Unskilled Workers in an Economy with Skill-Biased Technology

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first version: November, 1997; this version: April, 1999

Abstract

This paper contributes to the search theory of unemployment by endogenously deriving matching functions and surplus sharing rule for skilled and unskilled workers from a wage-posting game. In contrast to previous wage-posting models, here both sides of the market are heterogeneous and the resulted matching function can exhibit non-constant returns to scale. The model is capable of producing a positive skill premium and a positive wage differential among homogeneous unskilled workers. The skill premium arises from a skill-biased technology; the wage differential among unskilled workers sustains because a lower wage is compensated by a higher chance of getting the job. The model provides useful explanations for the observed dynamic patterns of within-skill and between-skill wage differentials in the 1970s and 1980s and for the relative cyclical volatility of hours of work by different skill groups of workers.

JEL classifications: C78, J31, J64.
Keywords: Wage-posting; Wage differentials; Skill-biased technology; Matching.

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1. Introduction

The U.S. labor market shows a number of interesting regularities of unskilled workers. First, there is sizable wage inequality within unskilled workers. The log weekly wage differential between the $50th$ percentile and the $10th$ percentile of workers in the U.S. was about 0.57 between 1964 and 1988, two thirds of which cannot be explained by skill or age/experience differences (Juhn et al., 1993, Table 2).[1] Second, the dynamic pattern of the wage differential within unskilled workers was in contrast with that of the education premium. While the education premium fell during the 1970s and then rose sharply in the 1980s, the within-group wage differential (unobserved skill price) rose rather steadily in both the 1970s and the 1980s (see Figure 1, reproduced from Juhn et al., 1993, p432). Third, over business cycles, hours of work by low-wage earners are much more volatile than those by high-wage earners, although both are procyclical (Rios-Rull, 1993).

In this paper I construct a theory that is useful for explaining the above facts. The theory has two fundamental elements. First, the labor market is frictional in the sense that not every worker or vacancy is guaranteed a match instantaneously. Second, and more important, firms post wages to attract workers and workers purposefully apply to jobs, endogenously producing matching functions for different skills. The matching rates and wages are different between skills in the same industry, implying skill premium, and different between industries for the same skill, implying a within-skill wage differential. Technological changes, by inducing non-uniform expansions or contractions of industries, generate different responses in the matching rates between skills and between industries and hence affect wage differentials.

The labor market studied here consists of many firms of different technologies (high or low) and many workers of different skills (skilled or unskilled). Skills are observable and complementary with the high technology in the sense that skilled workers' productivity is higher with the high technology than with the low technology. There is a search cost, implicit in the assumption that in any given period a worker can get at most one offer. Firms post wages to attract applicants

[1] See also Levy and Murnane (1992) for a survey.

and workers decide which job to apply to after observing the posted wages. To focus on unskilled workers, I abstract from match-specific productivity and set up the model deliberately so that skilled workers all get the same wage and work only in the high-technology industry.

A positive skill premium arises in this model because skilled workers are favored by high-technology firms for their higher productivity. There is also a wage differential among homogeneous unskilled workers. Since matching is frictional, a high-technology firm does not always succeed in getting a skilled worker, in which case the firm is willing to hire an unskilled worker. Because such unskilled workers have a lower chance of being employed in the high-technology industry than in the low-technology industry, they are compensated by a higher wage than what their identical peers get in the low-technology industry. The expected wage for unskilled workers is equalized between industries.

A skill-biased productivity increase generates a large increase in the skill premium and a moderate increase in wage inequality among unskilled workers. It also reduces skilled workers' unemployment rate and increases unskilled workers' unemployment rate. These effects arise because the skill-biased productivity increase induces an expansion of the high-technology industry and a contraction of the low-technology industry. The demand for skilled workers rises, reducing the unemployment rate of those workers and pushing up the skill premium. The change in the industry composition also attracts more unskilled workers to the high-technology industry. Since unskilled workers have a lower chance to be employed in the high-technology industry than in the low-technology industry, this shift of unskilled workers between industries increases the overall unemployment rate of unskilled workers. The shift also increases the wage inequality among unskilled workers by fattening the upper tail of the wage distribution among unskilled workers. However, the within-skill inequality increases by less than does the skill premium, because unskilled workers' productivity remains unchanged.

In contrast, an increase in the general productivity of all workers induces an expansion of both industries and increases the demand for both skilled and unskilled workers, thus reducing the unemployment rate for all workers. Unskilled workers' unemployment rate falls by more than does skilled workers', because the low-technology industry expands more than the high-technology industry as a result of a lower fixed (capital) cost. The non-uniform expansion attracts some unskilled workers from the high-technology industry to the low-technology industry. Similarly, the wage paid to unskilled workers in the low-technology industry increases by more than does the wage paid to such workers in the high-technology industry. Thus, opposite to the case of a skill-biased productivity increase, wage inequality among unskilled worker shrinks. Although the skill premium increases as in the case of a skill-biased productivity increase, the increase is caused more by the slow growth of unskilled workers' wages in the high-technology industry than by the fast growth of skilled workers' wages.

These results indicate that skill-biased technological progress is a valuable explanation for the concurrent increases in the skill premium and the within-skill wage inequality in the 1980s. In contrast, the opposite movements in the 1970s between the skill premium and the within-skill inequality seem consistent with a general productivity slowdown relative to the capital cost. Finally, consistent with the cyclical behavior of hours of work, unskilled workers' hours of work increase by more than do skilled workers' hours of work when the general productivity increases and decrease by more when the general productivity decreases.

1.1. Relation to the Literature

The patterns of wage inequality have been the subject of research recently. Greenwood and Yorukoglu (1997) and Violante (1996) have forcefully argued that skill-biased technological progress is the reason for the sharply rising skill premium in the 1980s and also for the increase in the within-skill wage inequality. To generate a within-group wage differential, these authors have relied excessively on match-specific productivity and/or complementarity between skilled and unskilled workers. To the extent that these elements are assumed exogenous, the explanation is incomplete. Also, it is unlikely that unskilled workers' productivity depends much on matches, although skilled workers' productivity might do. Thus, it is useful to find alternative explanations for the within-skill wage differential. It is also important to find one model that can explain both the concurrent movement in the 1980s and the opposite movements in the 1970s between the skill premium and the within-skill inequality. This paper takes up both tasks and the results have additional implications on the behavior of hours of work by differential skill groups.

There is also effort in the literature to explain the patterns of wage differentials using search frictions. Employing the standard search framework by Mortensen (1982) and Pissarides (1990), Acemoglu (1998) has shown that search frictions create the possibility that different skills are employed with a single technology (a pooling equilibrium). When the productivity of skills increases to pass a critical level, some firms switch to skill-biased technologies and employ only skilled workers (a separating equilibrium), increasing the skill premium. This switch in equilibrium is likely attributed to the exogenous matching function and the exogenous surplus sharing rule between firms and workers. In particular, in the separating equilibrium (in the static version of that model) no firm wants to hire low-skill workers and yet those workers continue to be matched with firms at the exogenous rate and demand for the fixed share of the match surplus. It is reasonable to think that, if firms have the ability to choose among different production technologies, they must be at least able to adjust their wages to influence their matches. When the matching functions are so endogenized, the wage share and the critical level of skill productivity that divides the separating equilibrium from the pooling equilibrium respond to technological changes. Then the equilibrium switching phenomenon may not occur.

Contributing to the search literature, this paper derives matching functions and surplus sharing rules endogenously from firms' and workers' strategic interplay. The key feature of the wage-posting model is that agents make a trade-off between wages and the associated probabilities. This trade-off seems realistic but is typically absent in the large literature on price/wage search, where workers discover a firm's offered wage only after visiting the firm (see McMillan and Rothschild, 1994, for a survey). The trade-off was first analyzed in a strategic context by Peters (1991) and Montgomery (1991) and subsequently used by Burdett, et al. (1996), Moen (1997), Acemoglu and Shimer (1998), and Shi and Wen (forthcoming).[2]

The main theoretical contribution of the current paper to this wage-posting literature is to derive matching functions for a market where firms and workers are both heterogeneous. In contrast, previous wage/price posting models have assumed that at least one side of the market is homogeneous. Heterogeneity on both sides of the labor market is necessary for discussing the skill premium and skill-biased technology. Matching functions in this environment cannot be obtained by simply adapting the results in previous models. In particular, the matching rate for an unskilled worker cannot be simply assumed as a function of the relative number of such workers to jobs, as done in Acemoglu and Shimer (1998) and Shi and Wen (forthcoming). The difficulty with this short-cut is that the matching rate for each unskilled worker depends on queues of *both* skilled and unskilled workers. The form of this dependence is unknown in the literature and a substantial portion of this paper is devoted to deriving it.

In fact, the average matching rate for an unskilled worker does not have any previously known form. It depends separately on the fraction of unskilled workers and the fraction of high-technology firms as well as on the overall worker/firm ratio and the ratio of skilled workers to high-technology jobs. Thus, the matching function for unskilled workers is not linearly homogeneous, in contrast to previous wage-posting models and traditional search models. Besides generating the desired responses of wages and employment to technological changes, these matching functions also help to reconcile wage-posting models with the paradoxical finding by Holzer et al. (1991) that jobs paying more than the minimum wage attract fewer applicants than do minimum wage jobs. In the current model it is possible for a worker in a short queue to obtain a higher wage than another identical worker in a long queue, provided that there are more workers in the short queue whose skills are above the reference worker's than in the long queue.

[2]Non-strategic analyses of the trade-off include Harris and Todaro (1970) and Carlton (1978). In the Harris-Todaro model the wage difference between sectors is exogenously assumed and agents only migrate slowly between sectors. The current paper endogenously generates such a wage differential from agents' strategic plays and shows that it sustains when firms can instantaneously switch between industries. The strategic analysis also contrasts with Carlton's analysis which exogenously assumes that agents' preferences have a smooth ordering over pairs of prices and service probabilities.

The source of the wage differential among unskilled workers is also different from that in previous models. For example, in Montgomery (1991) firms post different wages for homogeneous workers because some firms derive a higher value of output from the workers than other firms do (e.g., because of different product demands). In this paper all firms derive the same value of output from unskilled workers. Yet, wage inequality arises among these workers because of the skill-biased technology, as explained above. The dependence of the within-skill wage inequality on skill-biased technology is also in contrast to Lang (1991) and Burdett and Mortensen (1998), where homogeneous workers obtain different wages because some receive two wage quotes while others receive only one. In addition, these wage differential models have not studied the joint behavior of the skill premium and the within-group wage differential.

After this long introduction, I now describe the labor market in Section 2. Section 3 characterizes the equilibrium in the limit economy where the numbers of workers and firms approach infinity. Section 4 establishes differences in wages and matching rates among workers. Section 5 examines equilibrium responses to shocks and discusses the empirical facts. Section 6 extends the model. Section 7 concludes the paper and the appendix provides necessary proofs.

2. The Frictional Labor Market

Consider a labor market with N workers and M firms, where N and M are both large numbers. Let $n \equiv N/M$ be the worker/firm ratio. A fraction s of the workers are skilled and denoted with a subscript s; the remaining fraction are unskilled and denoted with a subscript u. Skills are perfectly observable. A fraction H of the firms use a high technology and are denoted with a subscript H; the remaining fraction of firms use a low technology and are denoted with a subscript L. Without loss of generality, let us assume that sN, $(1-s)N$, MH and $M(1-H)$ are all integers. Workers (firms) within each type are identical. Each firm wants to hire one and only one worker. To focus on search unemployment, I assume that workers and firms are both risk neutral, ruling out risk-sharing concerns.

Output depends on skill and technology as follows. An unskilled worker produces y units of output regardless of the technology used (but see Section 6.2). A skilled worker produces θy units of goods with the high technology and y units of goods with the low technology, where $\theta > 1$. Thus, skill and the high technology are complementary. θ is termed the skill-biased productivity and y is termed the general productivity.

The numbers n, M and H are determined in equilibrium by firms' entry, but s and N are fixed for now (see Section 6.1 for an extension). The fixed cost of entry is K_L for the low-technology industry and K_H for the high-technology industry, with $K_H > K_L$. The skill-biased productivity is assumed to be sufficient to cover the higher entry cost of a high-technology firm:

Assumption 1. $\theta > K_H/K_L$.

The matching process between firms and workers is time-consuming. This matching cost is captured here in the simplest way by assuming that each worker can apply to at most one firm in a period (although mixed strategies are allowed). To simplify, I restrict the time horizon to one period and argue in Section 6.3 that most of the results are also valid for a dynamic setting.

Firms and workers do not passively wait for matches dictated by an exogenous matching function as in Mortensen (1982) and Pissarides (1990). Instead, firms post wages to attract workers and workers observe the announced wages before applying. The strategic interactions between firms and workers endogenously generate both the matching function and the split of the match surplus between firms and workers.[3] There is no coordination among firms or workers. Some firms may fail to get any applicant while other firms may have more applicants than they can possibly hire, leaving some workers unemployed.

Given the large numbers of workers and firms, it is natural to focus on symmetric equilibria where ex ante identical firms or workers use the same strategy. Since skilled and unskilled workers have the same productivity in a low-technology firm, such a firm announces the same wage for both types of workers, denoted w_L.[4] A high-technology firm announces a wage w_{Hu}

[3] One can assume instead that each worker observes only two independently drawn wages (see Acemoglu and Shimer, 1997) or that firms announce only reserve wages and hold auctions after receiving applicants (see Julien et al., 1998). These alternative formulations complicate the analysis without changing the qualitative results much.

[4] When the number and composition of firms are fixed, it may be possible that low-technology firms pay different

for unskilled workers and w_{Hs} for skilled workers. Denote $w_H = (w_{Hs}, w_{Hu})$. The wages in the economy are $W \equiv (w_H, \cdots, w_H; w_L, \cdots, w_L)$. Observing the wages, each unskilled worker's application strategy is $P_u \equiv (p_{Hu}, \cdots, p_{Hu}; p_{Lu}, \cdots, p_{Lu})$, where p_{ju} is the probability that he applies to each firm in industry j ($j = H, L$). Similarly, a skilled worker's strategy is $P_s \equiv (p_{Hs}, \cdots, p_{Hs}; p_{Ls}, \cdots, p_{Ls})$. These probabilities depend on the posted wages and so $P_s = P_s(W)$ and $P_u = P_u(W)$. They must add up properly:

$$MH \cdot p_{Hu} + M(1 - H) \cdot p_{Lu} = 1, \tag{2.1}$$

$$MH \cdot p_{Hs} + M(1 - H) \cdot p_{Ls} = 1. \tag{2.2}$$

After workers have carried out their strategies, each firm that has received at least one applicant chooses one applicant (described below) to start production immediately. The worker is paid the specified wage. Unmatched firms and workers obtain nothing.

