^2) = \sigma_p^2,$$

where ρ_p is the persistence of the shock. Productivity in the steady state, z_p^*, is normalized to 1.

The gross rate of money growth between periods t and $t+1$ is

$$\gamma_t \equiv \hat{m}_{t+1}/\hat{m}_t = (\hat{m}_t + \tau_t)/\hat{m}_t,$$

where τ_t is the lump-sum monetary transfer the household receives at the end of period t. Money growth obeys the following stochastic process

$$\gamma_t = (1-\rho_m)\gamma^* + \rho_m \gamma_{t-1} + \varepsilon_{mt}, \tag{2.6}$$
$$E(\varepsilon_m) = 0, \qquad E(\varepsilon_m^2) = \sigma_m^2,$$

where γ^* is the steady state value of $\{\gamma_t\}$ and ρ_m is the shock's persistence.

2.3. The Household's Choice Problem

2.3.1. Choices

Consider a representative household of type h. The household functions as follows. At the beginning of each period t, the household chooses the search intensity for each buyer, s_t, and the number of vacancies for each firm, v_t. The household also prescribes the trading strategies to its members, described later. Then the members go to the market and are separated from each other until the end of the period. After exchange, the members bring their receipts of goods and money back to the household. The household chooses a consumption level, c_t, for each member, an employment plan for each firm in the next period, n_{t+1}, an inventory plan, i_{t+1}, and a plan of the money balance, m_{t+1}. Then, a lump-sum monetary transfer τ_t is realized.

The household's endogenous state variables at the beginning of period t are the household's total amount of money (m_t), inventory of goods ($a_p i_t$), and firms' labor input ($a_p n_t$). We denote the household's value function as $v(m_t, a_p i_t, a_p n_t)$, suppressing the dependence on aggregate state variables. Denote the expected marginal value of money in period $t+1$, discounted to t, as

$$\omega_{mt} = \beta E_t v_m(m_{t+1}, a_p i_{t+1}, a_p n_{t+1}), \tag{2.7}$$

where v_m is the derivative of v with respect to m. Similarly, define ω_{it} and ω_{nt} as the derivative with respect to the second and third argument, respectively.

2.3.2. Trading strategies

The household prescribes the terms of trade to each agent to propose and respond in a desirable match. In the labor market, we simplify the analysis by assuming that firms make take-it-or-leave-it offers. Thus, the household prescribes a real wage w_{t+1} for each matched firm to propose and a number $e_{nt} \in \{0,1\}$ for each matched worker to respond, where $e_n = 1$ means "accepting the trade" and $e_n = 0$ "rejecting the trade". Note that firms and workers that are matched in period t do not start producing in period t but rather in period $t+1$, and so w_{t+1} is negotiated in t but paid in $t+1$. The corresponding nominal wage is $\hat{p}_{t+1} w_{t+1}$, where $\hat{p}_{t+1}$ is the average price level of goods in period $t+1$.

Since the firm makes a take-it-or-leave-it offer, the wage gives the worker zero expected surplus. Let the disutility of working for one unit of time be $\varphi > 0$. The worker's expected surplus, discounted to period t, is $\beta[w_{t+1} E_t(\hat{\omega}_{m,t+1}\hat{p}_{t+1}) - \varphi]$. Setting this to zero, we have

$$w_{t+1} = \frac{\varphi}{E_t(\hat{\omega}_{m,t+1}\hat{p}_{t+1})}. \tag{2.8}$$

Similarly, facing a firm from other households that proposes a wage $\hat{w}_{t+1} = \varphi / E_t(\omega_{m,t+1}\hat{p}_{t+1})$, the worker's optimal strategy is to accept the offer, i.e., $e_{nt} = 1$.[8]

In a trade between a seller and a buyer in the goods market, both sides must obtain a positive surplus from the trade in order for inventory and money holdings to be positive.[9] For this, we assume that the buyer makes the offers but must give the seller a surplus greater than or equal to $\xi \hat{R}_t$, where $\xi \in (0,1)$ and $\hat{R}_t$ is the total surplus in a similar match (calculated below). This is a short-cut to a more elaborate sequential bargaining process where the seller is chosen to propose with probability ξ and the buyer with probability $(1-\xi)$ in each round of bargaining (e.g., Shi 2001). With the current formulation, the household prescribes to each buyer in a trade match the quantity of money to offer, x_t, and the quantity of goods to ask for, q_t. For each seller, the household prescribes a number $e_{pt} \in \{0,1\}$, where $e_p = 1$ means "accepting the proposal" and

[8] Although the worker receives zero surplus, a decision $e_{nt} < 1$ would not be robust, because a firm could always increase the wage slightly to induce the worker to accept the offer with probability one.

[9] If sellers had all the bargaining power, then money would not be valued; if buyers had all the bargaining power, then sellers would not want to maintain inventory.

$e_p = 0$ "rejecting the proposal".

A successful trade gives the seller's household the value of x_t units of money, $\hat{\omega}_{mt} x_t$. To calculate the opportunity cost for the seller, suppose that he holds onto the goods instead. Then, a fraction $(1 - \delta_i)$ of the goods will survive depreciation and each unit of goods has a discounted value $\hat{\omega}_{it}$. Thus, the expected cost of the trade is $(1 - \delta_i)\hat{\omega}_{it} q_t$ and the seller's expected surplus from the trade is $[\hat{\omega}_{mt} x_t - (1 - \delta_i)\hat{\omega}_{it} q_t]$. Since the buyer makes the offers, he will push the seller's surplus down to the minimum, $\xi \hat{R}_t$. Thus, the terms of trade satisfy

$$q_t = \frac{\hat{\omega}_{mt} x_t - \xi \hat{R}_t}{(1 - \delta_i)\hat{\omega}_{it}}. \tag{2.9}$$

The buyer's surplus is $[U'(c_t) q_t - \omega_{mt} x_t]$, where $U'(c_t)$ is the marginal utility of consumption.

The number $\hat{R}_t$ is the total surplus in a trade between an arbitrary pair of buyer and seller. In such a trade the terms of trade are $(\hat{q}_t, \hat{x}_t)$ which yield:

$$\hat{R}_t = \left[U'(\hat{c}_t) - (1 - \delta_i)\hat{\omega}_{it}\right] \hat{q}_t. \tag{2.10}$$

Now consider a seller of the representative household in a trade match. Facing a proposal $(\hat{q}_t, \hat{x}_t)$ by other households' buyers that satisfies a condition similar to (2.9), the seller will choose to accept, i.e., $e_{pt} = 1$. The price level implied by such a trade is $\hat{p}_t = \hat{x}_t / \hat{q}_t$.

2.3.3. The household's decision problem

Let us formulate the representative household's decision problem as a dynamic programming problem. Assume that the instantaneous utility function, $U(c)$, is strictly increasing and concave. Let $\Phi(s)$ be the cost of buyers' search intensity and $K(v)$ the cost of vacancies, both in terms of utility. The function Φ satisfies $\Phi' > 0$ and $\Phi'' > 0$ for $s > 0$, and $\Phi(0) = \Phi'(0) = 0$. The function K has similar properties.

In each period t, the endogenous state variables for the household are (m_t, i_t, n_t); the choice variables are $(c_t, s_t, v_t, m_{t+1}, i_{t+1}, n_{t+1})$ and $(q_t, x_t, e_{pt}, w_{t+1}, e_{nt})$. We set $e_{nt} = 1$ and $e_{pt} = 1$, because these choices are optimal for the household when other household's proposals satisfy conditions similar to (2.8) and (2.9). Taking the variables with a hat as given, the representative household solves:

$$(PH) \quad v(m_t, a_p i_t, a_p n_t) = \max \left\{ \begin{array}{r} U(c_t) - a_p \hat{n}_t \varphi - a_b \Phi(s_t) - a_p K(v_t) \\ +\beta E_t v(m_{t+1}, a_p i_{t+1}, a_p n_{t+1}) \end{array} \right\}.$$

The constraints are (2.8), (2.9) and the following:

$$a_b \hat{g}_{bt} s_t q_t \geq c_t; \tag{2.11}$$

$$\frac{m_t}{a_b} \geq x_t; \tag{2.12}$$

$$(1-\delta_n) n_t + \mu(\hat{v}_t) v_t \geq n_{t+1}, \quad \delta_n \in (0,1); \tag{2.13}$$

$$(1-\delta_i)\left[i_t + f(n_t) - \hat{g}_{st} \psi(i_t) \hat{q}_t\right] \geq i_{t+1}, \; \delta_i \in (0,1); \tag{2.14}$$

$$m_t + \tau_t + \left[a_p \hat{g}_{st} \psi(i_t) \hat{x}_t - a_b \hat{g}_{bt} s_t x_t\right] + a_p \hat{p}_t (\hat{w}_t \hat{n}_t - w_t n_t) \geq m_{t+1}. \tag{2.15}$$

The maximand consists of the utility of consumption, $U(c_t)$, the disutility of work, $a_p \hat{n}_t \varphi$, the disutility of buyers' search, $a_b \Phi(s_t)$, the cost of vacancy, $a_p K(v_t)$, and discounted future utility.

The constraints (2.8) and (2.9) come from the earlier discussion on the terms of trade. Constraint (2.11) states that the household cannot consume more than what the household's buyers obtain from exchanges in period t. Constraint (2.12) is a trading restriction: since household members are temporarily separated in the exchange, each buyer cannot spend more than the amount of money allocated to him.

The other three constraints, (2.13) – (2.15), are laws of motions of the three state variables – employment, inventory, and money holdings, respectively. Constraint (2.13) states that employment in period $t+1$ comes from retained workers and those newly recruited in t, where the job separation rate is a constant $\delta_n \in (0,1)$. Constraint (2.14) states that inventory at the beginning of $t+1$ is equal to the amount of goods that survived depreciation at the end of t. The amount of goods at the end of period t before depreciation consists of output in t and previous inventory minus the sales in t. Constraint (2.15) states that the increase in the household's money holdings between t and $t+1$, $(m_{t+1} - m_t)$, consists of the monetary transfer received at the end of t, the net money receipt from the goods market exchanges in t, and the net wage receipt in t.

