1 Cost curves

- Economic (opportunity) cost versus accounting cost. E.g. accounting cost values an input by what the firm paid.

- Sunk cost: Why sunk cost should not affect current decisions.

- Total cost of the firm in producing q units is the minimum cost of the firm in producing q units. Cost curves are flow concepts. I.e. cost of production for an interval of time, say a month.

- Let $f(k, L)$ be the production function, w the wage rate and r the rental price of capital.Long run total cost:

$$TC(q) = \min_{K,L} wL + rK$$
$$\text{s.t. } f(K, L) \geq q$$

- Fixed cost (FC) versus variable cost ($VC(q)$).

- Variable cost is never sunk and can be avoided by zero production.

- FC is cost with zero production. $VC(0) = 0$. E.g. firm contracts for K_0 units of capital.

$$FC = rK_0$$
$$VC(q) = wL \text{ s.t. } f(K_0, L) = q$$

- Marginal cost ($MC(q)$): Increase in cost from producing a marginal unit. $MC(q)$ depends on how much you are producing already.

- Average cost ($AC(q)$): Total cost divided by output.

$$TC(q) = FC + VC(q)$$
$$AC(q) = \frac{TC(q)}{q} = \frac{FC}{q} + \frac{VC(q)}{q}$$
$$MC(q)\Delta q = \Delta VC(q) = \Delta TC(q)$$

- Relationship between average cost and marginal cost:

$$\begin{aligned} AC(q)q &= FC + VC(q) \\ \Delta AC(q)q + AC(q)\Delta q &= \Delta VC(q) = MC(q)\Delta q \\ \Delta AC(q)q &= (MC(q) - AC(q))\Delta q \\ \Delta AC(q) &= \frac{(MC(q) - AC(q))\Delta q}{q} \end{aligned}$$

- Average cost is falling when average cost is above marginal cost.

- Average cost is rising when average costs is lower than marginal cost.

- Average cost is at its minimum when average cost is equal to marginal cost.

- Short run cost. Fixed cost is sunk (or contracted for). That is, at least one factor of production cannot be changed.

$$STC(0) = FC$$

- Long run cost. All factors of production variable. No sunk cost.

$$LTC(0) = 0$$

- Long run cost for output q is less than or equal to short run cost:

$$LTC(q) \leq STC(q)$$

The reason is that in the short run, at least one factor is fixed. So the firm can only adjust the rest of the factors to produce q. But in the long run, the firm can find the cheapest way to produce q units of output.

2 Choosing inputs to minimize cost

To minimize cost in producing q units, firm solves:

$$\min_{K,L} wL + rK$$
$$\text{s.t. } f(K, L) \geq q$$

- Iso cost curve: Combinations of L and K which gives the same cost C.

$$C = wL + rK$$
$$\Delta C = w\Delta L + r\Delta K$$
$$0 = w\Delta L + r\Delta K$$
$$\frac{\Delta K}{\Delta L} = -\frac{w}{r}$$

- What happens to slope of isocost curve as input prices change?

- Show cost minimizing choice of inputs in producing q: tangency of isocost curve and isoquant.

- Show what happens to choice of inputs when input prices change. All substitution effect. Use more of inputs whose relative price fell.

- At the optimum,

$$\frac{w}{r} = MRTS(K, L) = \frac{MPL(K, L)}{MPK(K, L)}$$

$$\frac{MPL(K, L)}{w} = \frac{MPK(K, L)}{r}$$

- Consider an initial choice of K and L , K_0 and L_0 which produces q. Now decrease K_0 by ΔK.