A low-technology firm is indifferent between all applicants. If the firm received k (≥ 1) applicants, each applicant gets the job with probability $1/k$. In contrast, a high-technology firm strictly prefers skilled applicants. Indeed, Section 3 shows that

$$\theta y - w_{Hs} > y - w_{Hu}. \tag{2.3}$$

That is, for a high-technology firm the ex post gain from hiring a skilled worker is higher than from hiring an unskilled worker. If the firm has received both skilled and unskilled applicants, only skilled applicants are considered and one of them is chosen (with equal probability). Unskilled applicants are considered only when the firm receives no skilled applicant, in which case the firm chooses one from the unskilled applicants it received with equal probability.

Condition (2.3) holds for the following reason. When the skill-biased productivity is high, as in Assumption 1, each high-technology firm tries to attract skilled workers. A wage w_{Hs} that

wages to skilled and unskilled workers. This is unlikely to happen here because the number and composition of firms are endogenous. Given the productive advantage of skilled workers in high-technology firms, there will be sufficiently many such firms looking for skilled workers. If there is any equilibrium where low-technology firms pay different wages to the two types of workers, they must pay more to skilled workers than to unskilled workers. This strategy of a low-technology firm, however, is self-defeating because the ex post profit for hiring a skilled worker is lower than hiring an unskilled worker, which puts skilled workers in a lower priority of selection and so does not attract skilled workers.

reverses the strict inequality in (2.3), although possibly very high, is not attractive to skilled workers because then the firm's ex post incentive is to prefer unskilled workers. A wage w_{Hs} that changes (2.3) into an equality makes the firm ex post indifferent between skilled and unskilled workers. But, in this case posting a marginally lower w_{Hs} would give skilled applicants a priority over unskilled applicants in the line of selection and would make the job much more attractive than before to skilled applicants. Therefore, the best way for a high-technology firm to attract skilled workers is to announce wages that satisfy (2.3).

Workers make a trade-off between the wage and the probability of obtaining it. Let q_{js} be the probability with which a skilled worker gets the job he applies to in industry j $(= H, L)$. Similarly, let q_{ju} $(j = H, L)$ be the corresponding probability for an unskilled worker. Define

$$f(p_1, p_2; a1, a2) \equiv \int_0^1 (1 - \phi p_1)^{a1} (1 - \phi p_2)^{a2} d\phi. \tag{2.4}$$

Lemma 2.1. *The probabilities q's are:*

$$q_{Ls} = f(p_{Ls}, p_{Lu}; sN - 1, (1 - s)N); \tag{2.5}$$

$$q_{Hs} = f(p_{Hs}, 0; sN - 1, (1 - s)N); \tag{2.6}$$

$$q_{Lu} = f(p_{Ls}, p_{Lu}; sN, (1 - s)N - 1); \tag{2.7}$$

$$q_{Hu} = (1 - p_{Hs})^{sN} \cdot f(0, p_{Hu}; sN, (1 - s)N - 1). \tag{2.8}$$

Moreover, $q_{Hs} > q_{Hu}$, *provided* Np_{Hs} *and* Np_{Hu} *are bounded above zero. Thus, when* $N, M \to \infty$, *there cannot be an equilibrium with* $w_{Hs} \geq w_{Hu}$ *if* $p_{Hs}, p_{Hu}, p_{Ls}, p_{Lu}$ *all lie in* $(0, 1)$.

Lemma 2.1 (proved in Appendix A) states that a skilled worker has a better chance of getting a job from a high-technology firm than does an unskilled worker, which is intuitive because of the skill-biased productivity. The additional term $(1 - p_{Hs})^{sN}$ in the formula of q_{Hu} is the probability that a high-technology firm to which an unskilled worker applies has received no skilled applicant, only in which case is the unskilled worker considered by the firm.

Lemma 2.1 also states that, if both types of workers mix in both industries, a skilled worker's wage in a high-technology firm must be lower than an unskilled worker's when the market gets large. To explain, note that the relative *expected* wage between skilled and unskilled workers must be the same in the two industries when both types of workers are indifferent between the two industries. In the low-technology industry, the relative expected wage between the two types of workers approaches unity when the numbers of firms and workers are sufficiently large, since the two types of workers are paid the same wage and in the limit have the same chance of getting the job there. Thus, in the high-technology industry the relative expected wage between the two types of workers must also approach unity. This is possible only when unskilled workers get a higher wage in the high-technology industry than do skilled workers, because unskilled workers have an inferior chance of getting a job there (even in the limit).

In reality skills command a premium, which can be generated in the current framework if skilled workers strictly prefer high-technology jobs, i.e., if $p_{Ls} = 0$, which will be the equilibrium analyzed in this paper. In this case, a high-technology firm can and will offer such wages that attract unskilled workers as well as skilled workers: Attracting only skilled workers would leave a high-technology firm empty-handed when no skill applicant shows up. This is stated below (The proof, presented in Appendix B, can be understood better after reading Section 3):

Lemma 2.2. *If $p_{Ls} = 0$, then $p_{Hu} > 0$ for sufficiently large N and M.*

It is easy to see that an equilibrium cannot be such that all workers apply only to the high-technology industry. Thus, $p_{Lu} > 0$. I can simplify the notation p_{Hs} to p_s, w_{Hs} to w_s and q_{Hs} to q_s. With $p_{Ls} = 0$, the probabilities q's can be explicitly computed from Lemma 2.1 as:

$$\begin{aligned} q_s &= \frac{1-(1-p_{Hs})^{sN}}{sNp_{Hs}}; \quad q_{Lu} = \frac{1-(1-p_{Lu})^{(1-s)N}}{(1-s)Np_{Lu}}; \\ q_{Hu} &= (1-p_{Hs})^{sN} \cdot \frac{1-(1-p_{Hu})^{(1-s)N}}{(1-s)Np_{Hu}}. \end{aligned} \tag{2.9}$$

Lemmas 2.1 and 2.2 indicate that equilibrium characterization is considerably simpler in the limit case $N, M \to \infty$ than in the finite case. In the finite case a single firm's decision affects the probability that workers apply to other firms, affects the probability that workers are chosen by other firms and so changes workers' expected payoffs from applying to other firms. This effect disappears when there are infinitely many firms and workers.

3. The Limit Equilibrium

3.1. Queue Lengths and Workers' Strategies

Now let $N, M \to \infty$ but let the worker/firm ratio remain at $n \in (0, \infty)$ and H lie in the interior of $(0, 1)$. In this limit each firm's decision has no effect on workers' expected payoff from other firms.[5] Let U_u be the expected utility that an unskilled worker gets in the market and U_s be the expected utility for a skilled worker. With the above qualification, U_s and U_u are taken as given by individual firms and are determined in equilibrium later. Note that a skilled worker has the option to apply to a low-technology firm, which yields an expected utility U_u. Since they strictly prefer applying to high-technology jobs, $U_s > U_u$.

In the limit, the probabilities p_s, p_{Hu} and p_{Lu} all approach zero but Np_s, Np_{Hu} and Np_{Lu} are finite and strictly positive. Since it is the latter which enter the calculation of firms' expected profits and worker's expected wages, it is convenient to use the *queue length* – the expected number of workers applying to a firm – in lieu of the probabilities. Let x_s be the queue length of skilled workers applying to a high-technology firm and x_{ju} be the queue length of unskilled workers applying to a firm in industry j $(= H, L)$. Then,[6]

$$x_s = sNp_s, \quad x_{Hu} = (1-s)Np_{Hu}, \quad x_{Lu} = (1-s)Np_{Lu}. \tag{3.1}$$

Since the x'es are simply the p'es rescaled, I will refer to $X_s \equiv (x_s, \cdots)$ as a skilled worker's

[5] In related environments Burdett et al. (1996) and Peters (1998) show that the equilibrium with this restriction is indeed the limit of the equilibrium in the finite economy without this restriction.

[6] For example,

$$x_s = \sum_{k=1}^{sN} kC_{sN}^{k}(p_s)^k (1-p_s)^{sN-k} = sNp_s \sum_{k=1}^{sN} C_{sN-1}^{k-1}(p_s)^{k-1}(1-p_s)^{sN-k} = sNp_s.$$

strategy and $X_u \equiv (x_{Hu}, \cdots; x_{Lu}, \cdots)$ as an unskilled worker's strategy, although the X'es are outcomes of aggregating workers' strategies. The adding-up constraints (2.1) and (2.2) can be rewritten as:

$$x_s = ns/H, \tag{3.2}$$

$$Hx_{Hu} + (1-H)x_{Lu} = n(1-s). \tag{3.3}$$

Each worker also gets the job he applies to with a strictly positive probability. Since $(1-p)^{sN} \to e^{-sNp}$, taking the limit $N, M \to \infty$ on (2.9) yields:

$$q_s = g(x_s),\ q_{Lu} = g(x_{Lu}),\ q_{Hu} = e^{-x_s} g(x_{Hu}),\ \text{ where } g(x) \equiv \frac{1-e^{-x}}{x}. \tag{3.4}$$

The function $g(\cdot)$ defined above is smooth and strictly decreasing. Also, $g(\cdot)$ is strictly convex, with $g(0) = 1$ and $g(\infty) = 0$.

3.2. Firms' Wage Posting Decisions

A firm's wage decision can be expressed as a trade-off between the wage w and the probability of a match, which enters through the queue length x. To characterize the trade-off, let us first consider a deviation by a single low-technology firm from w_L to w_L^d, while all other firms announce the same wages as before. For convenience, number the deviator as the first low-technology firm. The new wages are $W^d = (w_H, \cdots; w_L^d, w_L, \cdots)$. The deviation does not attract any skilled worker: It gives an expected wage U_u that is lower than what a skilled worker can get from applying to a high-technology firm. That is, skilled workers continue to apply only to high-technology firms and so x_s does not change.

Unskilled workers respond to the deviation. Each unskilled worker revises the probability of applying to the deviator from p_{Lu} to p_{Lu}^d, which results in a queue length x_{Lu}^d, where x_{Lu}^d is defined as in (3.1) with p_{Lu}^d replacing p_{Lu}. With large (infinite) numbers of firms and workers, the deviation has a negligible effect on the queue lengths of unskilled workers for other firms, x_{Hu} and x_{Lu}. Thus, an unskilled worker's strategy is $X_u^d = (x_{Hu}, \cdots; x_{Lu}^d, x_{Lu}, \cdots)$.

The deviation must leave an unskilled worker indifferent between the deviating firm and other firms, i.e., $g(x^d_{Lu})w^d_L = U_u$. This indifference curve of an unskilled worker can be rewritten as:

$$w^d_L = IND_{Lu}(x^d_L; U_u) \equiv \frac{U_u}{g(x^d_{Lu})}. \tag{3.5}$$

Since $g(x)$ is a decreasing function, the indifference curve $IND_{Lu}(\cdot; U_u)$ is upward sloping: A higher wage must be accompanied with a longer queue in order to make applicants indifferent between the deviator and a non-deviator. Also, $IND_{Lu}(x; U_u)$ is convex in x, with $IND_{Lu}(0; U_u) = U_u$ and $IND_{Lu}(\infty; U_u) = \infty$. In addition, $IND_{Lu}(x; U_u)$ is increasing in U_u.

Since the function $g(\cdot)$ is smooth, the indifference curve is smooth. A marginal increase in the wage offer by the deviating firm can only attract a marginal increase in the expected number of applicants. Workers do not increase the probability of application in a discrete fashion to respond to a marginally higher wage; If they did, each applicant would have almost zero probability of getting that wage. Similarly, a low-technology firm does not expect to lose all the applicants by cutting the wage offer marginally.

For given U_u, the deviating low-technology firm solves:

$$(PL)\ \max_{w^d_L}\ \pi^d_L = (y - w^d_L)\left(1 - e^{-x^d_{Lu}}\right), \text{ s.t. } w^d_L = IND_{Lu}(x^d_{Lu}; U_u).$$

The solution to this problem can be depicted geometrically. To do so, express the firm's iso-profit curve for any $\pi \in (0, y)$ as

$$w^d_L = ISP_L(x^d_{Lu}; \pi) \equiv y - \frac{\pi}{1 - e^{-x^d_{Lu}}}. \tag{3.6}$$

The iso-profit function $ISP_L(x; \pi)$ is strictly increasing in x, implying that a firm is compensated for a higher wage offer by a higher chance of a match. Also, $ISP_L(x; \pi)$ is concave in x, with $ISP_L(0; \pi) = -\infty$ and $ISP_L(\infty; \pi) = y - \pi$. In addition, $ISP_L(x; \pi)$ is decreasing in π. With the properties of the iso-profit curve and the indifference curve, the problem (PL) has a unique solution depicted by point L in Figure 2.

A deviation by a single high-technology firm can be examined similarly. Let a single high-technology firm deviate from $w_H = (w_s, w_{Hu})$ to $w^d_H = (w^d_s, w^d_{Hu})$, while all other firms continue to announce the same wages as before. Number the deviator as the first high-technology firm so the new wages are $W^d = (w_H^d, w_H, \cdots; w_L, \cdots)$. Observing the new wages, each skilled worker revises the strategy to $X_s^d = (x_s^d, x_s, \cdots)$ and each unskilled worker revises the strategy to $X_u^d = (x_{Hu}^d, x_{Hu}, \cdots; x_{Lu}, \cdots)$. Again, when there are infinitely many workers and firms, the expected numbers of skilled and unskilled applicants for a non-deviator do not change.

