1 Game Theory

- In a competitive industry, each firm assumes that its output decision will not influence the price that it faces (i.e. the output decisions of other competitors).

- A monopolist has no competitor and therefore does not worry about its decisions on other firms.

- An oligopoly has a small number of firms. Each firm expects its output decision to affect the output decisions of its competitors. Therefore, when it makes its decision, it takes into account how others will react to it.

- Consider a game (social situation) with two players, A and B, where each of them have to make a decision. In making its decision, A takes into account how B will react. In making its decision, B takes into account how A will react.

- How can we predict what they will do? Put another way, what is equilibrium behavior?

- E.g. Leaving a burning theater. 200 customers in a movie theater which catches fire. If all the customers file out orderly, 80% will survive. If they all rush and jam the exits, 50% will survive. What is the percent you predict will survive?

- Two firms in an industry can each produce half the monopoly output and each will enjoy half the monopoly profits. This is the maximum profits available to the industry. If they produce more, per firm profits will be lower. What do you predict each firm will do?

- If you go to a new country that you dont know anything about, will you predict the drivers all drive on the right, the left, or right and left?

- Do you expect the police in their effort to deter crime to follow a fixed route and schedule for making their patrols?

- Why can't NHL teams control their own spending on players?

- Should parents negotiate with kidnappers when their children are kidnapped? Will they negotiate?

2 The Prizoner's Dilemma

- There are two players, A and B. Each of them can take one of two decisions, cooperate or defect.

- The payoffs to the various combinations of strategies can be summarized by a matrix:

		A	
		cooperate	defect
B	cooperate	2,2	0,3
	defect	3,0	1,1

- The numbers in the cells are payoffs to the players.

- In any cell, which is chosen by the column of the column player (A) and the row of the row player (B), the first number refers to the payoff of the row player and the second number refers to the payoff of the column player. E.g. in cell (1,2). A defects and B cooperates. The payoff to A is 2 and B is zero.

- What do you think the players will do do?

- An n-player game has n players, numbered 1 to n. An arbitrary player is called player i.

- Let S_i denote the set of strategies available to player i (called i's strategy space), and let s_i denote an arbitrary member of this set.

- Let $(s_1, s_2, .., s_n)$ denote a combination of strategies, one for each player. Let u_i denote the payoff function to player i: $u_i(s_1, s_2, .., s_n)$.Note that i's payoff depends on the strategies of all the players.

- How do we predict what the agents in the game will behave? We assume that agents know the strategy space of all players and the payoff functions of all players. Every player just wants to maximize his own payoff. In the one shot game, they play the game once and there is no further interaction of any kind between the players.

3 Nash Equilibrium

- Conjecture that $s_1^*, s_2^*, .., s_n^*$ constitutes an equilibrium of the game.

- Then it must be that for any player i, his predicted equilibrium strategy s_i^* must be preferred to any other strategy (a best response) given that he believes that all the other players are playing $s_1^*, s_{\neq i}^*, .., s_n^*$.

- In otherwords, player i does not want to deviate from s_i^* as long as the other players do not deviate from their equilibrium play.

- If we can find such equilibrium strategies for all n players, then we have found a Nash Equilibrium.

- What is the NE in the prizoner's dilemma?

- Let us check if cooperate, cooperate is a NE.

- What about defect, defect?

- Note that the NE is not efficient. So NE's are not necessary efficient. But they are individually rational.

- NE's may not be unique.

- Look at a coordination game. Consider a road with n drivers. Drivers going the same direction can choose to drive on the right (r) or on the left (l). Each driver's utility is increasing in the number of drivers that drive on the same side as they do. Let n_r be the number of drivers on the right and n_l be the number of drivers on the left. The payoff of player i who drives on side s is $u_i(n_s)$ where u_i is increasing in n_s. Let n_s be the equilibrium number of drivers on side s. Player i will choose:

$$\max(u_i(n_r), u_i(n - n_r))$$

- If

$$n_s > n - n_s$$

  then player i will want to drive on side s.

- $n = n_r$ is a NE. $n = n_l$ is also a NE. So we have multiple NE. Which one applies? Multiple NE's imply the importance of history and conventions.