Reduction in capital cost

$$\Delta C_k = -r\Delta K$$

Output will fall by

$$\Delta q = MPK(K_0, L_0)\Delta K$$

Since firm has to produce q, it will have to increase labor by:

$$MPL(K_0, L_0)\Delta L = \Delta q$$
$$\Delta L = \frac{\Delta q}{MPL(K_0, L_0)}$$

Increase in labor cost

$$\Delta C_L = w\Delta L = w\frac{\Delta q}{MPL(K_0, L_0)}$$

So firm will increase K if reduction in capital cost exceeds increase in labor cost:

$$
\begin{aligned}
-r\Delta K + w\Delta L &< 0 \\
-r\frac{\Delta q}{MPK(K_0, L_0)} + w\frac{\Delta q}{MPL(K_0, L_0)} &< 0 \\
wMPK(K_0, L_0) - rMPL(K_0, L_0) &< 0
\end{aligned}
$$

Indifferent if

$$
\begin{aligned}
wMPK(K_0, L_0) - rMPL(K_0, L_0) = 0 \\
\frac{MPK(K_0, L_0)}{r} = \frac{MPL(K_0, L_0)}{w}
\end{aligned}
$$

- Expansion path of output determines $LTC(q)$. Expansion path may involved different ratios of capital and labor at different levels of output.

3 LTC versus STC

- Let input prices be w and r.

- Consider two levels of production, q_A and q_B where $q_A < q_B$.

- If all inputs are variable, then will optimally choose K_A and L_A to produce q_A at long run total cost $LTC(q_A)$, and K_B and L_B to produce q_B, at $LTC(q_B)$. In general

$$K_A \neq K_B$$

- Assume that the firm first has to choose K and L to produce at output level q_A. So it commits to K_A units of capital. Short run cost and long run cost is the same:

$$LTC(q_A) = STC(q_A, K_A)$$

- After committing to K_A capital, the firm is asked to produce q_B in the short run. Now short run cost must exceed long run cost. Why?

$$STC(q_B, K_A) > LTC(q_B)$$

- Now consider the reverse senario. Let the firm first choose K and L to produce q_B. Then short run cost at q_B is the same as long run cost. What happens to long run cost and short run cost at q_A after K_B is chosen?

- Picture of $LAC(q)$ and $SAC(q)$, $LMC(q)$ and $SMC(q)$.

4 Cobb Douglas

- Let input prices be w and r.

- Production function

$$q = AK^{\alpha}L^{\beta},\ \alpha + \beta < 1$$

- Long run cost minimization problem of firm in producing q units:

$$LTC(q, r, w) = \min_{K,L} wL + rK \text{ s.t. } AK^{\alpha}L^{\beta} \geq q$$

Marginal products of capital and labor

$$MPL = \beta AK^{a}L^{\beta-1} = \frac{\beta q}{L}$$
$$MPK = \alpha AK^{a-1}L^{\beta} = \frac{\alpha q}{K}$$

- At the optimum:

$$\frac{MPL(K^*, L^*)}{w} = \frac{MPK(K^*, L^*)}{r}$$
$$\frac{\beta q}{wL^*} = \frac{\alpha q}{rK^*}$$
$$K^* = \frac{\alpha w L^*}{\beta r}$$

- We also need:

$$q = A(K^*)^\alpha (L^*)^\beta$$
$$= A(\frac{\alpha w L^*}{\beta r})^\alpha (L^*)^\beta$$
$$L^* = (\frac{\beta r}{\alpha w})^{\frac{\alpha}{\alpha+\beta}} (\frac{q}{A})^{\frac{1}{\alpha+\beta}}$$

- Finally,

$$K^* = \frac{\alpha w}{\beta r}(\frac{\beta r}{\alpha w})^{\frac{\alpha}{\alpha+\beta}} (\frac{q}{A})^{\frac{1}{\alpha+\beta}}$$
$$= (\frac{\alpha w}{\beta r})^{\frac{\beta}{\alpha+\beta}} (\frac{q}{A})^{\frac{1}{\alpha+\beta}}$$

- $$\begin{aligned} LTC(q,r,w) &= wL^* + rK^* \\ &= [w(\frac{\beta r}{\alpha w})^{\frac{\alpha}{\alpha+\beta}} + r(\frac{\alpha w}{\beta r})^{\frac{\beta}{\alpha+\beta}}](\frac{q}{A})^{\frac{1}{\alpha+\beta}} \\ &= w^{\beta} r^{\alpha}[(\frac{\alpha}{\beta})^{\beta} + (\frac{\beta}{\alpha})^{\alpha}]\frac{q}{A},\ \alpha+\beta=1 \\ &= z(w,r,\beta,\alpha,A)q,\ \alpha+\beta=1 \end{aligned}$$