The indifference curves for each unskilled and skilled worker are

$$w_{Hu}^d = IND_{Hu}(x_{Hu}^d; U_u, x_s^d) \equiv \frac{U_u e^{x_s^d}}{g(x_{Hu}^d)}; \tag{3.7}$$

$$w_s^d = IND_s(x_s^d; U_s) \equiv \frac{U_s}{g(x_s^d)}. \tag{3.8}$$

These indifference curves have properties similar to those of IND_{Lu}. For given (U_s, U_u), a deviating high-technology firm's maximization problem is:

$$(PH) \max_{(w_s^d, w_{Hu}^d)} \pi_H^d = (\theta y - w_s^d)\left(1 - e^{-x_s^d}\right) + e^{-x_s^d}(y - w_{Hu}^d)\left(1 - e^{-x_{Hu}^d}\right) \text{ s.t. } (3.7), (3.8).$$

The first term of the expected profit is from hiring a skilled worker and the second term is from hiring an unskilled worker when no skilled worker applies to the firm.

It is useful to solve (PH) in two steps. First, for fixed $x_s^d \in (0, \infty)$, w_{Hu}^d solves

$$(PHu) \max_{w_{Hu}^d} \pi_{Hu}^d \equiv (y - w_{Hu}^d)\left(1 - e^{-x_{Hu}^d}\right) \text{ s.t. } (3.7).$$

This problem is similar to (PL) and the "iso-profit" curve has the same functional form $ISP_L(x; \pi)$ as in (3.6). Given (x_s^d, U_u), the unique solution for (PHu) is depicted by point H in Figure 2. Let the maximized value for π_{Hu} from (PHu) be $\pi_{Hu}(x_s^d; U_u)$, which depends on x_s^d because x_s^d affects an unskilled applicant's chance of getting the high-technology job through (3.7).

In the second step, w_s^d solves the following problem for given (U_u, U_s):

$$(PHs) \max_{w_s^d} \pi_H^d = (\theta y - w_s^d)\left(1 - e^{-x_s^d}\right) + e^{-x_s^d}\pi_{Hu}(x_s^d; U_u) \text{ s.t. } (3.8).$$

For any profit level π, the firm's iso-profit curve is

$$w_s^d = ISP_H(x_s^d; \pi, U_u) \equiv \theta y - \frac{\pi - e^{-x_s^d}\pi_{Hu}(x_s^d; U_u)}{1 - e^{-x_s^d}}. \tag{3.9}$$

With suitable restrictions, $ISP_H(x; \pi, U_u)$ is strictly increasing and concave in x. The solution to (PHs) is depicted in Figure 3 by point S, together with the solution to (PHu) (point H).

For the posted wages W and workers' strategies (X_s, X_u) to form an equilibrium, the deviations cannot be profitable and so w_L must solve (PL), w_{Hu} must solve (PHu) and w_s must solve (PHs). These solutions are functions of (U_s, U_u); so are the queue lengths.

3.3. Equilibrium: Definition, Existence and Uniqueness

In equilibrium the queue lengths (x_s, x_{Hu}, x_{Lu}) must satisfy the adding-up restrictions, (3.2) and (3.3), which can be used to solve for workers' expected wages (U_s, U_u). Also, (n, H) must be consistent with firms' entry, yielding zero net-profit in the two industries. That is,

$$\pi_L = K_L, \qquad \pi_H = K_H. \tag{3.10}$$

A *(mixed strategy) limit equilibrium* consists of the worker/firm ratio n, the fraction of high-technology firms H, workers' expected utilities (U_s, U_u), posted wages $W = (w_H, \cdots; w_L, \cdots)$, workers' strategies $X_s = (x_s, \cdots)$ and $X_u = (x_{Hu}, \cdots; x_{Lu}, \cdots)$ such that

(i) (2.3) is satisfied and $U_s > U_u$;

(ii) A skilled worker is indifferent between high-technology firms, i.e., $x_s \in (0, \infty)$; an unskilled worker is indifferent between all firms, i.e., $x_{Lu}, x_{Hu} \in (0, \infty)$;

(iii) Given (U_s, U_u) and other firms' wages, each firm's w_L solves (PL) and w_H solves (PH);

(iv) U_s and U_u, entering through (x_s, x_{Hu}, x_{Lu}), satisfy (3.2) and (3.3);

(v) The numbers (n, H) are such that firms earn zero net profit.

An equilibrium can be found by first solving the queue lengths and wages for given (n, H) and then invoking the zero net-profit conditions. Imposing the equilibrium requirements $x^d_{Lu} = x_{Lu}$, $x^d_{Hu} = x_{Hu}$ and $x^d_s = x_s$ in the first-order conditions of (PL), (PHu) and (PHs) yields:

$$x_{Lu} = \ln\left(\frac{y}{U_u}\right), \qquad w_L = \frac{U_u}{g(x_{Lu})}; \tag{3.11}$$

$$x_{Hu} = x_{Lu} - x_s, \qquad w_{Hu} = \frac{U_u e^{x_s}}{g(x_{Hu})}; \tag{3.12}$$

$$x_s = \ln\left(\frac{(\theta-1)y}{U_s - U_u}\right), \qquad w_s = \frac{U_s}{g(x_s)}. \tag{3.13}$$

The wages come directly from workers' indifference curves. The queue lengths can be interpreted as follows. Consider first an unskilled worker who applies to a low-technology firm. The wage share of output determined by the firm is $x_{Lu}/(e^{x_{Lu}}-1)$, which is intuitively a decreasing function of the queue length of such workers. Since the worker gets the job with probability $(1-e^{-x_{Lu}})/x_{Lu}$, the worker's expected wage is $e^{-x_{Lu}}y$. Equating this to U_u yields the expression for x_{Lu} in (3.11). If the unskilled worker applies to a high-technology firm, he faces a wage share $x_{Hu}/(e^{x_{Hu}}-1)$ and a probability of getting the job $e^{-x_s}(1-e^{-x_{Hu}})/x_{Hu}$. The expected wage is $e^{-(x_s+x_{Hu})}y$ which must be the same as that from applying to a low-technology firm, yielding $x_s + x_{Hu} = x_{Lu}$. Similarly, a skilled worker would be rewarded an expected wage $e^{-x_s}\theta y$ if he did not crowd out unskilled workers. But a skilled worker does crowd out unskilled workers and such crowding-out matters to the firm when the firm does not get any skilled applicant. The expected loss in profit from such crowding-out is $ye^{-x_s}(1-e^{-x_{Hu}})$, where $e^{-x_s}(1-e^{-x_{Hu}})$ is the probability that the firm receives some unskilled applicants but no skilled applicant. Taking this crowding-out effect into account, the firm rewards a skilled worker with an expected wage $e^{-x_s}\theta y - e^{-x_s}y(1-e^{-x_{Hu}})$. Equating this to U_s and substituting x_{Hu} yields the condition for x_s in (3.13).

Substituting (3.11) – (3.13) into the adding-up conditions (3.2) – (3.3) yields:

$$x_s = \frac{ns}{H}, \quad x_{Hu} = n - \frac{ns}{H}, \quad x_{Lu} = n; \tag{3.14}$$

$$U_u = ye^{-n}, \qquad U_s = y\left[e^{-n} + (\theta-1)e^{-ns/H}\right]. \tag{3.15}$$

Finally, substituting (3.14) and (3.15) into the zero net-profit conditions yields:

$$1-(1+n)e^{-n} = \frac{K_L}{y}; \tag{3.16}$$

$$1-\left(1+\frac{ns}{H}\right)e^{-ns/H} = \frac{K_H - K_L}{(\theta-1)y}. \tag{3.17}$$

Denote the left-hand side of (3.16) by $B(n)$ and its inverse function by $B^{-1}(\cdot)$. Then the left-hand side of (3.17) is $B(ns/H)$. Denote

$$\bar{s} \equiv \frac{B^{-1}\left((K_H - K_L)/[(\theta-1)y]\right)}{B^{-1}(K_L/y)}. \tag{3.18}$$

The following proposition is shown in Appendix C:

Proposition 3.1. *With Assumption 1 and $s < \bar{s}$, the limit equilibrium defined above exists and is unique. In particular, (2.3) is satisfied and $U_s > U_u$.*

The condition $s < \bar{s}$ ensures $H < 1$. Assumption 1 delivers $H > s$, which is necessary and sufficient for both x_s and x_{Hu} to be strictly positive (and finite). The same assumption delivers (2.3) and so high-technology firms prefer hiring skilled workers. The reason why a high θ is necessary for $x_{Hu} > 0$ is as follows. Only when the productivity advantage of skilled workers is high enough are there enough high-technology firms entering the industry to compete for skilled workers. In this case high-technology firms fail to find a skilled worker with a high probability, making it attractive for unskilled workers to apply to those firms.[7]

4. Properties of the Limit Equilibrium

4.1. Matching Rates and Unemployment Rates

The two types of workers experience different matching rates and unemployment rates. Let the average matching rate be α_s for a skilled worker and α_u for an unskilled worker. Then,

$$\alpha_s \equiv \frac{N_s}{sN} = \frac{\left(1 - e^{-ns/H}\right)}{ns/H}; \tag{4.1}$$

$$\alpha_u \equiv \frac{N_{Hu} + N_L}{(1-s)N} = \frac{1 - e^{-n} - H(1 - e^{-ns/H})}{n(1-s)}. \tag{4.2}$$

The following proposition can be shown directly and the proof is omitted:

Proposition 4.1. *Skilled workers have a higher matching rate than unskilled workers, i.e., $\alpha_s > \alpha_u$, and a lower unemployment rate.*

[7] In the more general environment (see Section 6.2) where an unskilled worker generates a higher value of product in a high-technology firm than in a low-technology firm, it is possible that $x_{Hu} > 0$ even when $H < s$.

As in reality, the endogenous matching functions generate a higher matching rate and hence a lower unemployment rate for skilled workers than for unskilled workers. The matching rates for the two skills differ not only in magnitudes but also in functional forms. Skilled workers' matching rate is a nice decreasing function of the ratio of the number of skilled workers to the number of high-technology firms (ns/H). In contrast, unskilled workers' matching rate depends separately on the skill composition s, the firm composition H and the overall worker/firm ratio n. The corresponding matching function does not exhibit constant returns to scale, in contrast to the standard search theory and previous wage-posting models.

Aggregate matching rates depend only on the overall worker/firm ratio and hence exhibit constant returns-to-scale. For workers, the aggregate matching rate is:

$$\alpha \equiv s\alpha_s + (1-s)\alpha_u = \frac{1-e^{-n}}{n}. \tag{4.3}$$

On the firms' side, since $x_s + x_{Hu} = x_{Lu} = n$, a firm gets the same expected number of applicants, regardless of which industry the firm is in, and so the matching rate is $1 - e^{-n}$ for all firms.

4.2. Wage Differentials

The equilibrium possesses positive wage differentials both between skills and within unskilled workers. By construction, there is no wage differential between skilled workers. Let us start with the wage differential within unskilled workers, which is summarized in the following proposition (see Appendix D for a proof):

Proposition 4.2. *$w_{Hu} > w_L$. That is, an unskilled worker in a high-technology firm is paid a higher wage than an identical unskilled worker in a low-technology firm.*

The explanation for the wage differential within unskilled workers is simple. An unskilled worker who applies to a high-technology job has a lower probability to get the job than does an identical unskilled worker who applies to a low-technology job. To compensate for this lower probability, high-technology firms must offer a higher wage to unskilled applicants than do low-technology firms. Figure 2 illustrates this wage differential. The indifference curve for an unskilled worker applying to a high-technology firm, $IND_{Hu}(x_{Hu}; U_u, x_s)$, lies above the indifference curve for an unskilled worker applying to a low-technology firm, $IND_{Lu}(x_{Lu}; U_u)$. Since the iso-profit curves in the two cases have the identical functional form, point H lies northwest of point L, yielding $w_{Hu} > w_L$.

Although the skill-biased technology does not increase unskilled workers' productivity, it is critical for the wage differential among unskilled workers: Such differential would not exist if $\theta = 1$. This contrasts to Montgomery (1991) and Lang (1991). The importance of skill-biased technology links the model to Greenwood and Yorukoglu (1997) and Violante (1996), but the fundamental reason for the within-skill inequality is different here. Unskilled workers in the high-technology industry earn higher wages than their peers in the low-technology industry not because they have additional match-specific productivity with the firms, nor because they are complementary with skilled workers in production, but because they bear a higher risk of failing to get the job.

The wage differential within unskilled workers is also a wage differential between industries. The existence of an inter-industry wage differential is consistent with the evidence in Katz and Summers (1989) but, in contrast to their interpretation of such a differential as an industry rent, here unskilled workers are indifferent between the two industries ex ante.

It should also be emphasized that, despite the higher wage which an unskilled worker gets in the high-technology industry than in the low-technology industry, the worker does not face a longer queue in the high-technology industry but rather a less favorable queue. Although the queue lengths of workers for a firm in the two industries are both equal to n, an unskilled worker faces a queue in the high-technology industry that has more skilled workers. Thus, failing to observe a positive correlation between the wage differential and the queue length differential does not necessarily imply that workers do not make the trade-off between the wage and the associated probability: To make this inference one must also ensure that the applicants queueing for different wages have the same quality. Therefore, the paradoxical finding in Holzer et al. (1991), that jobs paying more than the minimum wage attract fewer applicants than do minimum wage jobs, can be consistent with workers' trade-off between the wage and the associated probability if jobs paying more than the minimum wage attracts better applicants.