To find the optimality conditions, note first that (2.13)–(2.15) all hold with equality, provided that $(\omega_n, \omega_i, \omega_m)$ are positive. Use these equalities to substitute for n_{t+1}, i_{t+1} and m_{t+1} in the objective function of (PH). Also, (2.11) holds with equality and we use it to substitute for c. Next, substitute q from (2.9). Note that (2.12) holds for all buyers who have a trade match. Since the measure of buyers in trade matches is $a_b \hat{g}_{bt} s_t$, we let the shadow price of (2.12) be $a_b \hat{g}_{bt} s_t \lambda_t$. The first-order conditions of x_t, s_t and v_t are as follows:

$$U'(c_t) \frac{\hat{\omega}_{mt}}{(1-\delta_i)\hat{\omega}_{it}} = \omega_{mt} + \lambda_t; \tag{2.16}$$

$$\Phi'(s_t) = g_b(\hat{s}_t, \hat{\imath}_t)[U'(c_t)q_t - \omega_{mt} x_t]; \tag{2.17}$$

$$\omega_{nt} = K'(v_t)/\mu(\hat{v}_t). \tag{2.18}$$

These conditions can be interpreted as follows. In (2.16), the quantity $\hat{\omega}_{mt}/[(1-\delta_i)\hat{\omega}_{it}]$ is the amount of goods that can be bought with an additional unit of money (see (2.9)), and so the left-hand side of (2.16) is the marginal utility of consumption generated by the purchasing power of money. The right-hand side, measuring the cost of money to a buyer, consists of the opportunity cost of giving up the additional unit of money, ω_{mt}, and the cost of facing a tighter trading constraint (2.12), λ_t. In (2.17), the quantity $[U'(c)q - \omega_m x]$ is the buyer's surplus from a trade. So, (2.17) equates the expected marginal benefit from increased search intensity to the marginal cost of the search intensity. Eq. (2.18) equates the marginal cost of vacancy, $K'(v)$, to the expected benefit of an additional vacancy, $\mu(\hat{v})\omega_n$.

We can also derive the envelope conditions for m_t, n_t and i_t. Moving the time index forward by one period, these envelope conditions are

$$\omega_{mt} = \beta E_t[\omega_{m,t+1} + g_b(\hat{s}_{t+1}, \hat{\imath}_{t+1}) s_{t+1} \lambda_{t+1}]; \tag{2.19}$$

$$\begin{aligned} \omega_{nt} = {} & \beta E_t (1-\delta_n)\omega_{n,t+1} \\ & + \beta E_t[(1-\delta_i)\omega_{i,t+1} f'(n_{t+1}) - \omega_{m,t+1} \hat{p}_{t+1} w_{t+1}]; \end{aligned} \tag{2.20}$$

$$\begin{aligned} \omega_{it} = {} & \beta E_t (1-\delta_i)\omega_{i,t+1} \\ & + \beta E_t\{g_s(\hat{s}_{t+1}, \hat{\imath}_{t+1}) \psi'(i_{t+1})[\omega_{m,t+1} \hat{x}_{t+1} - (1-\delta_i)\omega_{i,t+1} \hat{q}_{t+1}]\}. \end{aligned} \tag{2.21}$$

Eqs. (2.19) – (2.21) have similar interpretations, each equating the "permanent income" to the "cash flow" for each state variable. For example, the permanent income from a marginal unit of money at the beginning of period $t+1$ is $(\omega_{mt} - \beta E_t \omega_{m,t+1})$ and the cash flow is the value that such a unit generates from relaxing the trading restriction (2.12) in $t+1$.

The assumptions $\psi' > 0$ and $\xi > 0$ are both necessary for there to be a meaningful inventory level in equilibrium. If $\psi' = 0$, the cash flow to inventory in (2.21) is zero; if $\xi = 0$, the surplus to a seller, $[\omega_m \hat{x} - (1-\delta_i)\omega_i \hat{q}]$, is zero (see (2.9)) and so the cash flow to inventory is again zero. In both cases, inventory is positive only when the expected value of inventory grows at a gross rate $1/(1-\delta_i)$. In this situation, however, the quantity of goods traded in a match approaches 0 for any finite value of money.

3. Equilibrium and Velocity

The following defines a symmetric search equilibrium:[10]

Definition 3.1. *For any given initial state (m_0, i_0, n_0) and the exogenous shock processes (2.5) and (2.6), a symmetric monetary search equilibrium consists of each household's choice variables $d_t \equiv (c_t, x_t, q_t, w_{t+1}, s_t, v_t, m_{t+1}, n_{t+1}, i_{t+1})$ and other households' choices $\hat{d}_t$ such that (i) d_t solves (PH) for each t taking $\hat{d}_t$ and (m_t, n_t, i_t) as given; (ii) $d_t = \hat{d}_t$ for all t; and (iii) $0 < \omega_{mt} m_{t+1} < \infty$, $0 < \omega_{nt} n_{t+1} < \infty$, $0 < \omega_{it} i_{t+1} < \infty$ for all $t \geq 0$.*

The conditions (i) and (ii) are straightforward. The condition (iii) requires that each of the stock variables be valued in equilibrium and that the total value of each state variable be bounded. A positive value of each stock variable is necessary for a meaningful analysis of that stable variable. A bounded value of each state variable is necessary for ensuring that the optimal choices are characterized by the first-order conditions.

We restrict our attention to the equilibrium where $\lambda > 0$, i.e., where the trading restriction (2.12) binds. The restriction $\lambda > 0$ requires that buyers prefer spending money to hoarding it in a trade match. If $\lambda = 0$, (2.19) would require that the expected value of money should grow at the discount rate, which is possible only if the stock of money shrinks at the discount rate.

[10] As is apparent from the formulation of (PH), each household directly takes into account the influence of its decisions (m, n, i) on the terms of trade. This equilibrium concept contrasts with that in Shi (1998), where each household ignores such influence.

Thus, the result $\lambda > 0$ can be achieved by restricting $\gamma_t > \beta$ for all t. With this restriction, $x_t = m_t/a_b$, and so the price level is $p_t = m_t/(a_b q_t)$. The gross rate of inflation between t and $t+1$ is $p_{t+1}/p_t = \gamma_t q_t/q_{t+1}$.

Next, we reduce the dimension of the equilibrium system. With symmetry, we suppress the hat on aggregate variables. Let $\omega_{rt} = p_t \omega_{mt}$ be the expected shadow value of the real money balance in $t+1$. Similarly denote $\lambda_{rt} = p_t \lambda_t$. We express the equilibrium equation system as one for $(n, i, v, \omega_i, \omega_r)$ and express other variables as functions of these five variables. First, $x = m/a_b$ and $p = x/q$ when $\lambda_r > 0$. Second, $w = \varphi/\omega_r$ from (2.8). Third, under symmetry, $\mu(\hat{v}) = \mu(v)$ and $\omega_n = k(v) \equiv K'(v)/\mu(v)$ by (2.18). Finally, using (2.11), (2.9), (2.16) and (2.17) we have

$$c_t = a_b s_t q_t g_b(s_t, i_t); \tag{3.1}$$

$$\omega_{rt} = \xi U'(c_t) + (1-\xi)(1-\delta_i)\omega_{it}; \tag{3.2}$$

$$U'(c_t) = (1-\delta_i)\omega_{it}(1 + \lambda_{rt}/\omega_{rt}); \tag{3.3}$$

$$\Phi'(s_t) = g_b(s_t, i_t)[U'(c_t) - \omega_{rt}]q_t. \tag{3.4}$$

Jointly solving (3.1) – (3.4), we express (c, s, q, λ_r) as functions of (i, ω_r, ω_i). Substituting these relationships into (2.13), (2.14) and (2.19) – (2.21), we have the following dynamic system:

$$\begin{cases} n_{t+1} = (1-\delta_n)n_t + v_t \mu(v_t); \\ i_{t+1} = (1-\delta_i)[i_t + f(n_t) - g_s(s_t, i_t)\psi_t q_t]; \\ \omega_{it} = \beta(1-\delta_i)E_t\left\{\omega_{i,t+1}\left[1 + \xi q_{t+1} g_s(s_{t+1}, i_{t+1})\psi'(i_{t+1})\frac{\lambda_{r,t+1}}{\omega_{r,t+1}}\right]\right\}; \\ k(v_t) = \beta E_t[(1-\delta_n)k(v_{t+1}) + (1-\delta_i)\omega_{i,t+1}f'(n_{t+1}) - \varphi]; \\ \omega_{rt} = \beta E_t\left\{\frac{q_{t+1}}{\gamma_t q_t}[\omega_{r,t+1} + g_b(s_{t+1}, i_{t+1})s_{t+1}\lambda_{r,t+1}]\right\}. \end{cases} \tag{3.5}$$

In this system, (i, n) are predetermined variables and others are jump variables.

Consumption velocity is defined and calculated as follows:

$$V_{ct} \equiv p_t c_t/m_t = g_t/a_b = z_0 B^{\alpha-1} s_t^{\alpha}[\psi(i_t)]^{1-\alpha}. \tag{3.6}$$

Consumption velocity is equal to the number of trades per buyer, which is a Cobb-Douglas function of buyers' search intensity and sellers' selling intensity. This formula indicates how money growth and productivity shocks might affect velocity. Consider first a positive money growth shock. The increase in the money growth rate reduces the value of the real money balance (see the last equation of (3.5)), and hence increases the buyer's surplus per unit of good received in the trade, $[U'(c_t) - \omega_{rt}]$. This higher match surplus stimulates buyers to increase the search intensity (see (3.4)), which leads to high velocity because inventory is predetermined. The higher search intensity also increases current sales, which reduces inventory and hence velocity in the next period. Because a positive money growth shock generates an initial increase and a subsequent fall in velocity, it increases the variability of velocity.

