

Jack owns a forest. He wants to auction the right to cut timber from his forest. So the highest bidder in an auction has exclusive right to harvest timber from his forest for the duration of the lease. If Jack wants to maximize the present value of profits from his forest, should he hold yearly auctions for one year leases or one auction for multi-year leases?

If Jack holds yearly auctions, the winner of the auction in the current year will harvest all the trees which will maximize its profits for the current year. It may harvest small trees, which if left unharvested will provide higher discounted profits when they are harvested in subsequent years. So the current winner of the yearly auction will not maximize the present value of the timber which can be harvested from the forest. It does not worry about the future profits from the forest because it has no right to those profits. A winner of a multi-year lease will take into account the consequences of its current actions on its future profits from the forest. It will not harvest those trees today, which if left unharvested will provide higher discounted profits when they are harvested in the future, since it has the right to harvest the forest in the future. The winner of a multi-year lease will choose its current and future actions to maximize the present value of profits from the multi-year lease. Thus Jack should hold one auction for a multi-year lease.

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Firms A and B want the government to build a road. Firm A derives a benefit of \$1000. from the road. Firm B derives a benefit of \$500. The government is willing to build the road only after the firms provide a study which shows that the total benefit to both firms exceeds \$1000. Such a study will cost \$400. Firm A chooses first how much it will contribute towards the financing of the study. Firm B sees firm A's contribution and then it chooses how much to contribute towards the financing of the study.

What do you expect A and B's equilibrium contributions to be?

Even if A contributes zero, firm B will contribute \$400 to do the study because the value of the road to it is \$500. Firm A anticipating that B will do the study by itself will optimally choose to contribute zero. After A's contribution of zero, B will optimally choose to contribute \$400. These contributions are the only subgame perfect equilibrium contributions.

3. There are two firms A and B in industry X. The inverse demand function is

$$p = 1 - y$$

where y is the total supply of the industry. If firm A produces y_A , the marginal revenue for firm B if it produces y_B is

$$MR(y_B) = 1 - y_A - 2y_B$$

Let the cost of production of both firms be zero. What is the equilibrium price of output in the industry if both firms engage in Cournot Nash competition?

Let y_A^* be the equilibrium contribution of firm A. Then the best response of B is to set

$$MR(y_B^*) = 1 - y_A^* - 2y_B^* = MC(y_B^*) = 0$$

Since both firms are identical, we can look for symmetric behavior which implies $y_A^* = y_B^* = y^*$.

Then we have $1 - y^* - 2y^* = 0$ which implies $y^* = 1/3$. Put another way, if Firm A produces $y_A = 1/3$, then firm B will choose to produce $y_B = 1/3$ and vice versa. The equilibrium price of output is then $p^* = 1 - (y^* + y^*) = 1/3$.

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Student no.:

4. There are two firms A and B in industry X. Let the cost of production of both firms be zero.

The inverse demand function for the industry is

$$p = 1 - y$$

where y is the total supply of the industry. If firm A produces y_A , the marginal revenue for firm B if it produces y_B is

$$MR(y_B) = 1 - y_A - 2y_B$$

(i) What is the equilibrium price of output in the industry if both firms engage in Bertrand competition?

(ii) What is the maximum price that firm B is willing to pay to merge with firm A? A merged firm can act as a monopolist.

(i) No price above zero can be a Nash equilibrium. Let firm A set a price $p_A > 0$ but below the monopoly price. If firm B set a price above p_A , it will get no demand.

So firm B's best response is to set a price just below it, say $p_A - \epsilon$. In this case, firm A will get no demand and firm B will get all the demand at its price. But if firm B sets its price at $p_A - \epsilon$, firm A will not want to set a price of p_A . Thus any price above zero is not an equilibrium price.

The only equilibrium price is zero and each firm will earn zero profit.

(ii) A monopolist will set $MR = MC$. To find MR for a monopolist, let the merged firm set A's output to zero. Then $MR(y_B) = 1 - 0 - 2y_B$ is the MR of a monopolist. Equating that to $MC = 0$ gives monopolist output of $1 - 2y_B^{**} = 0$. $y_B^{**} = 1/2$. Monopoly price is $1/2$. Monopoly profit is $1/4$. Since B is earning zero under Bertrand competition it will be willing to pay a maximum of $1/4$ to merge with firm A assuming that it gets to make all decisions and keep all profits after the merger.