Now let us turn to wage differentials between skills. The result $U_s > U_u$ in Proposition 3.1 states that a skilled worker obtains a higher *expected* wage from the market than does an unskilled worker. An important reason for this positive difference is that skilled workers have a better chance of getting a job. To generate a positive skill premium in terms of actual wages, θ must be large enough, as stated below (see Appendix D for a proof):

Proposition 4.3. *Skilled workers obtain higher expected wages than unskilled workers, i.e., $U_s > U_u$. In the high-technology industry, skilled workers obtain higher actual wages, i.e., $w_s > w_{Hu}$, if and only if $\theta > \max\{\theta_1, K_H/K_L\}$, where θ_1 is defined in Appendix D.*

Measures of wage differentials used in practice take into account of both the relative wage and the employment distribution. To define wage differentials, let N_s be the number of employed skilled workers, N_{Hu} be the number of unskilled workers employed in the high-technology industry, and N_L be the number of unskilled workers employed in the low-technology industry. Then,[8]

$$N_s = MH(1 - e^{-ns/H});\ N_{Hu} = MH(e^{-ns/H} - e^{-n});\ N_L = M(1-H)(1 - e^{-n}).$$

Let $\ln(AU)$ be the weighted average log wage of unskilled workers, calculated as:

$$\ln(AU) = \frac{N_{Hu}}{N_{Hu} + N_L} \ln w_{Hu} + \frac{N_L}{N_{Hu} + N_L} \ln w_L.$$

Denote RB as the log relative average wage between skilled and unskilled workers and RE as the log relative expected wage between skilled and unskilled workers. Denote RU as the log relative wage within unskilled workers between the two industries and RH as the log relative wage between skilled and unskilled workers in the high-technology industry. Then,

$$RB = \ln\left(\frac{w_s}{AU}\right);\ RE = \ln\left(\frac{U_s}{U_u}\right);\ RU = \ln\left(\frac{w_{Hu}}{w_L}\right);\ RH = \ln\left(\frac{w_s}{w_{Hu}}\right). \tag{4.4}$$

[8] For example, in the calculation of N_s, MH is the number of high-technology firms and $(1 - e^{-ns/H})$ is the probability with which each high-technology firm successfully hires a skilled worker.

Wage differentials are defined as standard deviations in log wages of the corresponding group of employed workers. Let DU be the wage differential within unskilled workers, DH be the between-skill wage differential in the high-technology industry, DB be the between-skill wage differential in terms of average log wages of the two types of workers, and DT be the overall wage differential. DU is a measure of within-skill differential, while DH and DB are between-skill wage differentials. DH is a narrower measure of skill premium than DB since it is a within-industry wage differential. Direct computation yields:

$$DU = \frac{(N_{Hu}N_L)^{1/2}}{N_{Hu} + N_L} RU; \tag{4.5}$$

$$DH = \frac{(N_s N_{Hu})^{1/2}}{N_s + N_{Hu}} RH; \quad DB = \frac{[N_s(N_{Hu} + N_L)]^{1/2}}{N_s + N_{Hu} + N_L} RB; \tag{4.6}$$

$$DT = \left[a_s(1 - a_s)(RH)^2 + 2a_s(1 - H)RH \cdot RU + H(1 - H)(RU)^2\right]^{1/2}, \tag{4.7}$$

where $a_s = N_s/(N_s + N_{Hu} + N_L)$. All wage differentials are positive.

5. Equilibrium Responses to Productivity Shocks

5.1. A Skill-Biased Productivity Increase

Consider an increase in the skill-biased productivity θ. The effects are summarized in the following proposition, whose proof is straightforward and omitted:

Proposition 5.1. *A skill-biased productivity increase has the following effects:*

$$\frac{dn}{d\theta} = 0, \ \frac{dH}{d\theta} > 0; \ \frac{dx_s}{d\theta} < 0, \ \frac{dx_{Lu}}{d\theta} = 0; \ \frac{d\alpha_s}{d\theta} > 0, \ \frac{d\alpha_u}{d\theta} < 0;$$

$$\frac{dU_s}{d\theta} > 0, \ \frac{dU_u}{d\theta} = 0; \ \frac{dw_L}{d\theta} = 0, \ \frac{dw_{Hu}}{d\theta} < 0, \ \frac{dw_s}{d\theta} > 0.$$

Let me explain these effects one at a time. The skill-biased technological progress increases the profit of high-technology firms and induces firms to enter the high-technology industry. (3.17) implies that the fraction of high-technology firms increases, but (3.16) implies that the overall worker/firm ratio is unchanged. Thus, the total number of firms is unchanged and the increase in the number of high-technology firms is matched one for one by the decrease in the number of low-technology firms. The skill-biased technological progress stimulates the high-technology industry at the expense of the low-technology industry.

Since there are now more high-technology firms, each attracts a smaller expected number of skilled applicants (x_s) and so the matching rate for skilled workers, α_s, increases. The relative expansion of the high-technology industry increases the probability with which unskilled workers get jobs there and so these workers increase the probability of applying to high-technology firms. This has two implications for unskilled workers' matching rate. First, the reduction in the application probability of unskilled workers to low-technology firms matches the reduction in the number of low-technology firms. Thus, the queue length of applicants for each low-technology firm is unchanged; so is each applicant's probability of getting a low-technology job. Second, as unskilled workers switch in application probability from an industry in which they are more likely to be employed to the other in which they have a lower priority of being selected, the average matching rate for unskilled workers, α_u, falls.

The overall matching rate in the economy is unchanged by the increase in θ, since the overall worker/firm ratio is unchanged. The increased matching rate for skilled workers is matched one for one by the fall in unskilled workers' matching rate. The queue length of workers for each firm does not change either, since it equals n in equilibrium.

The responses of wages are tied to those of the matching rates. First, since the queue length of workers for each low-technology firm does not change, as argued above, an applicant's trade-off between the wage and the probability of getting the low-technology job is the same as before. Since workers' productivity in the low-technology industry is also the same as before, the wage rate must be the same as before, i.e., w_L does not change. Since neither the wage nor the probability of getting a job in the low-technology industry changes, the expected wage for an unskilled worker, U_u, does not change (see (3.15)). The solution to a low-technology firm's problem continues to be depicted by point L in Figure 2.

Second, the wage posted by a high-technology firm for unskilled workers, w_{Hu}, falls; so does the average wage for employed unskilled workers. This is because the increased number of high-technology jobs makes it easier for an unskilled worker to obtain a high-technology job than before. High-technology firms can reduce the wage offered to unskilled workers and yet keep them indifferent between the two types of jobs. In Figure 2, a fall in x_s shifts southeast the indifference curve of an unskilled worker who applies to a high-technology job, inducing w_{Hu} to fall. Since w_L remains unchanged and w_{Hu} falls, the average wage for employed unskilled workers falls.

Third, the wage posted by high-technology firms for skilled workers, w_s, increases. So does the expected wage for skilled workers, U_s. The expected wage increases by more than does the actual wage because the probability for a skilled worker to get a job also increases when the number of high-technology firms increases.

The relative wage between skills in the high-technology industry, w_s/w_{Hu}, increases. Employment in the high-technology industry increases. So does the fraction of unskilled workers employed there, $N_{Hu}/(N_s + N_{Hu})$, as more unskilled workers apply to that industry. Thus, more workers in that industry are earning low wages, adding to the lower tail of the wage distribution in the high-technology industry. This change in the skill distribution re-enforces the increase in the relative wage w_s/w_{Hu} in generating a large increase in the between-skill wage differential in the high-technology industry, DH.

The wage differential within unskilled workers, DU, responds to θ ambiguously. On the one hand, the relative wage within unskilled workers, w_{Hu}/w_L, falls, which reduces the within-skill wage differential. On the other hand, there are more unskilled workers who are now employed in the high-technology industry, which adds to the upper tail of the wage distribution among unskilled workers and increases the corresponding wage differential. Analytically it is not clear whether the response of the relative wage or that of the wage distribution dominates.

To illustrate the wage differentials, let us consider a realistic example. Normalize $y = 10$. To circumvent the difficulty of precisely defining skill categories, I choose $s = 0.2$, match RU with the 50-10 percentile log relative wage and match RH with the 90-50 percentile log relative wage. Sample values (U.S. data) for these log relative wages can be found in Juhn et al. (1993, Table 2). The 50-10 percentile log relative wage is 0.50 in 1964 and 0.64 in 1988, with an average value 0.57. The 90-50 percentile log relative wage is 0.44 in 1964 and 0.54 in 1988, with an average value 0.49. According to the decomposition in Juhn et al. (1993, Table 4), about a third of the changes in the 50-10 percentile log relative wage is due to skill changes, which the measure RU does not capture. Thus, I match RU with the remainder, i.e., $RU = 0.57 \times 2/3 \approx 0.38$. Also, about 42% of the changes in the $90-50$ percentile log relative wage is due to factors other than skills. Since RH in the current model is generated solely by the skill difference, I set $RH = 0.49 \times 58\% \approx 0.285$. Finally, the overall wage/output ratio is set to the realistic value 0.64. The procedure yields: $K_L = 2.15$, $K_H = 3.51$, and $\theta = 1.912$, which satisfy Assumption 1.

Now I increase θ from its base value 1.912 to 2.062, with a step 0.015, and compute the equilibrium for each step. Figures 4.1 and 4.2 depict the responses of wage differentials and log relative wages. First, confirming the above analysis, the skill-biased productivity progress increases log relative wages between skills, RH and RB, and widens between-skill wage differentials, DH and DB. Second, the wage differential within unskilled workers, DU, increases, despite the fall in RU. This indicates that the shift in employment of unskilled workers from the low-technology industry to the high-technology industry generates a dominating effect on the wage differential within unskilled workers. Third, between-skill wage differentials increase by much more than does the within-skill wage differential. Finally, the overall wage differential increases.

These responses can be contrasted to those in Acemoglu (1998). First, the skill-biased productivity increase reduces the unemployment rate for skilled workers. Second, the expansion of the high-technology industry attracts unskilled workers as well as skilled workers. These effects seem natural, but the opposite occur in Acemoglu. The differences can be attributed to the exogenous matching function and exogenous wage share assumed there. In particular, when the high-technology industry expands in the current model, those firms still have incentive to employ some unskilled workers by offering a lower wage share to them. In Acemoglu (1998), firms are stuck with the exogenous wage share which eliminates the incentive for high-technology firms to hire unskilled workers.

5.2. A General Productivity Increase

Increasing the general productivity y has the following effects (see Appendix E for a proof).[9]

Proposition 5.2. *A general productivity increase has the following effects:*

$$\frac{dn}{dy} < 0, \ \frac{dH}{dy} < 0; \ \frac{dx_s}{dy} < 0; \ \frac{dx_{Lu}}{dy} < 0; \ \frac{d\alpha_s}{dy} > 0, \ \frac{d\alpha_u}{dy} > 0;$$
$$\frac{dU_s}{dy} > 0, \ \frac{d(U_u/U_s)}{dy} > 0; \ \frac{dw_s}{dy} > 0, \ \frac{dw_{Hu}}{dy} > 0; \ \frac{d(w_L/w_{Hu})}{dy} > 0.$$

The general productivity increase makes firms' entry profitable in both industries, generating a lower overall worker/firm ratio, n, and a lower ratio of skilled workers to high-technology firms, ns/H. Consequently, the matching rates for skilled and unskilled workers both rise, resulting in an increase in the overall matching rate for each worker. Since the demand for labor is higher and workers' productivity is higher now than before, expected wages for skilled and unskilled workers both rise. As indicated by (3.15), increases in U_u and U_s come from both the increase in y and the reductions in queue lengths $(n, ns/H)$.

The expansion is not uniform between industries. Proposition 5.2 indicates that the low-technology industry expands by more than does the high-technology industry and so the fraction of high-technology firms in the economy falls. Intuitively, with a lower fixed cost of entry, a low-technology firm's net profit responds by more proportionally to a multiplicative increase in the general productivity than does a high-technology firm's net profit, which must be eliminated in equilibrium by a relatively larger entry of new firms into the low-technology industry. Technically,

[9] The effects here should be more generally interpreted as those of uneven increases between productivity and capital costs (K_L, K_H). This exercise is useful because capital costs in the 1970s might have grown more rapidly than productivity, due to energy shocks and high inflation.

$B'(x)/B(x)$ is a decreasing function of x (see (3.16)). Since $x_s < x_{Lu}$ $(= n)$, a larger decrease in x_{Lu} is required than in x_s to eliminate the increase in profit brought about by the increase in y. That is, n decreases by more than ns/H does, implying a decrease in H.

Because of the non-uniform expansion, the matching rate for unskilled workers increases by more than does the matching rate for skilled workers. The expected wage for unskilled workers, U_u, increases by more than does the expected wage for skilled workers, U_s.

The wages, (w_s, w_{Hu}, w_L), all rise when y increases, but not in the same proportion. First, the relative wage within unskilled workers, w_{Hu}/w_L, falls. A simple explanation is that the relatively large increase in the revenue of a low-technology firm is shared by a relatively large increase in the corresponding wage. Specifically, the relatively large expansion of the low-technology industry increases the industry's relative demand for workers and, to attract applicants, low-technology firms increase wage offers by a large proportion. This higher wage induces the queue length of unskilled workers for each low-technology firm, n, to increase relative to that for each high-technology firm, $n - ns/H$, although both decrease in response to the increase in y. The response of the relative wage within unskilled workers can be seen from Figure 2: An increase in U_u shifts both IND_{Lu} and IND_{Hu} up northwest, but the shift in IND_{Hu} is smaller because x_s is smaller, reducing the relative wage w_{Hu}/w_L.

Second, the relative wage w_s/w_{Hu} increases. This is because unskilled workers switch in the application probability from the high-technology industry to the low-technology industry. This switch reduces the congestion of unskilled applicants in each high-technology firm relative to the congestion of skilled applicants. Conditional on applying to the high-technology industry, an unskilled worker's chance of getting a job increases by more than a skilled worker's chance does. To offset this relative change in the chance of getting a job, a skilled worker's wage must increase relative to an unskilled worker's in the high-technology industry. It is worthwhile emphasizing that the response of w_s/w_{Hu} is opposite to that of w_{Hu}/w_L and so the overall between-skill relative wage, RB, may either increase or decrease.

As unskilled workers switch in the application probability from the high-technology industry to the low-technology industry, the upper tail of the wage distribution within unskilled workers becomes thinner, which reinforces the fall in the relative wage w_{Hu}/w_L to narrow the wage differential within unskilled workers, DU. The same shift in employment reduces the lower tail of the wage distribution in the high-technology industry, which mitigates the increase in the relative wage between skills. The response of the between-skill wage differential in the high-technology industry, DH, is ambiguous analytically. So are the responses of the overall between-skill wage differential, DB, and the overall wage differential among all workers, DT.