Now consider a persistent increase in productivity. Such a shock increases the amount of goods that each seller can supply and reduces buyers' need to search intensively. Because inventory is predetermined, the immediate fall in buyers' search intensity reduces velocity. In the next period, employment and output rise. Inventory thus rises, yielding a higher number of trades and higher velocity. This positive effect on velocity persists, as the productivity increase is persistent.

The remainder of this paper illustrates the quantitative responses of velocity to these two types of shocks and details the correlations between endogenous variables.

4. Calibration

4.1. The Steady State and Parameter Values

To compute the model, we choose the following functional forms:

$$U(c) = \frac{c^{1-RA} - 1}{1 - RA}, \quad RA > 0;$$

$$\Phi(s) = \varphi(\varphi_0 s)^{1+\frac{1}{\varepsilon_\Phi}}, \ \varepsilon_\Phi > 0; \quad \psi(i) = (i^\eta - 1)/\eta, \quad \eta < 1.$$

The constant RA is the relative risk aversion. In the search cost function, φ is the disutility of employment, φ_0 is the efficiency units of a buyer's search intensity relative to a worker's time, and ε_Φ measures the elasticity of buyers' search intensity.

To account for fixed investment, we assume that it is a constant fraction, FI_k, of aggregate output per household. Then, (2.11) is revised as follows:

$$c_t = a_p B g_{bt} s_t q_t - FI_k \cdot a_p \hat{f}^*. \tag{4.1}$$

This revision does not change the forms of the first-order conditions, because each household takes aggregate output as given.

The steady state of the model is characterized in Appendix B. Interpreting the length of a period as a quarter, we identify the parameters by matching certain steady state predictions of the model with the data. The sample covers the period 59:II – 98:III. Because Hodrick et al. (1991) use the sub-sample, 59:II – 88:I, we also use this sub-sample to identify the parameters and include the values in brackets (see Appendix A for a description of the data).

Set $\beta = .9952$ (.9958) to match the sample mean of the quarterly real interest rate, .4809% (.4191%). The gross rate of money growth in the steady state matches the sample average $\gamma^* = 1.01724$ (1.02014). Set $A = .6$ and $\delta_n = .06$, both being consistent with the estimates in Blanchard and Diamond (1989). Set $\eta = -12$, so that the variance ratio of output to sales lies in the range $(1.03, 1.30)$ reported by Blinder and Maccini (1991). Let $RA = 4$, $\xi = .2$, $\alpha = .8$, $\varepsilon_\Phi = 2$ and $B = .5$ in the benchmark case. The value of RA is high relative to some other calibration exercises, but it is not higher than those in Hodrick et al. (1991), who experiment the range $(.5, 9.5)$. We will consider the range $(1.5, 8)$ for RA and later conduct sensitivity analyses with respect to $(B, \xi, \alpha, \varepsilon_\Phi, \eta)$.

The other parameters are identified using the following facts. (i) The labor participation rate (LP) is .6284 (.6150) and the unemployment rate (UR) is .0605 (.0612); (ii) The inventory/output ratio (IO) is .9 and the inventory investment/output ratio (IIO) is .0065; (iii) Income velocity of money (VI) is 1.7440 (1.6882); (iv) The labor share of income is .64 (Christiano 1988) and the hiring cost is 2% of the wage cost (which is in the range surveyed by Hamermesh 1993); (v) The shopping time of the population is 11.17% of the working time and the working time is 30% of agents' discretionary time (Juster and Stafford 1991); (vi) the steady state expenditure on fixed investment is 26.9% of output (Christiano 1988). In Appendix B we detail the procedure which solves the parameters using the above restrictions. Table 2 lists the parameter values and the steady state values of some endogenous variables.

Tables 2 and 3 here.

4.2. The Stochastic Processes and the Solution Method

We estimate the stochastic processes of money growth and productivity, using U.S. quarterly time-series data described in Appendix A. The monetary aggregate is chosen to be $M2$ rather than $M1$, for two reasons. One is that $M2$ appears stationary, a requirement that our model imposes, but $M1$ appears non-stationary (see Figure 1.1). The other is to make our results comparable with those of Hodrick et al. (1991), who focus on $M2$ velocity. Let $\gamma_t^d \equiv (\gamma_t - \gamma^*)/\gamma^*$ be the percentage deviation of the money growth rate from its sample average, γ. Then we can rewrite the money-growth process (2.6) as

$$\gamma_t^d = \rho_m \gamma_{t-1}^d + \varepsilon_{mt}; \qquad E(\varepsilon_m) = 0,\ E(\varepsilon_m^2) = \sigma_m^2. \tag{4.2}$$

Estimating (4.2) using the ordinary-least-square method, we obtain the standard deviation, $\hat{\sigma}_m$, and the persistence parameter, $\hat{\rho}_m$ (see Table 3).

To obtain the time series of productivity, transform the production function as follows:

$$\ln(f_t) = e_f \ln(n_t) + \ln(z_{pt}). \tag{4.3}$$

Interpreting $f(n)$ as aggregate labor income, we calculated it to be 64% of GDP (see Christiano 1988). With the parameter e_f being identified earlier and n being aggregate employment, we can use (4.3) to construct the series $\{z_{pt}\}$. From this series we compute the series of percent deviations of z_{pt}, denoted $\{z_{pt}^d\}$. According to the stochastic process assumed for productivity, z_{pt}^d obeys the following process:

$$z_{pt}^d = \rho_p z_{p,t-1}^d + \varepsilon_{pt}; \qquad E(\varepsilon_p) = 0,\ E(\varepsilon_p^2) = \sigma_p^2. \tag{4.4}$$

Regressing (4.4) by OLS gives estimated values of (ρ_p, σ_p^2) in Table 3.

To compute the dynamic responses of the equilibrium, we linearize the equilibrium dynamic system in Section 3 around the steady state and find the saddle-path of the linearized system that is consistent with rational expectations (see Blanchard and Kahn 1980). This saddle path is an approximation for equilibrium dynamics.[11]

[11] Because the solution procedure is standard, it is not detailed here but is available upon request.

5. Model Predictions

We simulate the model 1000 times and compute the unconditional moments of some key variables and their correlations that Hodrick et al. (1991) examine. Tables 4.1 and 4.2 report these moments, where the standard deviations of the moments over the simulations appear in brackets. In these tables, we choose five different values of RA (1.5, 2, 4, 6, and 8), where $RA = 4$ is the benchmark value.[12] Consumption velocity of money is calculated through (3.6). The variability of consumption velocity is measured by the coefficient of variation, defined as $cv(V_c) = \frac{\sigma(V_c)}{E(V_c)} \times 100$. Other major variables in Tables 4.1 and 4.2 are defined as follows:

inflation rate	$\pi_t = \gamma_t q_t / q_{t+1} - 1$
real interest rate	$ri_t = U'(c_t)/[\beta U'(c_{t+1})] - 1$
nominal interest rate	$ni_t = \omega_{mt}/(\beta \omega_{m,t+1}) - 1$
real money balance	$rm_t = a_b q_t$
consumption growth	$g_{ct} = c_{t+1}/c_t$
growth in real money balances	$g_{mt} = (M_{t+1}/p_{t+1})/(M_t/p_t)$.

The nominal interest rate is defined as the implicit rate of return to holding a claim to one dollar in the future, and the real interest rate as that to holding a claim to one unit of goods. Such definitions are standard, although the markets for such claims do not exist in this economy.[13]

Tables 4.1 and 4.2 here.

5.1. Basic Results

The Mean of consumption velocity. The mean of consumption velocity in our model matches the sample value well. The simulated mean of velocity is 1.2238 in the benchmark case ($RA = 4$) for the sample 59:II – 88:I, and it changes very little with the relative risk aversion. When RA changes from 1.5 to 8, velocity increases from 1.2234 to 1.2245. All these values of velocity are close to the sample mean 1.212. The reason for the close match is that we calibrated the

[12]When investigating the sensitivity of the numerical results to one parameter, we repeat the procedure in Appendix B to identify other parameters again. So, a change in one parameter may entail changes in other parameter as well, in order to satisfy the steady state restrictions in the identifying procedure. This may also require us to reconstruct the series $\{z_{pt}\}$, becuase the latter depends on the identified value of e_f.

[13]In these definitions, the inflation rate and interest rates are all ex post, rather than ex ante, concepts. Accordingly, we calculate the sample values of these variables using ex post numbers. The inflation rate in the sample is calculated using the GDP deflator.

steady-state income velocity to the sample average. This procedure yields a realistic mean of consumption velocity, provided that the ratio of consumption to output is realistic and stable. In Hodrick et al. (1991), in contrast, it is difficult to perform this matching procedure successfully. In particular, their model cannot generate a mean of consumption velocity larger than one, even after they incorporated credit goods into the cash-in-advance model.

The Variability of velocity. The simulated mean of $cv(V_c)$ is 1.9066 in the benchmark case, with a standard deviation 0.4246 across simulations. This is lower than the sample value 4.48. Also, the variability of velocity changes sensitively with the relative risk aversion. When RA increases from 1.5 to 8, the standard deviation of velocity increases from 0.0056 to 0.0365, resulting in an increase of $cv(V_c)$ from 0.4579 to 2.9746. This sensitivity is also present in Hodrick et al. (1991). In our model, such sensitivity can be explained as follows. When the relative risk aversion is high, households have a strong desire to keep consumption smooth. To achieve this goal, households adjust buyers' search intensity and sellers' inventory in large magnitudes to respond to the shocks to productivity and money growth, resulting in large variations in the number of trades per buyers and hence in velocity.

