1 Factor markets

- Consider the short run with one input factor, L.
- Price of output is p. Price of labor is w.
- production function is $q = f(K_0, L)$
- Revenue of firm is $R(q) = pq$.
- cost is $C(L, K_0) = wL - rK_0$
- Firm wants to solve:

$$\max_L pf(K_0, L) - wL - rK_0$$

- At the optimum,

$$p\frac{df(K_0, L)}{dL} = w$$

$$MRPL = VMPL = p * MPL = w$$

- How can we explain the optimal factor demand policy?

- Consider an initial choice of labor, L_0. What is the increase in cost due to the increase in labor by ΔL?

$$\Delta C = w * \Delta L$$

- What is the increase in revenue?

$$\Delta R = p * MPL * \Delta L$$

- Change in profits is

$$\begin{aligned} \Delta\pi &= \Delta R - \Delta C \\ &= p * MPL * \Delta L - w * \Delta L \\ &= (p * MPL - w)\Delta L \end{aligned}$$

- So you want to increase labor if $\Delta\pi > 0$ and vice versa. At the optimum if equal to zero.

- How does the MPL change as L increases? It increases and then decreases.

- When the $VMPL$ intersects w, we have the optimal amount of labor.

- How does the optimal amount of labor changes when w increases?

- How does the optimal amount of labor changes when p increases?

- What happens to MPL when K_0 increases?

- Long run demand

- Now both labor and capital are variable.

- Let the price of capital, K, be r.

- the rest of the problem is the same as in the short run. Now the firm solves:

$$\max_{K,L} \pi = pf(K, L) - rL - wL$$

Optimal demand will satisfy:

$$VMPK = p * MPK = r$$
$$VMPL = p * MPL = w$$

which implies

$$\frac{MPK}{MPL} = \frac{r}{w}$$

- We can explain why the value of the marginal product of factor i must be equal to the price of factor i to maximize profit for the firm.

- What happens to factor demands when the price of capital, r increases? Both capital and labor must adjust. You can actually show that the optimal demand for capital must fall. But the demand for labor may rise or fall depending on how substitutible labor is for capital. What if they are perfect complements or perfect substitutes?

- What happens to factor demands when the price of output increases? In general, optimal level of output will increase but not always.

2 Market demand

Market demand is just the horizontal demand for the input by all firms in all industries.

3 One market equilibrium

- Consider the market for workers in Canada.

- We have a market labor demand curve, $Q_C^d = g(w_C)$, $\frac{dQ_C^D}{dw} < 0$.

- We have a supply of labor curve, $Q_C^s = h(w_C)$, $\frac{dQ_C^S}{dw} > 0$.

- The market for labor clears when

$$Q_C^d = Q_C^s = Q_C^*$$
$$g(w_C^*) = h(w_C^*)$$

- What happens when there is a minimum wage, $w_m > w_C^*$.

4 Two markets equilibrium

- Let there be N_{US} doctors trained in the US and N_C doctors trained in Canada. So we have inelastic supplies in both countries.

- Let the demand for doctors in the US be $Q^d_{US} = g_{US}(w_{US})$.

- Let the demand for doctors in Canada be $Q^d_C = g_C(w_C)$.

- Let doctors be imobile. Then there is an equilibrium wage for doctors in the US, w^*_{US}, and an equilibrium wage for doctors in Canada, w^*_C, and the two market clears separately.

What happens if $w^*_{US} > w^*_C$ and doctors become mobile? What is the new equilibrium wage and equilibrium quantities?

5 Minimum wage in a two (market) sector model

- What if the minimum wage applies to only a subset of the economy? E.g. minimum wages do not apply to self employed individuals.

- Let the total number of workers by N.

- Let the demand for workers in the covered sector by $Q_C^d = g_C(w_C)$.

- Let the demand for workers in the uncovered sector be $Q_{UC}^d = g_{UC}(w_{UC})$.

- Consider an initial equilibrium without a minimum wage.

- Let the minimum wage in the covered sector be above the equilibrium wage.

- Is there any unemployment?

- Who benefits and who loses?