Let us consider the numerical example in the last subsection. Fix θ at the initial value, increase y from its base value 10 to 12.5, with a step 0.25, and compute the equilibrium for each step. Figures 5.1 and 5.2 illustrate the responses of wage differentials and log relative wages. First, as analyzed above, the log relative wage RU and the wage differential DU within unskilled workers both fall. Second, the log relative wage in the high-technology industry RH and the corresponding wage differential DH increase, indicating that the rise in the log relative wage RH outweighs the negative effect on DH of the change in the skill distribution in this industry. Third, the overall log relative wage between skills RB and the corresponding wage differential DB both fall, but the magnitudes are very small. Finally, the overall wage differential DT falls slightly.

5.3. Discussion

The above results are useful for explaining the empirical facts listed in the introduction. First, Juhn et al. (1993) have found that both the within-skill and between-skill wage differentials rose during the 1980s, with the skill premium rising much faster than the within-skill wage differential (see Figure 1). The current model shows that both the concurrent increase and the relative magnitude of changes in the two wage differentials can be generated by skill-biased technological progress (see Figure 4.1). The skill-biased technological progress causes the within-group wage differential among unskilled workers to rise because it induces an expansion of the high-technology industry relative to the low-technology industry and shifts unskilled workers from the low-technology to the high-technology industry. In contrast, a general productivity shock generates an expansion of the low-technology industry relative to the high-technology industry and causes the between-skill and within-skill wage differentials to move in opposite directions.

Second, the within-skill wage differential was rising while the skill premium was falling in the 1970s, with the overall wage differential rising slowly (Juhn et al., 1993). These opposite movements between the skill premium and the within-skill differential are in sharp contrast with the pattern in the 1980s. The opposite movements in the two wage differentials are inconsistent with skill-biased technological progress but consistent with a general productivity slowdown. If a decrease in y in the model is re-interpreted as a slowdown in the growth of general labor productivity relative to capital costs, Figure 5.1 shows that such a slowdown reduces the between-skill wage differential and increases the within-skill wage differential, while the overall wage differential rises by a magnitude much smaller than in the case of skill-biased technological progress.

Third, hours of work are procyclical and exhibit higher volatility for low wage earners than for high wage earners (Rios-Rull, 1993). The current model, suitably extended into a stochastic environment, is likely to deliver such a relative volatility if cycles are primarily driven by shocks to the general productivity. To see this, recall that an increase in the general productivity increases the matching rate for unskilled workers relative to skilled workers. Thus, low-wage earners' hours of work increase by more in good times and also decrease by more in bad times than do high-wage earners' hours of work.

The relatively more procyclical hours of work by unskilled workers are accompanied by counter-cyclical skill premium in average wages (Figure 5.2). That is, in good times skilled workers' average wage rises by less than unskilled workers' and in bad times it also falls by less. This counter-cyclical skill premium is realistic and has been found important for explaining the relative volatility of hours of work by different skill groups (Kydland, 1995). However, previous business cycle models typically do not distinguish between industries and so it is not clear whether the relative volatility of hours of work by different skill groups also entails a counter-cyclical skill premium within each industry. The current model provides a negative answer: When there is an increase in the general productivity, the skill premium in the high-technology industry, DH, increases rather than falls (Figure 5.1).

6. Extensions

The analysis so far has assumed a fixed fraction of skilled workers, a uniform productivity of unskilled workers across industries and a one-period setting. In this section I relax these restrictions one at a time to check the sensitivity of the results. Relaxing the second assumption also allows me to examine a sectorial shock.

6.1. The Supply of Skills

There can be many ways to endogenize the supply of skills. Since my purpose here is to check the sensitivity of the results, it suffices to adopt the following specification:

$$s = S\left(\frac{U_s}{U_u}\right) = b \cdot \ln\left(\frac{U_s}{U_u}\right), \quad b > 0. \tag{6.1}$$

This specification is intended to capture the following general features: (i) A higher relative expected wage for skilled workers attracts more workers to upgrade their skills ($S' > 0$); (ii) $s > 0$ only if $U_s > U_u$; (iii) The attraction of a higher expected wage diminishes as the relative expected wage increases ($S'' < 0$).

With this modification, I can examine the responses of the equilibrium to technological shocks and, to economize on space, only a skill-biased productivity increase is discussed here. Setting the initial value of s to the number 0.2 used in previous calculation yields $b = 0.27$. The responses of wage differentials and log relative wages to an increase in θ are very similar to those in Figures 4.1 and 4.2 and hence are not depicted here. The only difference is a slight change in magnitudes. In particular, the log relative wage within unskilled workers, RU, falls by less than when s is fixed. This is because the skill-biased technological progress increases the relative wage U_s/U_u and attracts more workers to become skilled. As the number of skilled workers increases, unskilled workers who apply to high-technology firms get jobs with a lower probability than in the case of a fixed s. For unskilled workers to be now indifferent between the jobs in the two industries, the relative wage w_{Hu}/w_L falls by less than before.

6.2. An Industry-Specific Productivity/Demand Increase

Let me now relax the assumption on productivity but retain the assumption of a fixed s. Allowing the products to be physically different between the two industries, I re-interpret y as the value of an unskilled worker's product in a low-technology firm and re-interpret θy accordingly for a skilled worker in a high-technology firm. An unskilled worker's value of product in a high-technology firm is denoted $y\theta_u$, where θ_u can differ from one. Since a worker's value of product depends on both the worker's productivity and the product demand, $\theta_u > 1$ indicates either that an unskilled worker is more productive in the high-technology industry than in the low-technology industry, or that the demand for the high-technology industry's product is higher, or both. This modification allows me to model an increase in the productivity/demand in the high-technology industry alone as simultaneous increases in θ and θ_u in the same proportion. The restriction $\theta > \theta_u$ is maintained to guarantee that in the same (high-technology) industry a skilled worker's value of product is higher than an unskilled worker's.

With this extension, one can re-formulate the firms' maximization problems and derive the equilibrium conditions. The exercise yields:

$$x_{Lu} = n - H \ln \theta_u;\ x_s = ns/H;\ x_{Hu} = n - \frac{ns}{H} + (1-H)\ln\theta_u;$$

$$U_u = y(\theta_u)^H e^{-n};\quad U_s = y\left[e^{-n}(\theta_u)^H(1+\ln\theta_u) + (\theta-\theta_u)e^{-ns/H}\right];$$

$$w_L = \frac{U_u}{g(x_{Lu})};\quad w_s = \frac{U_s}{g(x_s)};\quad w_{Hu} = \frac{e^{x_s}U_u}{g(x_{Hu})};$$

$$1-(\theta_u)^H e^{-n}\left(1+n-H\ln\theta_u\right) = K_L/y;$$

$$\theta - 1 - \left(1+\frac{ns}{H}\right)\left[e^{-n}(\theta_u)^H \ln\theta_u + (\theta-\theta_u)e^{-ns/H}\right] = (K_H - K_L)/y.$$

The last two equations solve for the distribution variables (n, H).

Consider a productivity/demand increase in the high-technology industry alone and start with the base values of parameters identified before, where $\theta_u = 1$ and $\theta = 1.912$. Increase θ

from its base value 1.912 to 2.062, with a step 0.015, and simultaneously increase θ_u in the same proportion so as to maintain the relation $\theta = 1.912\theta_u$.

The responses of wage differentials to the sector-specific productivity/demand increase are very similar to the responses to a skill-biased productivity increase and hence are not depicted here. The differences are in magnitudes. First, the wage differential within unskilled workers, DU, increases by more than in the case of a skill-biased productivity increase. This is because the value of product of unskilled workers in the high-technology industry increases relative to that in the low-technology industry. Second, for the same reason, the average wage of unskilled workers rises faster than in the case of a skill-biased productivity increase and so the overall skill premium (DB) rises by less in the current case.

The response of the skill distribution is slightly different in the current case. Recall that when θ_u is fixed at one, the skill-biased productivity increase does not change the total number of firms. This is no longer true for a sectorial shock. The improvement in the value of product for both skilled and unskilled workers in the high-technology industry makes a low-technology firm much less profitable than a high-technology firm. There are more unskilled workers who move from the low-technology industry to the high-technology industry than in the case of a skill-biased technological progress. As a result, the low-technology industry shrinks by more than the high-technology industry expands and the total number of firms decreases.

6.3. Dynamic Recruiting

Now I reset $\theta_u = 1$ but extend the time horizon to infinity. This extension is interesting because one would like to know whether the results are robust to firms' and workers' dynamic concerns. In particular, a high-technology firm that hired an unskilled worker may want to keep looking for a skilled worker in a dynamic setting. The description here is kept sketchy and the details are in Appendix F. In this dynamic setting, unmatched firms and workers try to get matched over time; matched workers and firms experience some exogenous separation. For the reason described below, there is also endogenous separation for unskilled workers employed by high-technology firms.

A high-technology firm still prefers skilled workers and, when there are no skilled workers forthcoming, the firm is willing to hire unskilled workers since there is a positive surplus for doing so relative to leaving the job vacant. In the next period, the firm may or may not want to keep the unskilled worker. I focus on parameter values that always give incentive for such firms to fire the unskilled worker and try to recruit again. That is, the turnover rate for unskilled workers in the high-technology industry is 100%. Admittedly this is not the most realistic case, but the exercise is meant to give high-technology firms the maximum benefit from the dynamic context and hence is useful for illustrating the different results generated by dynamic concerns.

Despite the dismal future outlook for unskilled workers, a steady state exists for the equilibrium where unskilled workers continue to apply to high-technology jobs. As a result, the wage offered to unskilled workers by high-technology firms relative to that offered by low-technology firms is much higher than in the static environment. That is, the relative wage between industries for unskilled workers is much higher now. Also, the log relative wage between skilled and unskilled workers, RH, is larger here than in the static environment, because the skill-biased productivity generates a benefit to the firm over a much longer horizon.

The higher relative wage between industries is accompanied by a decreased dispersion of skill employment in the high-technology industry. Since unskilled workers in the high-technology industry experience a 100% turnover rate, fewer of them are employed there in the steady state than in the one-period setting. Thus, there are fewer unskilled workers earning high wages, although they earn more now than in the one-period setting. These two opposite forces roughly cancel with each other, leaving the wage differential within unskilled workers, DU, roughly the same as in the one-period setting. Similarly, the wage differential between skills (DH) remains roughly the same as in the one-period setting. In contrast, the infinite horizon significantly increases the average between-skill differential DB and the overall wage differential DT. Except for these quantitative differences, the responses of the steady state to technological shocks are qualitatively the same as in the static environment.

7. Conclusion

I have constructed a wage-posting model that generates a positive skill premium and a positive wage differential within unskilled workers. The skill premium arises here because of a skill-biased technology. The wage differential within unskilled workers arises because the probability with which an unskilled worker gets a job differs in the two industries. When an unskilled worker applies to a high-technology job, he competes with skilled workers and has a lower chance of getting the job than if he applies to a low-technology job. To make unskilled workers indifferent between the two industries in terms of expected wages, the wage rate offered by high-technology firms must be higher. I have examined the responses of the wage differentials and matching rates to shocks to the skill-biased productivity, the general productivity and the sectorial productivity/demand. These responses provide useful explanations for the observed dynamic patterns of within-skill and between-skill wage differentials in the 1970s and 1980s and for the relative volatility of hours of work by different skill groups of workers over business cycles.

The model has been kept simple to emphasize the wage differential within unskilled workers. In particular, technologies and skills are such that there is no wage differential within skilled workers. This wage differential can be captured by allowing the skill-biased productivity θ to have different realizations depending on matches, since skilled workers' productivity is more likely to depend on specific matches than does unskilled workers'. This extension, although complicating the calculation considerably, would not change the qualitative results much.

The model has also abstracted from other important sources of wage differentials, such as the employer size. In a separate paper (Shi, 1997) I have used a similar price/wage posting framework to explain the size-wage differential among homogeneous workers. It remains to check how the size-wage differential interacts with the wage differentials examined here.

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Appendix

A. Proof of Lemma 2.1

The following lemma is useful for the proof of Lemma 2.1:

Lemma A.1. *Let f be defined in (2.4). For any probabilities (p_1, p_2) and positive integers $(a1, a2)$, denote $C_a^k = a!/[k!(a-k)!]$. Then,*

$$f(p_1, p_2; a1, a2) = \sum_{k1=0}^{a1} \sum_{k2=0}^{a2} \frac{1}{k1+k2+1} C_{a1}^{k1} (p_1)^{k1} (1-p_1)^{a1-k1} C_{a2}^{k2} (p_2)^{k2} (1-p_2)^{a2-k2}$$

Proof of Lemma A.1: Define a function of $\phi \in [0,1]$ as follows:

$$F(\phi) \equiv \sum_{k1=0}^{a1} \sum_{k2=0}^{a2} \frac{1}{k1+k2+1} C_{a1}^{k1} (\phi p_1)^{k1} (1-p_1)^{a1-k1} C_{a2}^{k2} (\phi p_2)^{k2} (1-p_2)^{a2-k2}.$$

Clearly, the double summation in Lemma A.1 is equal to $F(1)$. Also, since $k1$ and $k2$ are positive integers,

$$\begin{aligned} F(\phi) &< \sum_{k1=0}^{a1} \sum_{k2=0}^{a2} C_{a1}^{k1} (\phi p_1)^{k1} (1-p_1)^{a1-k1} C_{a2}^{k2} (\phi p_2)^{k2} (1-p_2)^{a2-k2} \\ &= \left(\sum_{k1=0}^{a1} C_{a1}^{k1} (\phi p_1)^{k1} (1-p_1)^{a1-k1} \right) \cdot \left(\sum_{k2=0}^{a2} C_{a2}^{k2} (\phi p_2)^{k2} (1-p_2)^{a2-k2} \right) \\ &= [1-(1-\phi)p_1]^{a1} [1-(1-\phi)p_2]^{a2} \le 1. \end{aligned}$$

Thus, $F(\phi)$ is uniformly bounded between 0 and 1 for $\phi \in [0,1]$. So is the function $\phi F(\phi)$. When computing the derivative $d[\phi F(\phi)]/d\phi$, I can then switch the order of the derivative with the summation in F. Carrying out the computation yields:

$$\frac{d}{d\phi}[\phi F(\phi)] = [1-(1-\phi)p_1]^{a1} [1-(1-\phi)p_2]^{a2}.$$

Note that $\phi F(\phi) = 0$ when $\phi = 0$. Integrating the above equation from 0 to 1 yields

$$F(1) = \int_0^1 [1-(1-\phi)p_1]^{a1} [1-(1-\phi)p_2]^{a2} \, d\phi.$$

A straightforward transformation of the integration variable yields the desired result. QED

Now I show Lemma 2.1. First, I compute the selection probabilities q's. Consider first a skilled worker, labeled worker A, who applies to a low-technology firm. If there are $k1$ other skilled applicants and $k2$ unskilled applicants for the same firm, worker A is chosen by the firm with probability $1/(k1+k2+1)$, since the low-technology firm is indifferent between all applicants.