Velocity is significantly more volatile in our model than in Hodrick et al. (1991), although it is still lower than in the data. In the models of Hodrick et al., the maximum of $cv(V_c)$ is 1.66 in their paper (see Table 6 therein), obtained under extreme parameter values $RA = 9.5$ and $\beta = .975$, which imply a quarterly real interest rate (2.6%) that is much higher than the sample value (.42%). We match the steady state real interest rate to the sample value. The contrast between the two papers shows that search frictions in the goods market and the labor market are an important explanation, though not a sufficient explanation, for the variability of velocity. We will elaborate on this in the next subsection.

The results are similar for the longer sample 59:II – 98:III (Table 4.2). Again, the mean of velocity in our model matches the sample mean well and the variability of velocity falls short. The gap between $cv(V_c)$ predicted by the model and the sample value is much wider for the longer sample than for the shorter sample. The reason for this wide gap is that we do not account for the structural break in velocity that seems to have occurred in mid 1980s (see Figure 1.1). In the period 59:II-86:IV, the average growth rate of nominal consumption is 2.0538% and the growth rate of $M2$ monetary aggregate is 2.0576%, implying a stable $M2$ velocity in this period. In the period 87:I-98:III, in contrast, the average growth rate of nominal consumption is 1.3862% and the growth rate of $M2$ monetary aggregate is 0.9359%, implying a much higher $M2$ velocity in this period than in the previous period. This structural change is likely caused by financial innovations which made it cheaper than before to transact with credits.

The Correlation between velocity and consumption growth. Velocity and real consumption growth are negatively correlated in our model, as in the data. Calibrated to the shorter sample, the model generates $corr(V_{ct}, g_{ct-1}) = -0.2498$ in the benchmark case, and this correlation changes very little with RA. The correlation is -0.3775 in the sample. (Notice that g_{ct-1} is defined as the consumption growth rate between $t-1$ and t.) Calibrated to the longer sample, the model generates $corr(V_{ct}, g_{ct-1}) = -0.2933$ in the benchmark case and, when RA changes from 1.5 to 8, the correlation changes from -0.3693 to -0.2867. The corresponding sample value is -0.3134. In comparison, Hodrick et al. (1991, Tables 5 and 6) fail to produce any number close to this sample correlation: in their model $corr(V_{ct}, g_{ct-1})$ ranges from -0.069 to 0.664.

The correlation between velocity and consumption growth is realistic in our model because a shock to money growth or productivity generates opposite responses in the intensive margin and the extensive margin of trade. To see this, recall that consumption depends on the number of trades (the extensive margin) and the quantity of goods traded in each match (q, the intensive margin), while velocity depends only on the number of trades per buyer. When there is a positive money growth shock, anticipated inflation induces buyers to search more intensively, thus increasing the number of trades and velocity. At the same time, higher inflation reduces the quantity of goods traded in each match. When the quantity of goods falls by more than the increase in the number of trades, consumption falls and hence is negatively correlated with velocity. To positive productivity shocks, the intensive margin and the extensive margin of trade also respond in opposite directions. An increase in productivity increases the quantity of goods traded in each match and reduces buyers' search intensity. Velocity and consumption growth are negatively correlated. Notice that the two shocks have opposite effects on trade. So, the negative correlation between consumption growth and velocity indicates that one shock dominates the other. In our model, this dominating shock is the productivity shock.

Interest rates and inflation. The real interest rate and inflation are negatively correlated with each other in our model, and more so than in the data. For the shorter sample, for example, this correlation ranges between -0.5942 and -0.8289 when RA changes between 1.5 and 8, in comparison with the sample value -0.3531. This negative correlation between the real interest rate and inflation implies that consumption is likely to respond to shocks in a hump-shaped fashion, and so there is a strong internal propagation mechanism from the shocks to consumption.

To see how this propagation works, consider a positive money growth shock. As argued above, this shock increases inflation and induces buyers to search more intensively. Because buyers respond to both the current inflation and expected future inflation, search intensity increases by the most at the time of the shock. The higher search intensity increases the number of trades and mitigates a large part of the negative effect of inflation on consumption through the quantity of goods traded in each match. In the subsequent period, however, expected inflation is not as high as when the shock was first realized, because money growth shocks are not very persistent. This induces a large fall in buyers' search intensity, although not completely back to the level before the shock. As a result, consumption falls further, yielding a negative correlation between the real interest rate and inflation.

A positive productivity shock also generates hump-shaped consumption responses, but the mechanism is different. When a positive productivity shock is first realized, consumption increases as a result of the increase in the quantity of goods traded in each match. Most of this increase in consumption comes from inventory, instead of new output, because employment responds slowly to the shock due to the labor market friction. Since the productivity shock is persistent, employment increases in the next period, which further increases the quantity of goods traded in each match and hence increases consumption. Because inflation falls when there is a positive productivity shock, there is a negative correlation between the real interest rate and inflation.

Note that the negative correlation between the real interest rate and inflation is much stronger in our model than in the data. This strong negative correlation is responsible for two other results. One is that the nominal interest rate and velocity are less positively correlated with each other than in the data. In the benchmark case, for example, the correlation between the nominal interest rate and inflation is 0.1780, while the sample value is 0.6502; the correlation between the nominal interest rate and velocity is 0.0968, while the sample value is 0.702. The other is that, in most cases, the nominal interest rate and inflation are less positively correlated with each other than in the data. Nevertheless, it is remarkable that the nominal interest rate and inflation are positively correlated in our model, despite the fact that there is no bond market to transmit the shocks.

Related to the above discussions, interest rates are more volatile in the model than in the data. Also, because prices are perfectly flexible, inflation responds to shocks immediately, and so inflation is more volatile in the model than in the data. For the same reason, there is virtually no growth in the real money balance. The mean of g_m is close to 0, with a relatively large standard deviation across simulations.

One way to improve the quantitative matches on the above statistics is to allow money growth to respond positively to productivity shocks. Since money growth is likely to affect consumption growth in the opposite direction to a positive productivity shock, such endogenous money growth attenuates the negative correlation between the real interest rate and inflation. This can deliver quantitatively more realistic correlations among the nominal interest rate, velocity and inflation.

Volatility of sales and output. Output is more volatile than sales in most cases (not listed in Tables 4.1 and 4.2). In the benchmark case, the relative variance of output to sales is 1.279. This variance ratio is realistic, as Blinder and Maccini (1991) find. The reason for the larger variance in output than in sales is that productivity shocks are the predominant chocks in the model, which affect employment and output first before affecting sales.

5.2. Shocks, Search Intensity, and Inventory

We stated earlier that search frictions are the primary reasons why our model generates more variable velocity than the models in Hodrick et al. (1991) do. To see more clearly how velocity responds to the two shocks, we now detail how the two determinants of velocity, inventory and search intensity, are correlated with money growth and productivity. To simplify discussion, we will focus on the shorter sample 59:II – 88:I. Tables 5.1 and 5.2 exhibit the cross correlations.

Tables 5.1 and 5.2 here.

Examine the cross correlations between (s_t, i_t) and productivity z_p first. Table 5.1 shows that buyers' search intensity is negatively correlated strongly with past, present and future levels of productivity. The negative correlation comes from the fact that higher productivity increases the supply of goods and hence reduces the need for buyers to search intensively. Inventory is positively correlated with past, current and future levels of productivity, because higher productivity leads to higher output and inventory. Among these correlations between (s_t, i_t) and productivity, the strongest in the absolute value is the one with one-period past productivity. This is because an increase in productivity affects output with one-period delay, as employment is predetermined in the period where the shock takes place and starts to respond a period later. Moreover, the correlations between (s_t, i_t) and z_p remain at high levels across time because productivity shocks are very persistent. These cross correlations indicate that a positive productivity shock reduces velocity immediately by reducing buyers' search intensity, and then increases velocity in future periods by increasing inventory. The immediate fall and the subsequent rise in velocity produce the variability of velocity.

Notice that the correlations between inventory and productivity are much stronger in our model than in the data. One reason is that our model treats the money growth process independently of productivity shocks. If one allows money growth to increase sufficiently to respond to a positive productivity shock, then buyers' search intensity falls less precipitously than in the current model, in which case inventory will rise less precipitously in response to a positive productivity shocks and so the model will yield quantitatively more realistic correlations between inventory and productivity.

Now examine the cross correlations between (s_t, i_t) and money growth. Table 5.2 shows that buyers' search intensity in period t is positively correlated with the money growth between $t-1$ and t, i.e., with γ_{t-1}, and with γ_t. Search intensity s_t is also positively correlated with money growth two periods in the past, γ_{t-2}, but the correlation is smaller than with γ_{t-1}. This confirms our earlier analysis that a positive money growth shock increases buyers' search intensity by the most at the time of the shock, and then such a positive response declines over time. However, inventory at the beginning of period t is positively correlated with money growth in the previous period, which seems to contradict our earlier analysis that positive money growth shocks reduce inventory. Moreover, most of the correlations between (i, s) and money growth are indistinguishable from zero, once the standard deviations across simulations are taken into account.

The explanation for these seemingly puzzling results is that productivity shocks are the dominating shocks in the model. The correlations in Table 5.2 arise from the responses of (s, i) to both shocks. Because a positive productivity shock changes inventory and search intensity opposite to what a positive money growth shock does, it obliterates or even reverses the responses of these variables to money growth shocks. In particular, a positive productivity shock raises inventory by more than the reduction in inventory caused by a positive money growth shock, thereby inducing a positive correlation between inventory and money growth.[14] Again, allowing money growth to respond positively to productivity shocks may produce more realistic correlations between (s, i) and money growth shocks.

To conclude this section, we make two remarks. The first is that output is endogenous in the current model, but exogenous in the models in Hodrick et al. (1991). However, we believe that simply making output endogenous in Hodrick et al. (1991) will not increase the variability of velocity much, if the markets are kept to be Walrasian. The non-Walrasian features of the goods market and the labor market are important for the variability of velocity in our model.