- What about $n_r = n_l = \frac{1}{2}n$? This is also a NE which is also inefficient. Can we argue that we should not observe this NE? This is called refining NE.

- You can also get stuck in a pareto inferior equilibrium. E.g. the imperial measurement system in North America.

- Is there a pure strategy NE in every game. No. What about matching pennies? We both have to show a penny. If we match faces, you win \$1. If we don't match, I win \$1.

		me	
		head	tail
you	head	1,-1	-1,1
	tail	-1,1	1,-1

- There is no pure strategy NE in this game. Assume h,h is NE. Then I want to chose t. Assume h,t is NE. Then you want to choose t.

- But there is a mixed strategy NE in which you choose h with prob .5 and I choose h with prob .5.

4 Continuous strategy space: NHL hockey salaries

- Consider a league with two teams, A and B.

- Team A spends resources r_A and team B spends resources r_B.

- Profits for team i is:

$$\pi(r_i, r_j) = a + br_i - cr_i^2 - dr_j - e(r_i - r_j)^2$$

- $b > 0$, $c > 0$, imply profits is increasing at a decreasing rate due to own spending. Fans like it when the team gets better players.

- $d > 0$ imply that profits fall when the competitor gets better players.

- $e > 0$ is the competitive balance effect. If one team spends more than the other team, it makes the games less competitive and fans dont like it.

- We want to find the NE of this spending game.

- Let $\{r_A^*, r_B^*\}$ be pair of equilibrium strategies.

- Given r_j^*, team i best response is:

$$b - 2cr_i^* - 2e(r_i^* - r_j^*) = 0$$

$$r_i^* = \frac{b + 2er_j^*}{2(c+e)}$$

Then:

$$r_A^* = \frac{b + 2er_B^*}{2(c+e)}$$

$$r_B^* = \frac{b + 2er_A^*}{2(c+e)}$$

$$r_B^* = r_A^* = \frac{b}{2c}$$

What is joint profit maximization?

$$\pi(r_A, r_B)+\pi(r_B, r_A) = 2a+(b-d)(r_A+r_B)-c(r_A^2+r_B^2)-2e($$

Marginal profit from increasing r_i is:

$$(b-d) - 2cr_i - 2e(r_i - r_j)$$

Equate marginal profit of r_A and r_B to zero:

$$(b-d) - 2cr_A^{**} - 2e(r_A^{**} - r_B^{**}) = 0$$
$$(b-d) - 2cr_B^{**} - 2e(r_B^{**} - r_A^{**}) = 0$$

$$r_B^{**} = r_A^{**} = \frac{b-d}{2c} < \frac{b}{2c} = r_B^{*} = r_A^{*}$$

- So the two non-cooperative teams spend too much resources relative to joint profit maximization.

- Why? Because it does not take into account that its spending causes the other firm to lose money. Note that r_j^* does not depend on d, the rate at which your spending damages the other team.

- So the NHL wants to penalize each team if it spends beyond the joint profit maximization amount of resources. But players union want the teams to spend more.

- What is the tax on spending that will get the teams to spend the optimal amount. Consider a tax t on each unit of resources spent. Then each team's profit function will look like:

$$\pi(r_i, r_j, t) = a + (b - t)r_i - cr_i^2 - dr_j - e(r_i - r_j)^2$$

In this case, equilibrium spending will be:

$$\widehat{r_B} = \widehat{r_A} = \frac{b - t}{2c}$$

- So if we have a tax on spending of $t = d$, we will get optimal spending per team.

- We can also have a salary cap on spending of r_i^{**} per team.

- What do fans want? We have to figure out fan's demand curve and optimal pricing of tickets.

5 Extensive form games

- We now turn to sequential games, where there is a temporal aspect to the game.

- The extensive form specifies the order of play (who moves when), the information and choices available to a player when it is his turn to play, the payoffs for all players (contingent on all players' choices) and possibly a probability distribution for moves by "nature".

- The "tree" of the game depicts this extensive form.