Because there are $(sN-1)$ other skilled workers, each applying to the same firm with probability p_{Ls}, and $(1-s)N$ unskilled workers, each applying to the same firm with probability p_{Lu}, worker A is chosen by the firm to which he applies with the following probability:

$$q_{Ls} \equiv \sum_{k1=0}^{sN-1} \sum_{k2=0}^{(1-s)N} \frac{1}{k1+k2+1} C_{sN-1}^{k1} (p_{Ls})^{k1} (1-p_{Ls})^{sN-1-k1} C_{(1-s)N}^{k2} (p_{Lu})^{k2} (1-p_{Lu})^{(1-s)N-k2},$$

where $C_J^I = J!/[I!(J-I)!]$ for integers I and J $(\geq I)$. The long expression following $1/(k1+k2+1)$ is the probability that exactly $k1$ other skilled workers and $k2$ unskilled workers apply to the same low-technology firm to which worker A applies. Applying Lemma A.1 yields $q_{Ls} = f(p_{Ls}, p_{Lu}; sN-1, (1-s)N)$, as in (2.5).

If worker A (skilled) applies to a high-technology firm, his only competitors are other skilled applicants, since high-technology firms prefer skilled applicants to unskilled ones. Since there are $(sN-1)$ other skilled workers in the market and each applies with probability p_{Hs} to a high-technology firm, worker A will be chosen by the firm with the following probability $q_{Hs} = f(p_{Hs}, 0; sN-1, (1-s)N)$.

Similarly, one can compute the selection probabilities for an unskilled worker and verify that q_{Lu} is given by (2.7) and q_{Hu} is given by (2.8).

Now I show $q_{Hs} > q_{Hu}$. Since $f(0, p_{Hu}; sN, (1-s)N-1) \leq 1$, $q_{Hs} > q_{Hu}$ if $q_{Hs} > (1-p_{Hs})^{sN}$, which is equivalent to the following inequality after the integral for q_{Hs} is computed:

$$1-(1-p_{Hs})^{sN} - sNp_{Hs}(1-p_{Hs})^{sN} > 0.$$

The left hand side of this inequality is a strictly increasing function of p_{Hs} for any $p_{Hs} \in (0,1]$ and has a value zero when $p_{Hs} = 0$. Hence the inequality holds for all $p_{Hs} \in (0,1]$, yielding $q_{Hs} > q_{Hu}$. Note that this inequality holds for arbitrarily large N and M as long as Np_{Hs} and Np_{Hu} are bounded above zero.

Finally, I show $w_{Hs} < w_{Hu}$. When $M, N \to \infty$, the probability with which a worker visits each firm is close to zero in a mixed strategy equilibrium, i.e., p_{Hs}, p_{Hu}, p_{Ls}, $p_{Lu} \to 0$. Then, $q_{Ls} \to q_{Lu} \to f(p_{Ls}, p_{Lu}, sN, (1-s)N)$ and so $q_{Ls}w_L \to q_{Lu}w_L$. That is, in the limit skilled and unskilled workers have the same expected payoff from applying to a low-technology firm. Since $p_{Hs} \in (0,1)$ requires $q_{Hs}w_{Hs} = q_{Ls}w_L$, $p_{Hu} \in (0,1)$ requires $q_{Hu}w_{Hu} = q_{Lu}w_L$, and $q_{Ls}w_L \to q_{Lu}w_L$, then $p_{Hs}, p_{Hu} \in (0,1)$ implies $q_{Hs}w_{Hs} \to q_{Hu}w_{Hu}$. Since $q_{Hs} > q_{Hu}$ in the limit, as shown above, $w_{Hs} < w_{Hu}$. This completes the proof of Lemma 2.1. QED

B. Proof of Lemma 2.2

Suppose, contrary to the lemma, that $p_{Hu} = 0$. Then $p_{Lu} = 1/[(1-H)M]$. Since $p_{Ls} = 0$, $p_{Hs} = 1/(HM)$. Let $x_s^\infty = \lim_{N,M\to\infty} sNp_{Hs}$ and $x_u^\infty = \lim_{N,M\to\infty}(1-s)Np_{Lu}$. With $p_{Ls} = p_{Hu} = 0$, one can follow the calculation in Section 3 to show that in the limit $N, M \to \infty$ the expected profit is $\theta y[1-(1+x_s^\infty)e^{-x_s^\infty}]$ for a high-technology firm and $y[1-(1+x_u^\infty)e^{-x_u^\infty}]$ for a low-technology firm. In equilibrium these profits must be equal to the corresponding entry costs and so Assumption 1 implies

$$\frac{\theta[1-(1+x_s^\infty)e^{-x_s^\infty}]}{1-(1+x_u^\infty)e^{-x_u^\infty}} = \frac{K_H}{K_L} < \theta.$$

Since the function $1-(1+x)e^{-x}$ is increasing in x, $x_s^\infty < x_u^\infty$ and so $s < H$.

Consider a single high-technology firm that offers the same wage w_{Hs} as other firms do to skilled workers but a different wage w_{Hu}^d to unskilled workers, where

$$w_{Hu}^d = \left[\frac{1-(1-p_{Lu})^{(1-s)N}}{(1-s)Np_{Lu}} w_{Lu} + \varepsilon\right] \Big/ (1-p_{Hs})^{sN},$$

and ε is a sufficiently small positive number. Note that the first term in the square brackets is the expected wage that the unskilled worker gets from applying to a low-technology firm. An unskilled worker who applies to w_{Hu}^d when no other unskilled worker applies to w_{Hu}^d gets the wage with probability $(1-p_{Hs})^{sN}$. Thus, he obtains a strictly higher expected wage from applying to w_{Hu}^d and so the wage w_{Hu}^d attracts unskilled workers.

The wage w_{Hu}^d is feasible to a high-technology firm when N and M are sufficiently large. When $N, M \to \infty$, $w_{Hu}^d = e^{x_s^\infty - x_u^\infty} y + \varepsilon e^{x_s^\infty}$. Since $x_u^\infty > x_s^\infty$ and ε is sufficiently small, $w_{Hu}^d < y$ for sufficiently large N and M. For given $w_{Hs} < \theta y$, let $\hat{w} = w_{Hs} - (\theta - 1)y$ and $w_{Hu}^{dd} = \max\{w_{Hu}^d, \hat{w} + \delta\}$, where δ is an arbitrarily small positive number. Then w_{Hu}^{dd} is less than y, satisfies (2.3), and attracts unskilled workers. Thus, a high-technology firm that offers w_{Hu}^{dd} to unskilled workers does not lose any skilled workers and yet attracts unskilled workers. As a result, this firm gets a higher expected profit than other high-technology firms, contradicting to the equilibrium requirement. Therefore, $p_{Hu} > 0$. QED

C. Proof of Proposition 3.1

The two equations (3.16) and (3.17) solve for a unique pair (n, H):

$$n = B^{-1}\left(\frac{K_L}{y}\right); \quad ns/H = B^{-1}\left(\frac{K_H - K_L}{(\theta-1)y}\right).$$

The assumption $s < \bar{s}$ implies $H < 1$. Other variables can be solved by substituting the solutions for (n, H) back into (3.11) – (3.15). The equilibrium requires x_s, x_{Hu} and x_{Lu} all to lie in the interior of $(0, \infty)$. To verify these requirements, note first that $x_{Lu} = n \in (0, \infty)$. Second, $\theta > K_H/K_L$ is necessary and sufficient for $H > s$, which in turn implies $x_s \in (0, n)$ and $x_{Hu} \in (0, n)$.

The equilibrium also requires (2.3) to be satisfied and $U_s > U_u$. With (3.15) it is easy to verify $U_s > U_u$. To verify (2.3), substitute the solutions for (w_s, w_{Hu}) to rewrite the condition as

$$\theta - 1 > e^{-n+ns/H}\left(1 - \frac{e^{ns/H}-1}{ns/H} \cdot \frac{n - ns/H}{1-e^{-n+ns/H}}\right) \Big/ \left(\frac{e^{ns/H}-1}{ns/H} - 1\right). \tag{C.1}$$

Since $e^a > 1+a$ and $a > 1-e^{-a}$ for any $a > 0$, then $e^{ns/H}-1 > ns/H$ and $n-ns/H > 1-e^{-n+ns/H}$ for $H > s$. The right-hand side of (C.1) is negative and so (C.1) is satisfied for $H > s$. QED

D. Proofs of Propositions 4.2 and 4.3

For Proposition 4.2, compare w_L in (3.11) with w_{Hu} in (3.12). Substituting $x_{Lu} = n$ yields: $w_{Hu} > w_L \iff (n - x_s)(1 - e^{-n}) - n(e^{-x_s} - e^{-n}) > 0$. I show that this inequality holds in the feasible region $x_s \in (0, n)$. For any arbitrary $n > 0$, temporarily denote the left-hand side of the above inequality by $LHS(x_s)$. Since $LHS(0) = LHS(n) = 0$, $LHS(x_s) > 0$ for all $x_s \in (0, n)$ if $LHS(\cdot)$ is concave in the interval, but the concavity of $LHS(\cdot)$ can be verified directly.

For Proposition 4.3, substituting (3.13) and (3.12) yields: $w_s > w_{Hu} \iff$

$$\theta > 1 + \frac{e^{ns/H} - 1}{ns/H} \cdot \frac{n - ns/H}{e^{n-ns/H} - 1} - e^{-n+ns/H}. \tag{D.1}$$

Since $H > s$ under the assumption $\theta > K_H/K_L$, the right-hand side of (D.1) is an increasing function of s/H. Since the solution for s/H is a decreasing function of θ, there is a unique θ_1 such that (D.1) holds with equality and that the strict inequality holds if and only if $\theta > \theta_1$. The value of θ_1 is not necessarily greater than one. QED

E. Proof of Proposition 5.2

In equilibrium, $x_s = ns/H$. Temporarily drop the subscript s on x and denote ns/H by x. The left-hand side of (3.16) is $B(n)$ and the left-hand side of (3.17) is $B(x)$. Differentiating the two zero-profit conditions with respect to y yields

$$\frac{dn}{dy} = -\frac{B(n)}{yB'(n)}; \ \frac{dx}{dy} = -\frac{B(x)}{yB'(x)}; \ \frac{dH}{dy} = \frac{nB(x)\,B'(n) - xB'(x)\,B(n)}{nB'(n)xB'(x)\,y/H}.$$

Since $B' > 0$, clearly $dn/dy < 0$, implying $dx_{Lu}/dy < 0$ and $dU_u/dy > 0$. Also, $dx/dy < 0$, implying $d\alpha_s/dy > 0$. To show $dH/dy < 0$, temporarily denote the numerator of the expression for dH/dy by $RHS(n)$ for any fixed x. Then $dH/dy < 0$ if and only if $RHS(n) < 0$. I show that indeed $RHS(n) < 0$ in the feasible region $n \in (x, \infty)$. Since $RHS(x) = 0$, it suffices to show $RHS'(n) < 0$. Compute

$$\begin{aligned} RHS'(n) &= (2-n)[1-(1+x)e^{-x}] - x^2e^{-x} \\ &< (2-x)[1-(1+x)e^{-x}] - x^2e^{-x} = 2 - x - (2+x)e^{-x}. \end{aligned}$$

The inequality follows from $n > x$ and $1 - (1+x)e^{-x} > 0$. The function $2 - x - (2+x)e^{-x}$ has a value zero when $x = 0$, a derivative $-[1-(1+x)e^{-x}] < 0$, and hence is negative for all $x > 0$. Thus, $RHS'(n) < 0$ for all $n > x$.

The matching rate for an unskilled worker, α_u, can be shown to be a decreasing function of (n, H). Since (n, H) both fall with y, $d\alpha_u/dy > 0$. The responses of wages stated in the proposition can be verified directly. QED

F. Details of the Dynamic Model in Subsection 6.3

Three elements are added for the dynamic analysis: a recruiting cost $c > 0$, which a firm incurs each time the firm recruits, endogenous (100%) separation for high-technology firms hiring unskilled workers, and an exogenous job separation rate $\sigma > 0$ for other pairs (see Pissarides (1990)). Except for lost production and wages, there is no other cost of job separation.

The sequence of events is as follows. At the beginning of each period, new firms enter and existing high-technology firms that have hired unskilled workers fire them and look for new workers (on-the-job search is excluded here for simplicity). Other firms do not have incentive to fire their current workers. On the supply side of the market, the skill composition s is determined according to (6.1). These decisions by firms and workers result in M firms and N workers in the recruiting market, with a fraction H of them being high-technology firms. Then, firms post their wages to attract applicants. After hiring, production takes place and wages are paid. Finally, each firm has probability σ of losing its current worker, after which a period of time elapses and the sequence of actions starts anew in the next period. The discount factor is $\beta \in (0, 1)$ for both firms and workers.

Firms' and workers' decisions are now based on gains in present values rather than one-period gains. For firms, let V_i be the present value of a vacancy in industry i $(= H, L)$ and F_j be the present value of a job (before production) that is filled by a worker, where $j = L$ indicates a filled job in the low-technology industry, $j = Hu$ indicates a job in the high-technology industry filled by an unskilled worker, and $j = s$ indicates a job in the high-technology industry filled by a skilled worker. For workers, let U_u now be the value function of an unskilled applicant and U_s be the value function of a skilled applicant. Let J_j be the value function for an employed worker (before the current wage is paid), where $j = L$, Hu, s as above. I focus on a stationary equilibrium where all these value functions are constant.

Restriction 1. $\beta V_H < F_{Hu} < V_H$.