[14]We have investigated this dominating role of productivity shocks by abstracting from money growth shocks (not reported in the tables). Productivity shocks alone generate $cv(V_c) = 1.8745$ in the case $RA = 4$, which is very close to the number (1.9066) that we obtained in the presence of both types of shocks.

Because trades are decentralized in the goods market, the number of trades becomes important for aggregate demand, in addition to the quantity of goods in each trade. By affecting the number of trades, the search intensity and inventory affect velocity. On the other hand, the friction in the labor market makes it costly to adjust employment immediately. This delays the responses of employment and inventory, thus making velocity adjust non-monotonically to the shocks.

The second remark is that the strength of monetary propagation in the current model is much weaker than in a similar model by Shi (1998). Two differences in the modelling might be responsible for this quantitative differences between the two models. One is that inventory appears in the matching function in the current model but not in Shi (1998). As a result, changes in buyers' search intensity alone cause smaller changes in the number of trades in the current model than in Shi (1998). The other reason is that the two models define an equilibrium differently. In Shi (1998), it is assumed that each household ignores its influence on the terms of trade when choosing buyers' search intensity. In the current model, in contrast, each household takes into account such influence. In the event of a positive money growth shock, households will take into account the fact that the shock will reduce the quantity of goods traded in each match and hence reduce the gain from trade. Such considerations attenuate the response of search intensity to money growth shocks.

6. Sensitivity Analysis

In this section we examine the sensitivity of the quantitative results to five parameters, $(B, \xi, \varepsilon_\Phi, \alpha, \eta)$. The benchmark values of these parameters are: $B = 0.5$, $\xi = 0.2$, $\varepsilon_\Phi = 2$, $\alpha = 0.8$, and $\eta = -12$. In the sensitivity analysis we keep the relative risk aversion at $RA = 4$. Each time we change the value of one parameter, we calibrate the model again to identify other parameters. The results are reported in Tables 6.1 and 6.2, for the shorter sample.

Table 6.1 and 6.2 here.

The mean of velocity is not sensitive to the five parameters. When any of these parameter changes, the mean of velocity barely changes. This is because, for each new parameter value, we calibrate the model again to match the steady state income velocity with the sample value.

The variability of velocity is also insensitive to changes in the buyer/seller ratio, B. When B increases from .35 to .65, the coefficient of variation in velocity changes only slightly, from 1.9132 to 1.9017. To explain this insensitivity, note that an increase in the number of buyers creates two types of externality. One is that the increase in the number of buyers increases the congestion for buyers and reduces their search intensity. This negative externality reduces the number of matches per buyer and reduces velocity. The other is that the increase in the number of buyers gives each seller a higher matching rate than before, which induces sellers to increase production and inventory. This positive externality increases the number of matches per buyer and increases velocity. Similar to Hosios (1990), the two externalities cancel out with each other when the two sides of the market are rewarded according to their contributions to the match formation. The latter condition is $\xi = 1 - \alpha$, which is satisfied in the benchmark model. As a result, an increase in the number of buyers does not change much the number of matches per buyer or velocity.

The variability of velocity is moderately sensitive to changes in $(\varepsilon_\Phi, \xi, \alpha)$, becoming larger when any of these parameters increases. The positive dependence of $cv(V_c)$ on ε_Φ and α is intuitive. A higher ε_Φ makes buyers' search intensity more elastic, thereby making the number of trades more responsive to the shocks, while a higher α increases the elasticity of the number of trades with respect to buyers' search intensity. Both increase the responsiveness of velocity to shocks. The dependence of the variability of velocity on ξ is more complicated. A larger ξ increases the surplus share for the seller in a trade and reduces the share for the buyer. Thus, sellers' inventory becomes more responsive to shocks and buyers' search intensity less responsive to shocks. The sensitivity of $cv(V_c)$ to ξ indicates that the increase in the responsiveness of inventory dominates the reduced responsiveness of buyers' search intensity.[15]

Now consider the sensitivity of the results with respect to η, a parameter in the function ψ.

[15]Ideally, we would like to pin down the value of ξ by matching the mark-up ratio in our model with that in the data. Unfortunately, the markup in our model varies too little with respect to ξ, which makes it difficult to match the range of values in the data. The markup in our model can be defined as $[p(q - wn)\omega_m - K(v)]/(qU')$, where the numerator is the sales revenue minus wage cost and vacancy cost, all measured in utility, and the denominator is the utility value of sales. When ξ varies from 0.01 to 0.99, this markup varies between 0.4426 and 0.4489. In contrast, markups in the US data vary from the range $(0.4, 0.7)$ estimated by Domowitz et al. (1988) to 1.5 by Hornstein (1993).

Since the function ψ did not appear in previous search models, we experiment a wide range of values of η, from 0.5 to -15. As η decreases from 0.5 to -15, the mean of velocity does not change much. Neither do some other statistics, such as the correlations between velocity and consumption growth, and between velocity and interest rates. However, the absolute value of the correlation between inflation and the real interest rate increases when η decreases.

An interesting result is that velocity becomes more variable when η decreases. The coefficient of variation in velocity increases from 1.0532 to 1.9665 when η decreases from 0.5 to -15. This result is puzzling, because a smaller η makes ψ less elastic with respect to inventory which should make the number of trades and velocity less volatile, not more volatile. The resolution to the puzzle lies in the calibration process. Since the parameter values must satisfy other restrictions in order to match the data, the steady state level of ψ depends positively on the value of η (see (B.14) in Appendix B). When η decreases from 0.5 to -15, ψ^* decreases from 0.5498 to 0.058. Thus, for smaller η, a given change in ψ represents a larger variation relative to the mean. To see these two effects of η on the variability of velocity, we list the statistics of sales. As η decreases, the standard deviation of sales becomes smaller, reflecting the smaller elasticity of ψ with respect to inventory. The mean of sales also falls, reflecting the decrease in ψ. The second effect dominates for most of the changes in η, as the coefficient of variation of sales increases for most of the reductions in η.

Another interesting fact is that, as η decreases, the mean and variance of output both fall but the variance of output falls by more. Thus, the variance ratio of output to sales decreases as η decreases. For η greater than or equal to -5, the variance ratio of output to sales exceeds 1.30, the upper bound found in Blinder and Maccini (1991). This is why we use a rather large negative number (-12) for η in the benchmark case.

7. Conclusion

We construct a dynamic search model to examine the behavior of velocity. The prominent feature of the model is the presence of costly search in both goods and labor markets. Incorporating money growth shocks and productivity shocks, we calibrate the model to the US time series data. Our model generates significantly more volatile velocity than some other dynamic equilibrium models, e.g. Hodrick et al. (1991). Search in the goods market is important for this higher volatility of velocity, because it makes the number of trades a function of sellers' inventory and buyers' search intensity which respond to shocks. Search in the labor market is also important for the high volatility of velocity, because it delays the response of employment and output to shock and so prolongs the effects of shocks.

The high volatility of velocity in our model is also related to the propagation mechanism. Shocks have persistent effects on consumption, and hence on consumption growth. Because of costly search in the labor market, a positive productivity shock increases the supply of goods by more in the future than in the current period, thus making consumption peak several periods after the shock. Because of costly search in the goods market, a positive money growth shock induces a larger increase in buyers' search intensity in the current period than in the future, thus preventing consumption from dipping to the bottom immediately. The persistent variation in consumption growth is an alternative illustration of how our model generates a larger volatility of velocity than the models of Hodrick et al. (1991) do.

The high volatility of velocity in our model is an encouraging result, although it is still smaller than in the data. Since there is no other asset for agents to substitute away from money in our model, it is not clear a priori whether velocity can be volatile at all. It is then somewhat surprising that our model does yield higher volatility of velocity than the models in Hodrick et al. (1991), which allow for the substitution between money and credit or other assets.

However, our model fails to match some aspects of the data, thus pointing to corresponding extensions. First, the volatility of velocity in our model is still smaller than in the sample, particularly in the sample including the 1990s. This might be caused by the absence of other assets in our model, such as bonds, and the absence of financial innovation. Second, the negative correlation between the real interest rate and inflation is much stronger than in the data; so is the positive correlation between inventory and productivity. These results arise because productivity shocks are the overwhelming shocks in our model. To improve the quantitative results for such correlations, it seems necessary to allow money growth to respond positively to productivity shocks. Finally, the positive correlation between inflation and money growth is much stronger in our model than in the data. To reduce such a correlation one may need to introduce some mechanism to delay the response of nominal prices to money growth.

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A. Data Sources

Data used in our paper mainly came from the following sources (all seasonally adjusted).

1. Citibase (Acronyms in bracket)

   - M1 Aggregate (FM1), monthly. The quarterly data is calculated from the average of three months.
   - M2 Aggregate (FM2), monthly. The quarterly data is calculated from the average of three months.
   - Real M2 Aggregate (FM2DQ), monthly. The quarterly data is calculated from the average of three months.
   - Nominal interest rates (FYGM3), 3-month Treasury bill yield, monthly. The quarterly data is calculated from the average of three months.
   - Population (GPOP), quarterly.
   - Gross Domestic Product: Implicit Price Deflator (GDPD), index, 92=100.
   - Consumer Price Index, Urban Area, All Items, $82-84=100$, monthly. The quarterly data is calculated from the average of three months.

2. Database, the Federal Reserve Bank of St. Louis

   - Civilian Employment (16 years and older), monthly.
   - Civilian participation rate, monthly.
   - Unemployment rate, monthly.

3. National Income and Products Accounts (NIPA), University of Virginia

   - Real gross domestic product, in 1992 dollar, quarterly.
   - Nominal gross domestic product, quarterly.
   - Personal expenditure on non-durable goods, nominal, quarterly.
   - Personal expenditure on service, nominal, quarterly.
   - Government consumption, nominal, quarterly.
   - Real inventory of farm industry, in 1992 dollar, quarterly.
   - Real inventory of non-durable goods, non-farm industry, in 1992 dollar, quarterly.
   - Real final sales of domestic business, in 1992 dollar, quarterly.