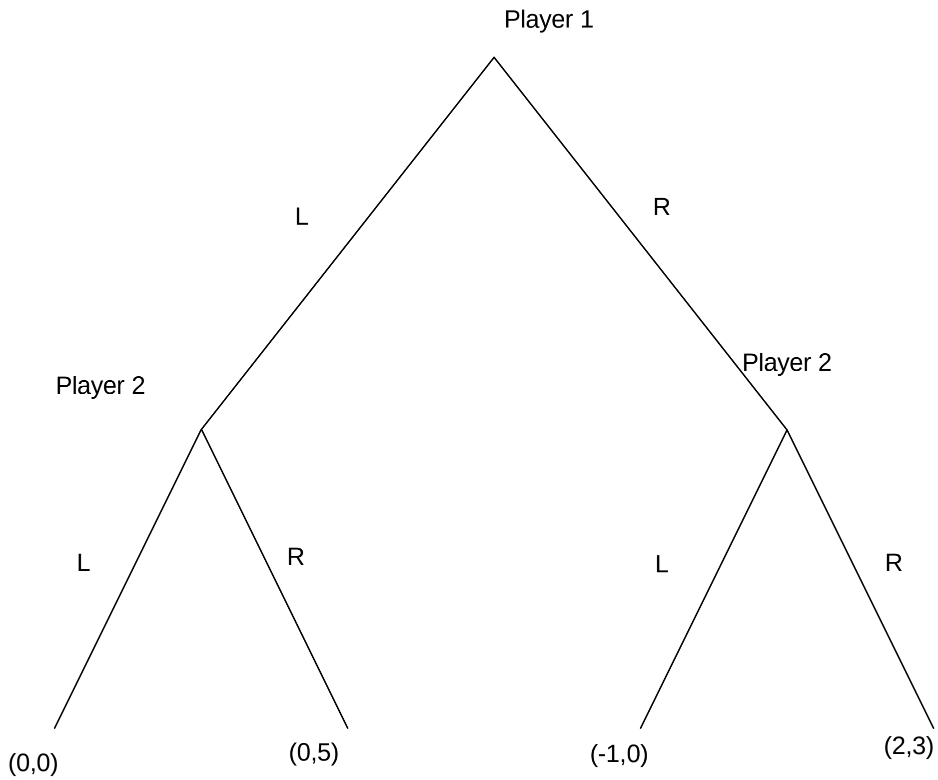
- At time t=1, only player one makes a decision (L or R). At t=2, player 2 makes a decision (L or R) after he observes player's 1 decision.

- The players' payoffs are listed at bottom of tree. So if one chooses L and 2 chooses L, player 1 gets 0 and player 2 gets 5.

- Player 2 observes 1 before moving. So 1 is committed before 2 has to move.

- What are all the NE's in this game?

- All extensive form games can be put in normal form as in Figure below.



$1\backslash 2$	a_2^1 ={L,L}	a_2^2 ={L,R}	a_2^3 ={R,L}	a_2^4 ={R,R}
a_1^1 =L	0,0	0,0	0,5(ne)	0,5
a_1^2 =R	-1,0	2,3(ne)	-1,0	2,3(spne)

- A strategy is a specification of what a player will do at every node in the tree that he finds himself in.

- Player 1 has two pure strategies, L or R.

- Player 2 has four pure strategies: {L,L}, {L,R}, {R,L}, {R,R}. Consider {L,R}. If 1 moves L then 2 moves L. If 1 moves R then 2 moves R.

- This sequential move game has 3 NE, $\{a_1^1, a_2^3\}$, $\{a_1^2, a_2^2\}$, $\{a_1^2, a_2^4\}$.

- Why is $\{a_1^1, a_2^3\}$ a NE? But it is based on an incredible threat.

- So how do we solve for the subgame perfect NE? We work backwards. Find the NE's for the last stage of the game, then work backward.

- E.g. if player 1 moves left, player 2 moves R. If player 1 moves R, player 2 moves R. So player 1 when chosing where to move knows that if he moves L, he will get 0 and if he moves R he will get 2. Knowing this, he will move R.

- There are other NE's in this game which is not sub-game perfect based on incredible threats.

6 Dealing with kidnappers

- A kidnapper has to decide whether to kidnap a child or not.