The restriction $\beta V_H < F_{Hu}$ ensures that a high-technology firm is willing to hire an unskilled worker when there is no skilled applicant; the restriction $F_{Hu} < V_H$ ensures that a high-technology firm that hired an unskilled worker will fire the worker next period and recruit. With these restrictions, agents' strategies can be analyzed by going through the following steps.

Step 1: The value functions. Denote the wage that a firm posts today by w and the wage that a firm posts next time the firm recruits by w' (although the two are equal to each other in a stationary equilibrium, the distinction is useful for analyzing firms' decisions). When the numbers M and N are sufficiently large, a low-technology firm's matching rate is $1 - e^{-x_{Lu}}$, a high-technology firm's matching rate with a skilled worker is $1 - e^{-x_s}$, and a high-technology firm's matching rate with an unskilled worker is $e^{-x_s}(1 - e^{-x_{Hu}})$. The value functions V's and

F's are given as follows:

$$V_L(w_L) = -c + (1 - e^{-x_{Lu}})F_L(w_L) + e^{-x_{Lu}}\beta V_L(w_L'); \tag{F.1}$$

$$\begin{aligned} V_H(w_s, w_{Hu}) &= -c + (1 - e^{-x_s})F_s(w_s) \\ &+ e^{-x_s}[(1 - e^{-x_{Hu}})F_{Hu}(w_{Hu}) + e^{-x_{Hu}}\beta V_H(w_s', w_{Hu}')]; \end{aligned} \tag{F.2}$$

$$F_L(w_L) = \frac{y - w_L + \sigma\beta V_L(w_L')}{1 - (1-\sigma)\beta}; \tag{F.3}$$

$$F_{Hu}(w_{Hu}) = (y - w_{Hu}) + \beta V_H(w_s', w_{Hu}'); \tag{F.4}$$

$$F_s(w_s) = \frac{\theta y - w_s + \sigma\beta V_H(w_s', w_{Hu}')}{1 - (1-\sigma)\beta}. \tag{F.5}$$

These equations can be easily interpreted. For example, (F.3) states that the present value of a filled job in the low-technology industry is the discounted sum of the surplus $(y - w_L)$ and the remaining value when the job is separated, the latter of which is the value of a future vacancy. The discounting takes into account of the possibility of job separation. Notice that the restriction $F_{Hu} > \beta V_H$ in Restriction 1 ensures that the term in $[\cdot]$ of (F.2) is greater than βV_H in a stationary equilibrium and so a high-technology firm will hire an unskilled worker rather than leaving the job vacant. The restriction $V_H > F_{Hu}$ in Restriction 1 implies that the effective job separation rate for a high-technology firm that hired an unskilled worker is one, which explains the different appearance of (F.4) in comparison with (F.3) and (F.5).

For job applicants, the value functions U's depend on all posted wages, and when the market is big, a single posted wage has very little influence on the U's. In contrast, the value functions J's depend on the worker's particular wage. These value functions are given as follows:

$$(1-\beta)U_u = \frac{1 - e^{-x_{Lu}}}{x_{Lu}}[J_L(w_L) - \beta U_u]; \tag{F.6}$$

$$(1-\beta)U_u = \frac{e^{-x_s}(1 - e^{-x_{Hu}})}{x_{Hu}}[J_{Hu}(w_{Hu}) - \beta U_u]; \tag{F.7}$$

$$(1-\beta)U_s = \frac{1 - e^{-x_s}}{x_s}[J_s(w_s) - \beta U_s]; \tag{F.8}$$

$$J_L(w_L) = \frac{w_L + \sigma\beta U_u}{1 - (1-\sigma)\beta}; \tag{F.9}$$

$$J_{Hu}(w_{Hu}) = w_{Hu} + \beta U_u; \tag{F.10}$$

$$J_s(w_s) = \frac{w_s + \sigma\beta U_s}{1 - (1-\sigma)\beta}. \tag{F.11}$$

These equations are parallel to (F.1)–(F.5) and can be interpreted similarly. In particular, (F.6) states that the value function of an unskilled worker is the discounted value of expected gains from finding a job in the low-technology industry, where the gains are the ex post gains $J_L - \beta U_u$

times the probability of getting such a job. (F.7) calculates the expected gains to an unskilled applicant from his application to a high-technology job, since he must be indifferent between the two jobs. (F.8) is a skilled applicant's indifference curve.

Step 2: Firms' wage posting decisions. I analyze only the determination of w_L, since the analyses for w_s and w_{Hu} are similar. Consider a one-period deviation by a low-technology recruiting firm to a wage w_L^d. Observing the deviation applicants modify their application strategies so that the queue length (of unskilled applicants) for the deviator is x_{Lu}^d. The present value of this vacancy is $V_L(w_L^d)$ and, if it is filled, the present value is $F_L(w_L^d)$; the value function for a worker who gets this job is $J_L(w_L^d)$. The value function $V_L(w_L^d)$ is given by (F.1), $F_L(w_L^d)$ by (F.3) and $J_L(w_L^d)$ by (F.9), with w_L and x_{Lu} being replaced by w_L^d and x_{Lu}^d, respectively. Since the deviation is a one-period deviation, the notation w_L' in these equations stays the same.

The best deviation that the firm can have is the solution to the following problem:

$$\max\ V_L(w_L^d) \text{ s.t. } \frac{1-e^{-x_{Lu}^d}}{x_{Lu}^d}\left[J_L(w_L^d)-\beta U_u\right] \geq (1-\beta)U_u.$$

If w_L is the equilibrium wage posted by a low-technology firm, the above deviation cannot be profitable and so $w_L^d = w_L$ must be the solution to the above problem. The wage decisions by a high-technology firm can be analyzed similarly. Solving these decision problems and imposing stationarity yields the following proposition.

Proposition F.1. *In a stationary equilibrium with dynamic recruiting, wages and queue lengths are:*

$$w_L = (1-\beta)U_u\left[(1-\sigma)\beta + (1-(1-\sigma)\beta)\frac{x_{Lu}}{1-e^{-x_{Lu}}}\right]; \tag{F.12}$$

$$x_{Lu} = \ln\left(\frac{y-(1-\sigma)(1-\beta)\beta(V_L+U_u)}{[1-(1-\sigma)\beta](1-\beta)U_u}\right); \tag{F.13}$$

$$w_{Hu} = y\cdot\frac{x_{Hu}}{e^{x_{Hu}}-1}; \tag{F.14}$$

$$x_{Hu} = \ln\left(\frac{y}{(1-\beta)U_u}\right) - x_s; \tag{F.15}$$

$$w_s = (1-\beta)U_s\left[(1-\sigma)\beta + (1-(1-\sigma)\beta)\frac{x_s}{1-e^{-x_s}}\right]; \tag{F.16}$$

$$x_s = \ln\left(\frac{[\theta-1+(1-\sigma)\beta]y-(1-\sigma)(1-\beta)\beta(V_H+U_s)}{[1-(1-\sigma)\beta](1-\beta)(U_s-U_u)}\right). \tag{F.17}$$

These formulas approach the corresponding ones in (3.11) – (3.13) when $\beta \to 0$.

A stationary equilibrium can be defined by requiring that (i) firms' wage posting decisions obey (F.12), (F.14) and (F.16); (ii) applicants' strategies obey (F.13), (F.15), (F.17), and the adding-up restrictions (3.2) – (3.3); (iii) Restriction 1 is satisfied in addition to $F_s > F_{Hu}$ and

$U_s > U_u$; (iv) s is determined by (6.1) and firms' entry decisions satisfy $V_H = K_H$ and $V_L = K_L$; and (v) the distribution variables (n, H, s) are stationary.

Requirements (i) and (ii) are self-explanatory. The use of Restriction 1 was explained before. The condition $F_s > F_{Hu}$ ensures that a high-technology firm chooses to hire a skilled worker when both skilled and unskilled applicants are available; the condition $U_s > U_u$ ensures $s > 0$ and guarantees that a skilled worker applies only to high-technology jobs. As before, firms' entry conditions help determine the distribution variables (n, H). Finally, the stationarity requirement (v) helps determine the employment distribution. The numbers (N_s, N_{Hu}, N_L), with the same meanings as before, are now given by:

$$N_s = \frac{MH}{\sigma}(1 - e^{-x_s}); \; N_{Hu} = MHe^{-x_s}(1 - e^{-x_{Hu}}); \; N_L = \frac{M(1-H)}{\sigma}(1 - e^{-x_{Lu}}). \tag{F.18}$$

For example, the number of employed skilled workers (N_s) must be such that in each period the number of newly recruited skilled workers equals the number of separated skilled workers, σN_s. Note that the effective job separation rate is one for unskilled workers employed in high-technology firms. Wage differentials DU, DH, DB, and DT are defined in the same way as before using the above employment distribution.

The steady state of the equilibrium exists for the following realistic parameter values. First, let $y = 10$, $\theta = 1.912$ and $b = 0.27$ as before. Interpret a period as a year and let the discount factor β be 0.95 to give an annual real interest rate 5%. Second, set the ratio of K_L to the present value of an unskilled worker's product, which is now $y/(1-\beta)$, to the same value 0.215 as before. Similarly, set the ratio of K_H to $y/(1-\beta)$ to the same value 0.351 as before. The annual job separation rate is set to $\sigma = 0.2$, which is reasonable given that the quarterly job separation rate is about 10%. The recruiting flow cost c is set at $0.1y$ so that the initial stationary equilibrium satisfies Restriction 1, $F_s > F_{Hu}$, and $U_s > U_u$. The distinct features of this steady state are summarized in the text.

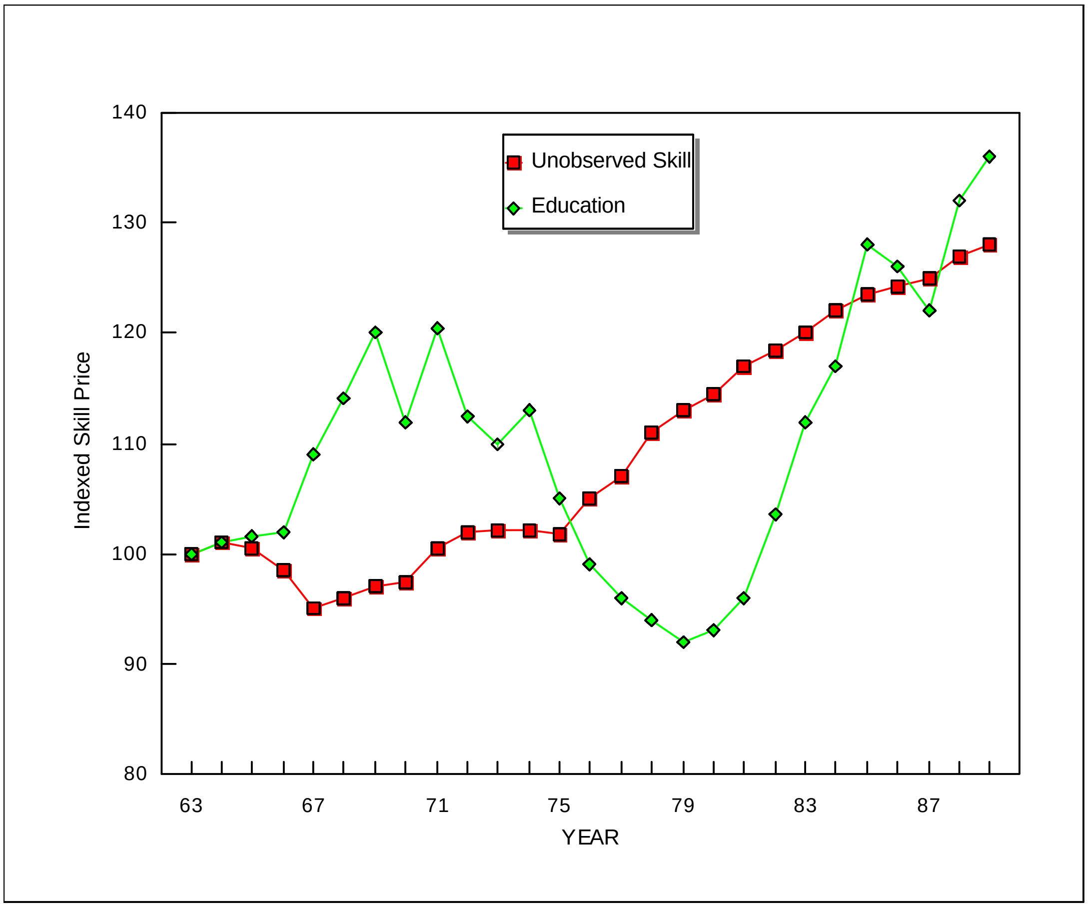

Figure 1. Skill price indexes for men, 1963-89 (1963/64 index equals 100)