B. Identification of Parameters

We first list the steady state equations of the model. Denoted steady state values of the variables by adding an asterisk. Setting the shocks to zero and requiring all real variables to be stationary, we obtain the following equations from (2.8) – (2.10) and (3.1) – (3.5):

$$c^* = a_b g_b(s^*, i^*) s^* q^* - F I_k a_p f^*; \tag{B.1}$$

$$R^* = [U'(c^*) - (1-\delta_i)\omega_i^*] q^* = (1-\delta_i)\omega_i^* q^* \frac{\lambda_r^*}{\omega_r^*}; \tag{B.2}$$

$$\omega_r^* = \xi U'(c^*) + (1-\xi)(1-\delta_i)\omega_i^*; \tag{B.3}$$

$$\delta_n n^* = v^* \mu^*; \tag{B.4}$$

$$\delta_i i^* = (1-\delta_i)[f(n^*) - g_s(s^*, i^*)\psi^* q^*]; \tag{B.5}$$

$$U'(c^*) = (1-\delta_i)\omega_i^* \left(1 + \frac{\lambda_r^*}{\omega_r^*}\right); \tag{B.6}$$

$$\omega_n^* = \frac{K'(v^*)}{\mu^*}; \tag{B.7}$$

$$\frac{\gamma^*}{\beta} - 1 = g_b(s^*, i^*) s^* \frac{\lambda_r^*}{\omega_r^*}; \tag{B.8}$$

$$\frac{1}{\beta(1-\delta_i)} - 1 = \xi g_s(s^*, i^*) \psi'(i^*) q^* \frac{\lambda_r^*}{\omega_r^*}; \tag{B.9}$$

$$[1 - \beta(1-\delta_n)]\omega_n^* = \beta\left[(1-\delta_i)\omega_i^* f'(n^*) - \varphi\right]; \tag{B.10}$$

$$\frac{\Phi'(s^*)}{g_b(s^*, i^*)} = (1-\xi) R^*. \tag{B.11}$$

Next, we solve the parameters using the restrictions (i) – (vi) in section 4.1, together with the values for $(\beta, \gamma^*, RA, A, \delta_n, \xi, \alpha, \varepsilon_\Phi, B)$. Restriction (i) leads to the following equations:

$$a_p(1+n^*) + u = LP; \quad \frac{u}{a_p(1+n^*) + u} = UR.$$

Hence $u = UR \times LP$ and $a_p(1+n^*) = LP(1-UR)$. Since $IO = i^*/f^*$ and $IIO = \delta_i i^*/f^*$, Restriction (ii) solves $\delta_i = IIO/IO$ and leaves an equation $i^*/f^* = IO$ to be utilized later. Computing the income velocity of money, we can write (iii) as

$$a_p f^* \frac{p}{m} = \frac{a_p f^*}{a_b q^*} = \frac{f^*}{Bq^*} = VI. \tag{B.12}$$

To use this equation, we divide (B.9) by (B.8) and substitute (g_b^*, g_s^*) by (2.4) and (2.3). Then,

$$\Delta \equiv \frac{[\beta(1-\delta_i)]^{-1} - 1}{\xi(\gamma^*/\beta - 1)} = \frac{z^* \psi'(i^*) q^*}{s^*}. \tag{B.13}$$

Substituting $z^* = a_b s^*/(a_p \psi(i^*))$ and using (B.12), we have $i^* \psi'(i^*)/\psi^* = \Delta \times IO \times VI$. Under the functional form of ψ, we have $i^{*\eta} = \eta\psi^* + 1$, and so $i^* \psi'(i^*)/\psi^* = \eta + 1/\psi^*$. Thus,

$$\psi^* = (\Delta \times IO \times VI - \eta)^{-1}. \tag{B.14}$$

As Δ is known by now, this yields ψ^*, which implies $i^* = (\eta\psi^* + 1)^{1/\eta}$.

Also, using (B.5), we can solve for $g_s^*\psi^* q^*$ as

$$g_s^*\psi^* q^* = f^* - \frac{\delta_i}{1-\delta_i} i^* = \frac{1-(1+IO)\delta_i}{1-\delta_i} f^*.$$

Since q^*, f^* and ψ^* are known now, this equation solves for g_s^*. Substituting this solution for g_s^* in (B.9), we can solve for λ_r^*/ω_r^*.

Restriction (iv) helps identify e_f, a_p and n^*. To see this, calculate the wage/output ratio and the hiring cost/wage ratio. Using (2.8), Restriction (iv) implies:

$$\frac{W^* n^*}{f^*} = \frac{\varphi n^*}{\omega_r^* f^*} = .64 \Longrightarrow \varphi = .64 \times \omega_r^* \frac{f^*}{n^*}; \tag{B.15}$$

$$\frac{K_0 v^{*2}}{\omega_r^* W^* n^*} = \frac{K_0 v^{*2}}{.64 \times \omega_r^* f^*} = .02 \Longrightarrow K_0 = .0128 \times \omega_r^* \frac{f^*}{v^{*2}}. \tag{B.16}$$

Substituting (B.16) and (B.4), (B.7) becomes:

$$\omega_n^* = \frac{2K_0 v^*}{\mu^*} = \frac{0.0256 \times \omega_r^* f^*}{\delta_n n^*}. \tag{B.17}$$

Using (B.3) and (B.6), we have

$$(1-\delta_i)\omega_i^* = \omega_r^* \bigg/ \left(1 + \xi \frac{\lambda_r^*}{\omega_r^*}\right).$$

Since $f(n^*) = z_p^*(n^*)^{e_f}$ and $z_p^* = 1$,

$$(1-\delta_i)\omega_i^* f'(n^*) = \frac{e_f \omega_r^* f^*/n^*}{1 + \xi \frac{\lambda_r^*}{\omega_r^*}}. \tag{B.18}$$

Substituting (B.15), (B.17) and (B.18) into (B.10), we obtain:

$$e_f = .64 \left(1 + \xi \frac{\lambda_r^*}{\omega_r^*}\right) \left\{[1-\beta(1-\delta_n)] \frac{2 \times .02}{\beta \delta_n} + 1\right\}. \tag{B.19}$$

Because λ_r^*/ω_r^* has been solved already, this equation identifies e_f. Then we can solve for n^* using $f^* = z_p^*(n^*)^{e_f}$. With the earlier restriction, $a_p(1+n^*) = LP(1-UR)$, we can identify a_p.

Restriction (v) helps identify z_0, z^* and s^*. It implies

$$s^* = .1117 \times .3 \times a_p(1+n^*)/a_b = .03351 \times (1+n^*)/B,$$

where n^* is calculated above and $B = 0.5$. Then we can calculate $z^* = Bs^*/\psi^*$ and $z_0 = g_s^*/z^{*\alpha}$.

Restriction (vi), together with previous restrictions, helps identify the rest of the parameters and steady state values. In the symmetric equilibrium, aggregate output per household ($a_p \hat{f}^*$) is equal to $a_p f^*$. So, Restriction (vi) and (4.1) solve for c^* as follows:

$$c^* = a_p B g_b^* s^* q^* - .269 a_p f^*.$$

Then, we can calculate ω_i^* from (B.6) and ω_r^* from (B.3). Since λ_r^*/ω_r^* is now known, we can retrieve λ_r^*. Also we can pin down φ, K_0 and ω_n^* from (B.15)-(B.17) and φ_0 from (B.11). Table 2 summarizes the identified parameters and steady-state values of variables.

Table 2: Parameter Values and the Steady State

Sample 1 (1959:II – 1998:III)						**Sample 2 (1959:II – 1988:I)**					
β	.9952	a_p	.2479	c^*	.2230	β	.9958	a_p	.2400	c^*	.2184
γ^*	1.01724	e_f	.6694	q^*	1.4239	γ^*	1.02014	e_f	.6693	q^*	1.4879
z_p^*	1	δ_i	.0072	s^*	.1596	z_p^*	1	δ_i	.0072	s^*	.1612
RA	4.0	φ	230.34	v^*	.0550	RA	4.0	φ	248.57	v^*	.0558
FI_k	.2690	φ_0	.5936	n^*	1.3817	FI_k	.2690	φ_0	.6487	n^*	1.4056
B	.5	ε_Φ	2.0	i^*	1.1175	B	.5	ε_Φ	2.0	i^*	1.1303
z_0	11.44	α	.8	ω_r^*	400.51	z_0	10.89	α	.8	ω_r^*	434.67
δ_n	.06	K_0	2103.4	λ_r^*	5.1195	δ_n	.06	K_0	2246.5	λ_r^*	6.3349
A	.6	ξ	.2	ω_i^*	402.40	A	.6	ξ	.2	ω_i^*	436.56
u	.0380	η	-12	ω_n^*	153.56	u	.0376	η	-12	ω_n^*	165.71

Table 3

Sample I (1959:II – 1998:III)			**Sample II (1959:II – 1988:I)**		
γ^*	$\hat{\rho}_m$	$\hat{\sigma}_m$	γ^*	$\hat{\rho}_m$	$\hat{\sigma}_m$
1.01724	.359835	.0099707	1.02014	.187904	.0098569
z_p^*	$\hat{\rho}_p$	$\hat{\sigma}_p$	z_p^*	$\hat{\rho}_p$	$\hat{\sigma}_p$
1	.990509	.0077935	1	.982578	.0085531

Table 4.1: Simulation Results vs. Sample Values (1959:II – 1988:I)