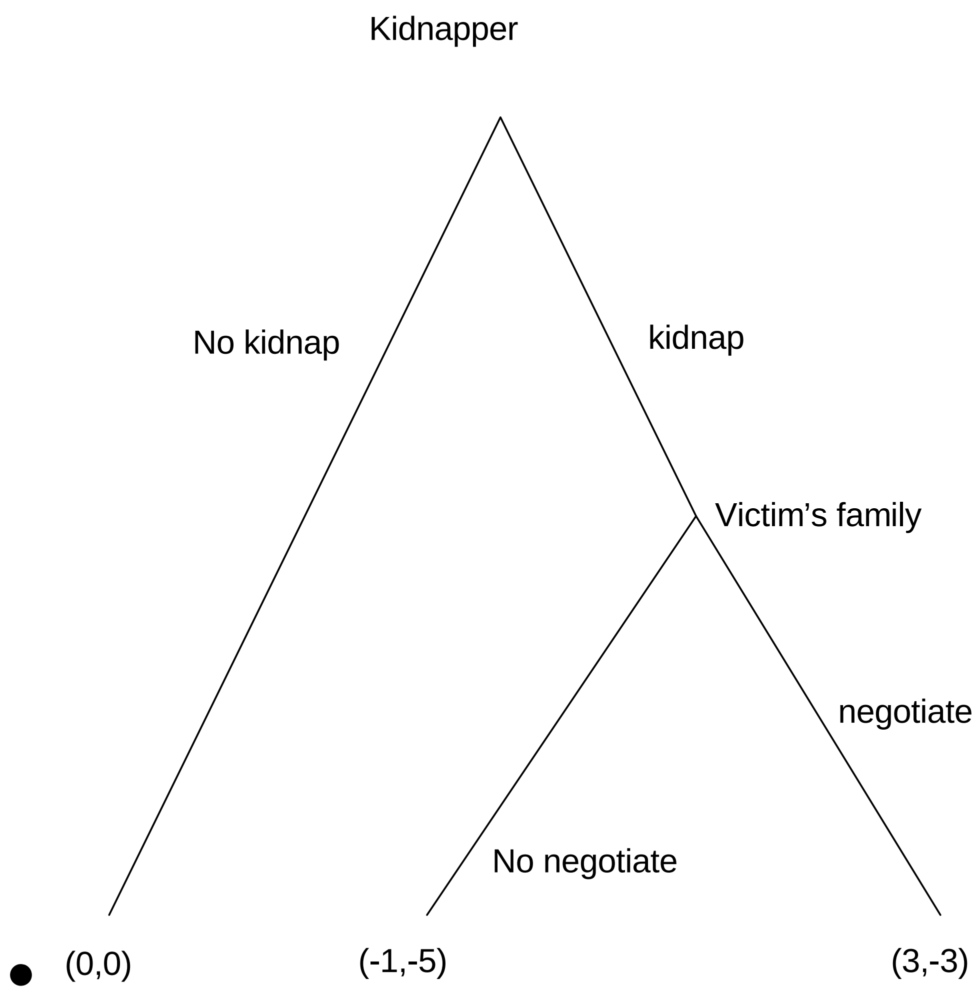
- If the child is kidnapped, the parents have to decide whether to negotiate and pay the kidnapper or not.

- The extensive form is below:

- The normal form payoffs are below:

kidnapper\parents	$a_p^1 = \{0, NN\}$	$a_p^2 = \{0, N\}$
$a_k^1 = NK$	0,0 (NE)	0,0
$a_k^2 = K$	-1,-5	2,-3 (SPNE)

- Kidnapper has two pure strategies.


Kidnapper
No kidnap
kidnap
Victim's family
negotiate
No negotiate
(0,0)
(-1,-5)
(3,-3)

- Parents have two pure strategies. If the child is kidnapped, they can choose to not negotiate or to negotiate. If the child is not kidnapped, the parents do not have to do anything.

- Note that $(NK, \{0, NN\})$ is a NE. Why? Is this credible?

- $(K, \{0, N\})$ is the SPNE. Note that society is worse off. So society wants to deter parents from negotiating.