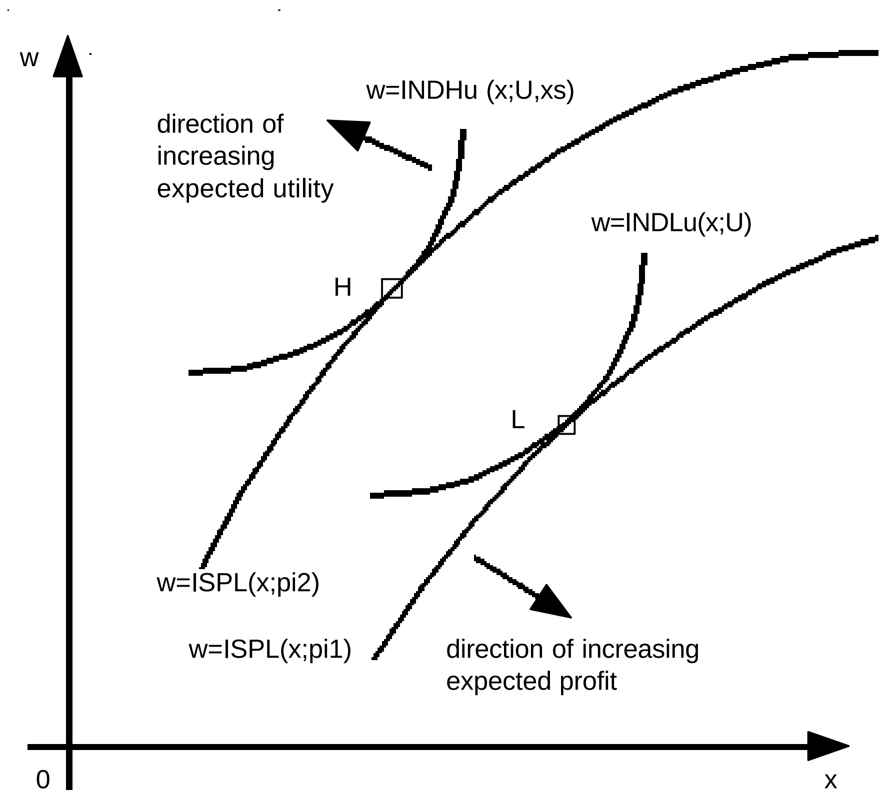

Figure 2. Unskilled workers' trade-off between wage and queue length

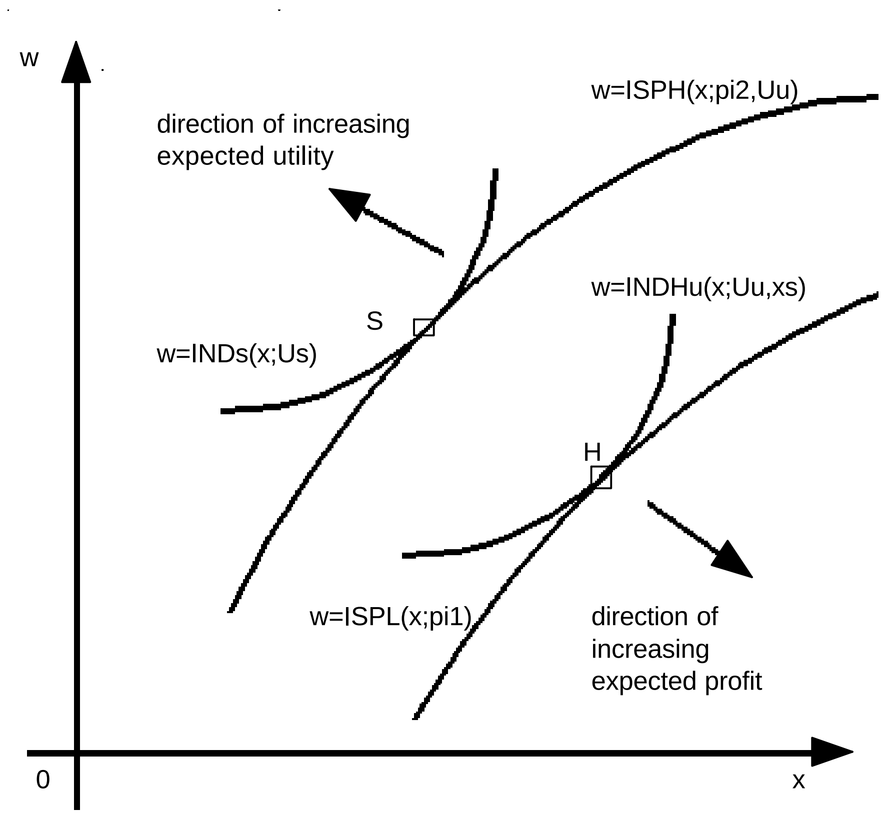

Figure 3. Trade-off between wages and queue lengths
in the high-technology industry

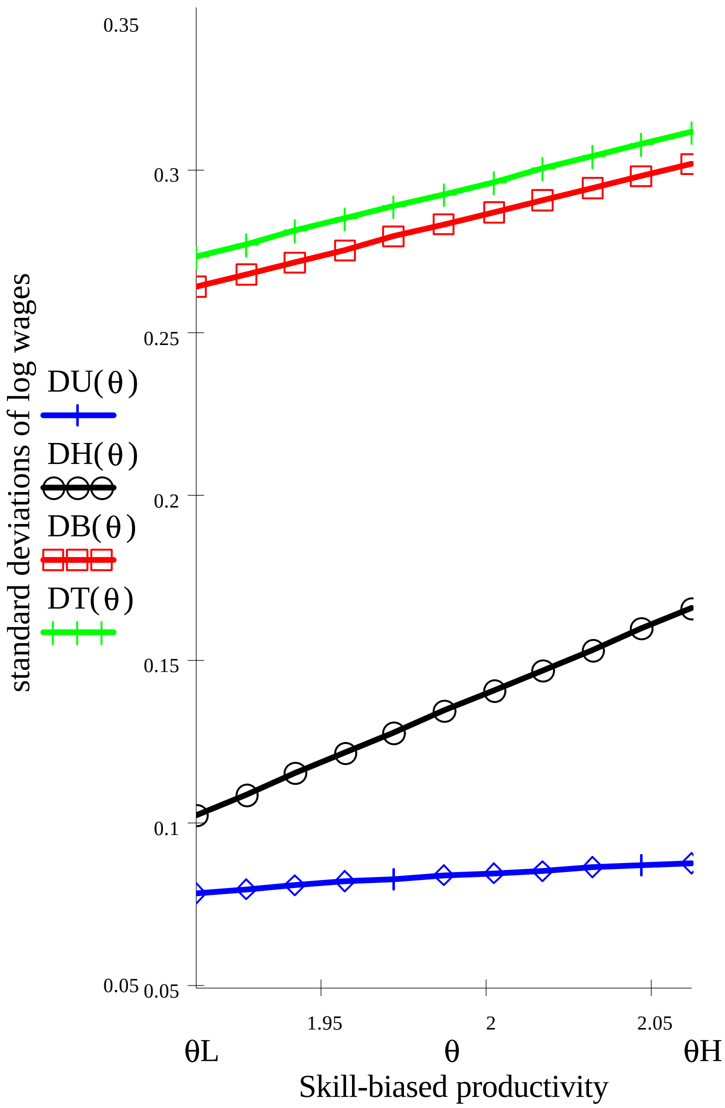

Figure 4.1. Responses of wage differentials to skill-biased productivity increases

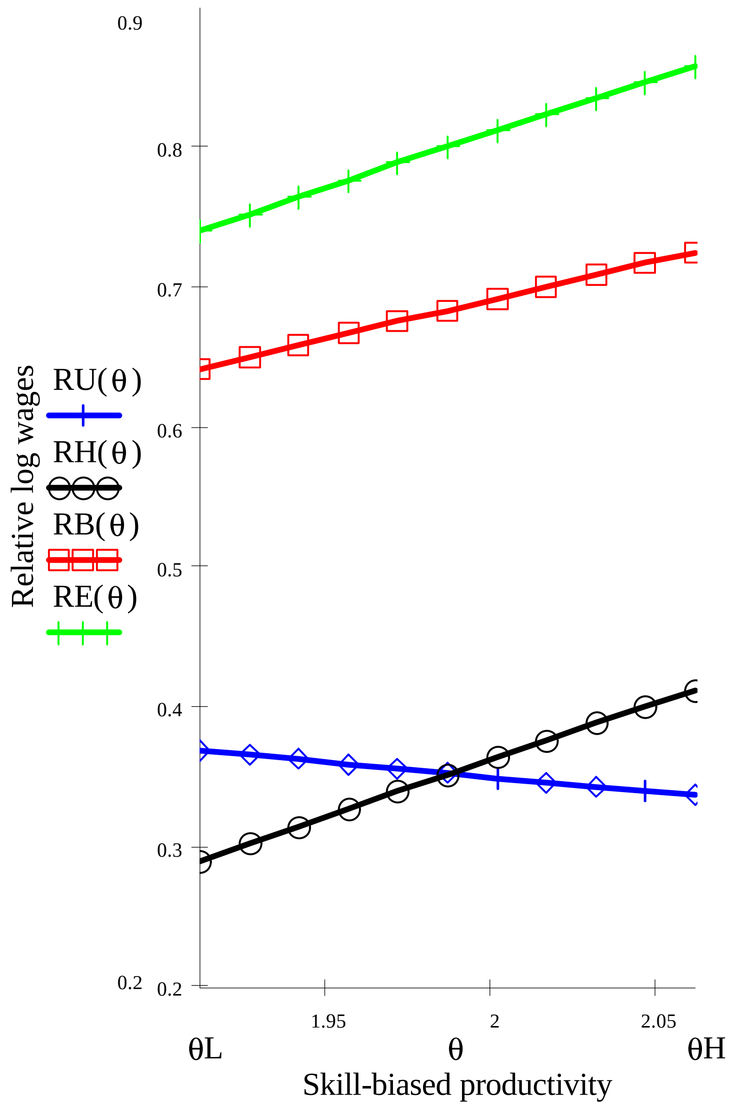

Figure 4.2. Responses of relative wages to skill-biased productivity increases

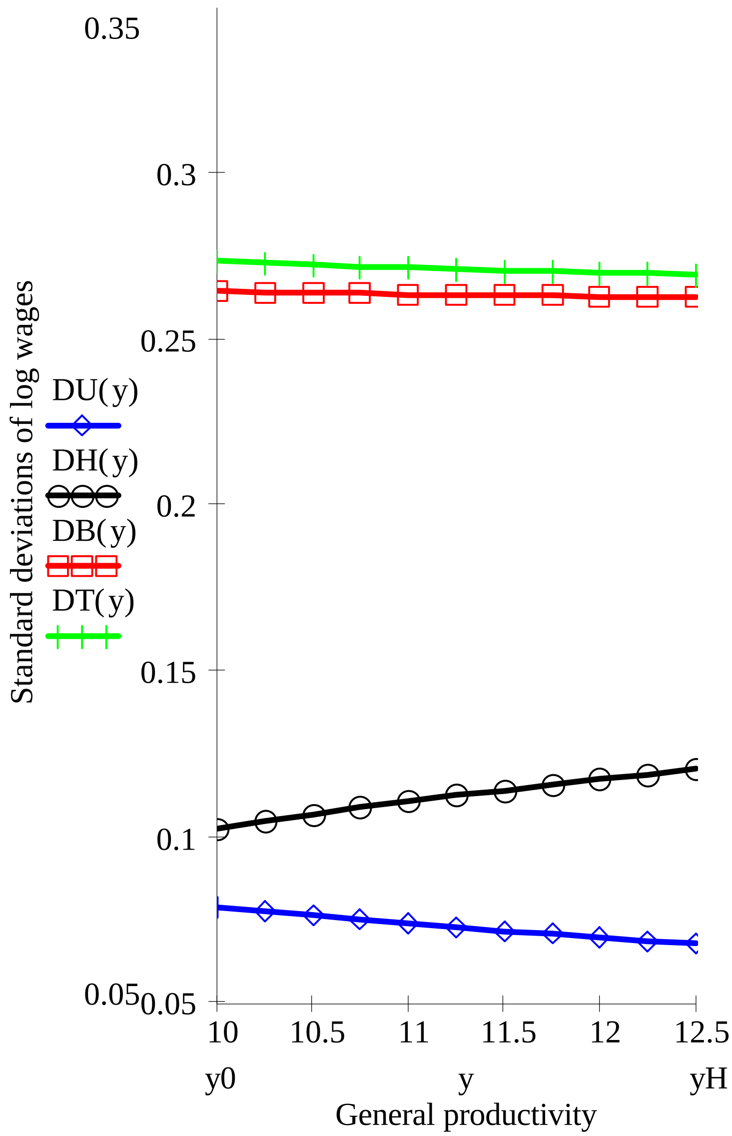

Figure 5.1. Responses of wage differentials to general productivity increases

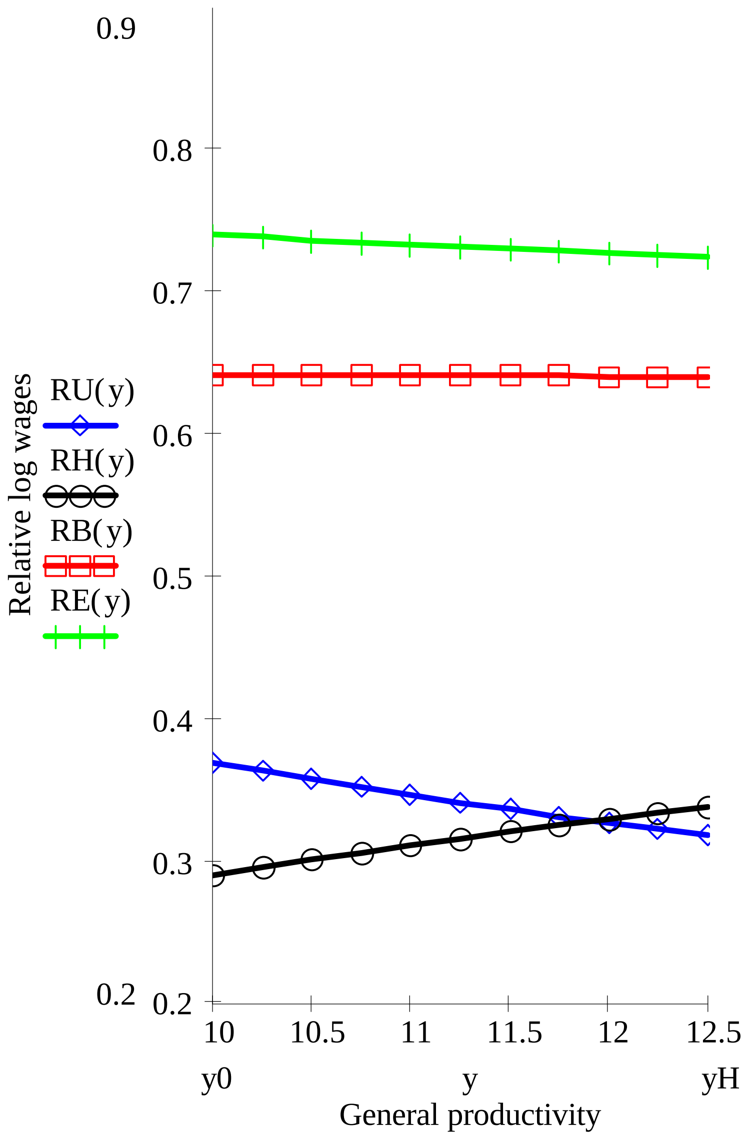

Figure 5.2. Responses of relative wages to general productivity increases