	Simulation results					sample
RA	1.5	2	4	6	8	values
$E(V_c)$	1.2234 (.0055)	1.2235 (.0105)	1.2238 (.0204)	1.2251 (.0251)	1.2245 (.0283)	1.2120
$\sigma(V_c)$	.0056 (.0013)	.0105 (.0030)	.0234 (.0053)	.0313 (.0069)	.0365 (.0077)	.0544
$cv(V_c)$	.4579 (.1096)	.8671 (.2458)	1.9066 (.4246)	2.5534 (.5466)	2.9746 (.6052)	4.48
$corr(V_{ct}, g_{ct-1})$	-.2507 (.0718)	-.2359 (.0678)	-.2498 (.0567)	-.2573 (.0544)	-.2597 (.0518)	-.3775
$corr(V_{ct}, ni_t)$	.0189 (.0900)	.0476 (.0913)	.0968 (.0867)	.1149 (.0852)	.1282 (.0811)	.7020
$corr(\pi_t, \gamma_t)$	.7872 (.0360)	.7539 (.0395)	.6481 (.0554)	.5850 (.0614)	.5427 (.0648)	.1054
$corr(\pi_t, ni_t)$	.6646 (.0639)	.5316 (.0785)	.1780 (.1180)	.0043 (.1299)	-.1033 (.1240)	.6502
$corr(\pi_t, ri_t)$	-.5942 (.0606)	-.6357 (.0583)	-.7485 (.0436)	-.8014 (.0336)	-.8289 (.0297)	-.3531
$E(\pi)$	.0202 (.0012)	.0202 (.0013)	.0202 (.0013)	.0203 (.0013)	.0203 (.0012)	.0114
$\sigma(\pi)$	.0153 (.0010)	.0160 (.0011)	.0187 (.0013)	.0207 (.0014)	.0220 (.0016)	.0068
$E(ri)$	.0043 (.0006)	.0043 (.0007)	.0045 (.0008)	.0046 (.0010)	.0046 (.0010)	.0042
$\sigma(ri)$	.0114 (.0008)	.0140 (.0010)	.0211 (.0014)	.0255 (.0018)	.0283 (.0019)	.0059
$E(ni)$	.0245 (.0011)	.0244 (.0012)	.0245 (.0012)	.0246 (.0013)	.0245 (.0012)	.0155
$\sigma(ni)$	.0126 (.0009)	.0131 (.0009)	.0147 (.0012)	.0158 (.0016)	.0165 (.0018)	.0073
$E(g_m)$	.000041 (.00046)	.000043 (.00049)	.000138 (.00050)	.000143 (.00055)	.000161 (.00056)	.0088
$\sigma(g_m)$	.0094 (.0007)	.0104 (.0008)	.0140 (.0010)	.0166 (.0012)	.0182 (.0013)	.0103

Table 4.2: Simulation Results vs. Sample Values (1959:II – 1998:III)

	Simulation results					sample
RA	1.5	2	4	6	8	values
$E(V_c)$	1.2638 (.0072)	1.2643 (.0139)	1.2660 (.0258)	1.2650 (.0316)	1.2659 (.0351)	1.2705
$\sigma(V_c)$	.0087 (.0013)	.0119 (.0026)	.0231 (.0053)	.0303 (.0068)	.0349 (.0071)	.1179
$cv(V_c)$	.6843 (.0977)	.9442 (.1998)	1.8197 (.4045)	2.3903 (.5171)	2.7547 (.5432)	9.28
$corr(V_{ct}, g_{ct-1})$	-.3693 (.0709)	-.3370 (.0703)	-.2933 (.0604)	-.2866 (.0569)	-.2867 (.0526)	-.3134
$corr(V_{ct}, ni_t)$	.0018 (.0951)	.0198 (.0938)	.0668 (.0933)	.0871 (.0863)	.0912 (.0896)	.0767
$corr(\pi_t, \gamma_t)$	.8236 (.0268)	.8017 (.0302)	.7182 (.0437)	.6640 (.0480)	.6286 (.0563)	.2009
$corr(\pi_t, ni_t)$	.7963 (.0449)	.7060 (.0643)	.4357 (.1140)	.2696 (.1284)	.1809 (.1425)	.6542
$corr(\pi_t, ri_t)$	-.4711 (.0719)	-.5165 (.0684)	-.6382 (.0590)	-.7041 (.0517)	-.7358 (.0453)	-.3774
$E(\pi)$	.0173 (.0015)	.0174 (.0015)	.0174 (.0016)	.0174 (.0016)	.0175 (.0015)	.0102
$\sigma(\pi)$	.0176 (.0012)	.0182 (.0013)	.0203 (.0015)	.0220 (.0016)	.0232 (.0017)	.0064
$E(ri)$	.0049 (.0007)	.0049 (.0007)	.0050 (.0009)	.0051 (.0010)	.0051 (.0010)	.0048
$\sigma(ri)$	.0106 (.0008)	.0130 (.0009)	.0198 (.0014)	.0240 (.0017)	.0267 (.0018)	.0054
$E(ni)$	.0222 (.0015)	.0223 (.0014)	.0223 (.0016)	.0222 (.0016)	.0223 (.0015)	.0150
$\sigma(ni)$	.0159 (.0011)	.0162 (.0011)	.0174 (.0013)	.0183 (.0016)	.0191 (.0019)	.0066
$E(g_m)$	.000036 (.00051)	.000054 (.00049)	.000116 (.00050)	.000122 (.00054)	.000131 (.00053)	.0070
$\sigma(g_m)$	.0105 (.0008)	.0113 (.0009)	.0144 (.0011)	.0166 (.0013)	.0181 (.0014)	.0101

Table 5.1: Cross correlations of search intensity and inventory with productivity[16]
(1959:II – 1988:I)

	corr of s_t with z_p			*corr* of i_t with z_p			sample values
	RA =2	RA =4	RA =6	RA =2	RA =4	RA =6	
$z_{p,t-4}$	-.7302 (.1159)	-.6183 (.1284)	-.5545 (.1349)	.7046 (.0961)	.5598 (.1015)	.5200 (.1121)	.2580
$z_{p,t-3}$	-.8109 (.0790)	-.7154 (.0914)	-.6616 (.0981)	.7966 (.0627)	.6666 (.0790)	.6276 (.0938)	.2504
$z_{p,t-2}$	-.8872 (.0443)	-.8118 (.0566)	-.7682 (.0646)	.8693 (.0404)	.7520 (.0679)	.7139 (.0847)	.2378
$z_{p,t-1}$	-.9487 (.0180)	-.8965 (.0325)	-.8633 (.0427)	.8996 (.0326)	.7924 (.0628)	.7559 (.0789)	.2120
$z_{p,t}$	-.8975 (.0481)	-.8503 (.0491)	-.8174 (.0546)	.8522 (.0536)	.7520 (.0695)	.7197 (.0797)	.1419
$z_{p,t+1}$	-.8489 (.0799)	-.8074 (.0742)	-.7739 (.0786)	.8074 (.0810)	.7143 (.0878)	.6861 (.0913)	.0473
$z_{p,t+2}$	-.8029 (.1086)	-.7674 (.0989)	-.7323 (.1045)	.7651 (.1071)	.6786 (.1094)	.6541 (.1076)	.0121
$z_{p,t+3}$	-.7603 (.1331)	-.7291 (.1217)	-.6929 (.1286)	.7253 (.1306)	.6451 (.1299)	.6239 (.1245)	-.0068
$z_{p,t+4}$	-.7201 (.1539)	-.6924 (.1427)	-.6564 (.1496)	.6874 (.1519)	.6133 (.1486)	.5954 (.1406)	-.0184

[16]Note: (1) The variables i, γ and z_p represent inventory, money growth rate and productivity disturbance, respectively; (2) the data used for inventory is the sum of real inventory of farm industry and real inventory of non-durable goods, non-farm industry.

Table 5.2: Cross correlations of search intensity and inventory with money growth[17]

(1959:II – 1988:I)

	corr of s_t with γ			*corr* of i_t with γ			sample values
	RA =2	RA =4	RA =6	RA =2	RA =4	RA =6	
γ_{t-4}	-.0126 (.1115)	-.0034 (.1111)	-.0104 (.1102)	.0009 (.1091)	.0042 (.1114)	.0046 (.1092)	-.0382
γ_{t-3}	-.0099 (.1103)	-.0030 (.1108)	-.0092 (.1117)	.0047 (.1097)	.0050 (.1100)	.0041 (.1080)	-.0573
γ_{t-2}	.0128 (.1104)	.0062 (.1119)	.0004 (.1119)	.0115 (.1112)	.0066 (.1098)	.0039 (.1082)	-.0902
γ_{t-1}	.1509 (.1136)	.0717 (.1117)	.0515 (.1105)	.0173 (.1115)	.0073 (.1112)	.0040 (.1084)	-.0988
γ_t	.0198 (.1102)	.0118 (.1097)	.0032 (.1096)	.0005 (.1107)	.0024 (.1110)	.0030 (.1085)	-.0984
γ_{t+1}	-.0057 (.1078)	-.0022 (.1092)	-.0027 (.1088)	-.0023 (.1114)	.0004 (.1093)	.0043 (.1074)	-.1393
γ_{t+2}	-.0099 (.1068)	.0002 (.1097)	-.0027 (.1075)	-.0037 (.1117)	.0001 (.1083)	.0044 (.1074)	-.1578
γ_{t+3}	-.0094 (.1093)	-.0002 (.1090)	-.0009 (.1072)	-.0045 (.1113)	.0001 (.1086)	.0046 (.1081)	-.1903
γ_{t+4}	-.0093 (.1107)	-.0014 (.1109)	-.0036 (.1055)	-.0042 (.1109)	-.0003 (.1084)	.0043 (.1096)	-.1423

[17]Note: (1) The variables i, γ and z_p represent inventory, money growth rate and productivity disturbance, respectively; (2) the data used for inventory is the sum of real inventory of farm industry and real inventory of non-durable goods, non-farm industry.

Table 6.1: Sensitivity to B, ε_Φ, α, and ξ (59:II – 88:I)
(using $M2$ as money aggregate)

B	.35	.40	.50	.60	.65
$E(V_c)$	1.2236	1.2238	1.2238	1.2240	1.2239
	(.0212)	(.0204)	(.0204)	(.0204)	(.0216)
$\sigma(V_c)$	.0234	.0234	.0234	.0234	.0233
	(.0056)	(.0058)	(.0053)	(.0056)	(.0054)
$cv(V_c)$	1.9132	1.9130	1.9066	1.9097	1.9017
	(.4513)	(.4675)	(.4246)	(.4515)	(.4310)
ξ	.1	.2	.3	.4	.5
$E(V_c)$	1.2247	1.2238	1.2251	1.2253	1.2245
	(.0202)	(.0204)	(.0232)	(.0214)	(.0223)
$\sigma(V_c)$	.0213	.0234	.0246	.0249	.0252
	(.0050)	(.0053)	(.0057)	(.0060)	(.0061)
$cv(V_c)$	1.7397	1.9066	2.0040	2.0307	2.0533
	(.3991)	(.4246)	(.4598)	(.4805)	(.4848)
ε_Φ	1.0	1.5	2.0	2.5	3.0
$E(V_c)$	1.2246	1.2235	1.2238	1.2233	1.2245
	(.0178)	(.0191)	(.0204)	(.0214)	(.0226)
$\sigma(V_c)$	.0200	.0220	.0234	.0242	.0246
	(.0046)	(.0052)	(.0053)	(.0057)	(.0061)
$cv(V_c)$	1.6324	1.7935	1.9066	1.9751	2.0109
	(.3682)	(.4160)	(.4246)	(.4594)	(.4899)
α	.6	.7	.8	.85	.9
$E(V_c)$	1.2244	1.2243	1.2238	1.2253	1.2263
	(.0148)	(.0178)	(.0204)	(.0227)	(.0244)
$\sigma(V_c)$	.0160	.0201	.0234	.0249	.0267
	(.0037)	(.0049)	(.0053)	(.0061)	(.0063)
$cv(V_c)$	1.3058	1.6389	1.9066	2.0277	2.1752
	(.3010)	(.3989)	(.4246)	(.4856)	(.5013)

Table 6.2. Sensitivity to η (59:II – 88:I)
(using $M2$ as money aggregate)

η	.5	.1	-1	-5	-12	-15
$E(V_c)$	1.2237	1.2237	1.2231	1.2243	1.2238	1.2240
	(.0131)	(.0139)	(.0157)	(.0193)	(.0204)	(.0222)
$\sigma(V_c)$	.0128	.0139	.0159	.0202	.0234	.0241
	(.0035)	(.0039)	(.0043)	(.0052)	(.0053)	(.0054)
$cv(V_c)$	1.0532	1.1370	1.3002	1.6501	1.9066	1.9665
	(.2822)	(.3188)	(.3455)	(.4156)	(.4246)	(.4312)
$corr(V_{ct}, g_{ct-1})$	-.2268	-.2276	-.2264	-.2349	-.2498	-.2557
	(.0645)	(.0635)	(.0601)	(.0604)	(.0567)	(.0572)
$corr(V_{ct}, ni_t)$	.1071	.1009	.1054	.1012	.0968	.0954
	(.0846)	(.0857)	(.0861)	(.0862)	(.0867)	(.0870)
$corr(\pi_t, \gamma_t)$	.8176	.8068	.7787	.7090	.6481	.6310
	(.0310)	(.0332)	(.0380)	(.0448)	(.0554)	(.0567)
$corr(\pi_t, ni_t)$	.3287	.3205	.2890	.2296	.1780	.1665
	(.0931)	(.0967)	(.0992)	(.1059)	(.1180)	(.1258)
$corr(\pi_t, ri_t)$	-.5539	-.5697	-.6101	-.6895	-.7485	-.7639
	(.0641)	(.0612)	(.0586)	(.0499)	(.0436)	(.0404)
E(sales)	1.4846	1.4588	1.4068	1.3092	1.2435	1.2277
	(.0195)	(.0194)	(.0186)	(.0180)	(.0158)	(.0167)
σ(sales)	.0192	.0190	.0185	.0177	.0168	.0166
	(.0084)	(.0084)	(.0081)	(.0071)	(.0059)	(.0059)
cv(sales)	1.2933	1.3024	1.3150	1.3520	1.3510	1.3521
E(output)	1.4988	1.4728	1.4201	1.3213	1.2547	1.2386
	(.0195)	(.0192)	(.0184)	(.0177)	(.0159)	(.0168)
σ(output)	.0232	.0230	.0220	.0203	.0190	.0186
	(.0047)	(.0050)	(.0047)	(.0045)	(.0039)	(.0039)
$\frac{var(\text{output})}{var(\text{sales})}$	1.4600	1.4633	1.4142	1.3154	1.2792	1.2555

Supplementary Appendix

C. The Solution Method

The solution method is similar to Blanchard and Khan (1980). Below we describe the method. Notice that some of the symbols used here do not represent the same meanings as the ones in the text.

The dynamic system has exogenous state variables $Y_z \equiv (\gamma, z_p)^T$, two endogenous variables $Y_s \equiv (i, n)^T$, and three jump variables $Y_d \equiv (\omega_i, v_t, \omega_r)^T$. All other variables can be expressed as deterministic functions of these variables, as discussed in Section 3. The exogenous state variables are characterized by (2.6) and (2.5), while the dynamics of the other five endogenous variables are described by (3.5). Stack Y_d, Y_s and Y_z and denote the resulted 7×1 vector by Y. Then the dynamic system can be written in the following form:

$$F(Y_t, Y_{t+1}) = 0.$$

The steady state of this system Y^* such that $F(Y^*, Y^*) = 0$. Log-linearize the dynamic system, we have:

$$D \begin{bmatrix} y_t \\ \\ y_{t+1} \end{bmatrix} = 0,$$

where the ith element of y_t is $y_{it} \equiv (Y_{it} - Y_i^*)/Y_i^*$, the percentage deviation of the variable Y_{it} from its steady state ($i = 1, 2, ..., 7$). Define the vectors y_s, y_d and y_z similarly. By definition, the steady state value of y is $y^* = 0$.

To solve the saddle path of this linearized system, rewrite it as follows:

$$\begin{bmatrix} y_{st+1} \\ \\ y_{dt+1} \end{bmatrix} = W \begin{bmatrix} y_{st} \\ \\ y_{dt} \end{bmatrix} + Q y_{zt} + R y_{zt+1}, \tag{C.1}$$

where W is a 5×5 matrix; Q and R are 5×2 matrices. For exogenous $\{y_{zt}\}$, this system has two predetermined variables and three jump variables. For the system to be saddle-path stable, the matrix W must have two stable eigenvalues (i.e., those whose absolute values are less than one) and three unstable eigenvalues (i.e., those whose absolute values are greater than one). The calibrated parameter values indeed generate such eigenvalues.

Let J_1 be a 2×2 diagonal matrix whose diagonal elements are the two stable eigenvalues, and J_2 be a 3×3 diagonal matrix whose diagonal elements are the three stable eigenvalues (the eigenvalues are ordered in increasing absolute values along the diagonals of J_1 and J_2). Denote $J = diag(J_1, J_2)$. Write W as $W = C^{-1} J C$, where C^{-1} is the eigenvector matrix corresponding to J. Decompose the matrices C, C^{-1}, Q and R as follows:

$$C = \begin{bmatrix} \underset{(2\times 2)}{C_{11}} & \underset{(2\times 3)}{C_{12}} \\ \\ \underset{(3\times 2)}{C_{21}} & \underset{(3\times 3)}{C_{22}} \end{bmatrix}, \quad C^{-1} = \begin{bmatrix} \underset{(2\times 2)}{B_{11}} & \underset{(2\times 3)}{B_{12}} \\ \\ \underset{(3\times 2)}{B_{21}} & \underset{(3\times 3)}{B_{22}} \end{bmatrix}$$

$$Q = \begin{bmatrix} \underset{(2\times 2)}{Q_1} \\ \\ \underset{(3\times 2)}{Q_2} \end{bmatrix}, \quad R = \begin{bmatrix} \underset{(2\times 2)}{R_1} \\ \\ \underset{(3\times 2)}{R_2} \end{bmatrix}.$$

For given y_{s0}, the saddle-path solution to (C.1) is:

$$\begin{aligned} y_{st} &= B_{11}J_1B_{11}^{-1}y_{st-1} + Q_1 y_{zt-1} + R_1 E_{t-1}y_{zt} \\ &\quad -(B_{11}J_1C_{12} + B_{12}J_2C_{22})C_{22}^{-1} \\ &\quad \times \textstyle\sum_{j=0} J_2^{-j-1}\left[(C_{21}Q_1 + C_{22}Q_2)E_{t-1}y_{zt+j-1} + (C_{21}R_1 + C_{22}R_2)E_{t-1}y_{zt+j}\right], \end{aligned} \tag{C.2}$$

$$\begin{aligned} y_{dt} &= -C_{22}^{-1}C_{21}y_{st} - C_{22}^{-1} \\ &\quad \times \textstyle\sum_{j=0} J_2^{-j-1}\left[(C_{21}Q_1 + C_{22}Q_2)E_t y_{zt+j} + (C_{21}R_1 + C_{22}R_2)E_t y_{zt+j+1}\right]. \end{aligned} \tag{C.3}$$

The exogenous processes (2.6) and (2.5) can be written as $y_{zt+1} = \Gamma y_t + \varepsilon_t$, where ε_t is a vector of *iid* random variables. Then $E_t(y_{zt+j}) = \Gamma^j y_{zt}$ for all $j \geq 0$. Given a draw of innovations and initial values y_{s0}, one can calculate the time path of y_{st} and y_{dt} according to (C.2) and (C.3).