

# 1 Intertemporal markets.

- As interest falls, will you spend more today as a consumer or less?
- What does the price of oil look like over time? Will we run out of oil? Does it matter?
- What does the price of price of paper look like over time? Are we running out of forests?

- There is no new technique here - we interpret our tools in a new way.
- We want to compare income today with income in future periods of time.
- How much are you willing to pay today to receive \$100 per year forever starting one year from today?
- Let the real rate of interest be  $r$  which is the interest rate at which you can borrow and lend.
- The put  $p$  in bank. You will get  $p(1 + r)$  at the end of one year.
- You need

$$p(1 + r) = 100 + p$$
$$p = \frac{100}{r}$$

- So if  $r = 0.02$ , you need  $p = \$5000$ .
- So if you have a project that pays  $y_1, \dots, y_i, \dots, y_n$  where period 1 is today. Note that  $y_i$  may be negative which means that you pay into the project.
- The present value of the project today is:

$$PV = \sum_{i=0}^n \frac{y_i}{(1+r)^i}$$

- So if you have different projects to invest in, you should pick the project with the largest present value.
- Have to account for risk if income streams are uncertain.

- Some analysts use internal rate of return  $\hat{r}$  to evaluate the project.  $\hat{r}$  is the rate of return such that

$$\sum_{i=0}^n \frac{y_i}{(1 + \hat{r})^i} = 0$$

- Then if  $\hat{r} > r$ , do the project and do not do it if  $\hat{r} < r$ . Is this always right?

## 2 Borrowing and lending

- 2 period model - consumption ( $c_1$ ) today vs consumption tomorrow ( $c_2$ )

$$U(c_1, c_2)$$

Let  $r$  be the real rate of interest.

$r$  is a number like 0.1 for 10%.

$$\beta = \frac{1}{1 + r}$$

$\beta$  is the discount factor. As  $r$  increases,  $\beta$  falls.

Let the consumer have  $y_1$  and  $y_2$  respectively.

We normalize the price of output to 1 in each period.

So there is no inflation.

Budget constraint

$$c_1 + \beta c_2 \leq y_1 + \beta y_2$$

Present value of consumption is less than or equal to the present value of income.

$\beta$  is the price of consumption tomorrow relative to the price of consumption today.

In general, we know that optimal consumption will satisfy:

$$\frac{MU(c_1)}{1} = \frac{MU(c_2)}{\beta}$$

This is shown in figure. Is the person a borrower or a lender?

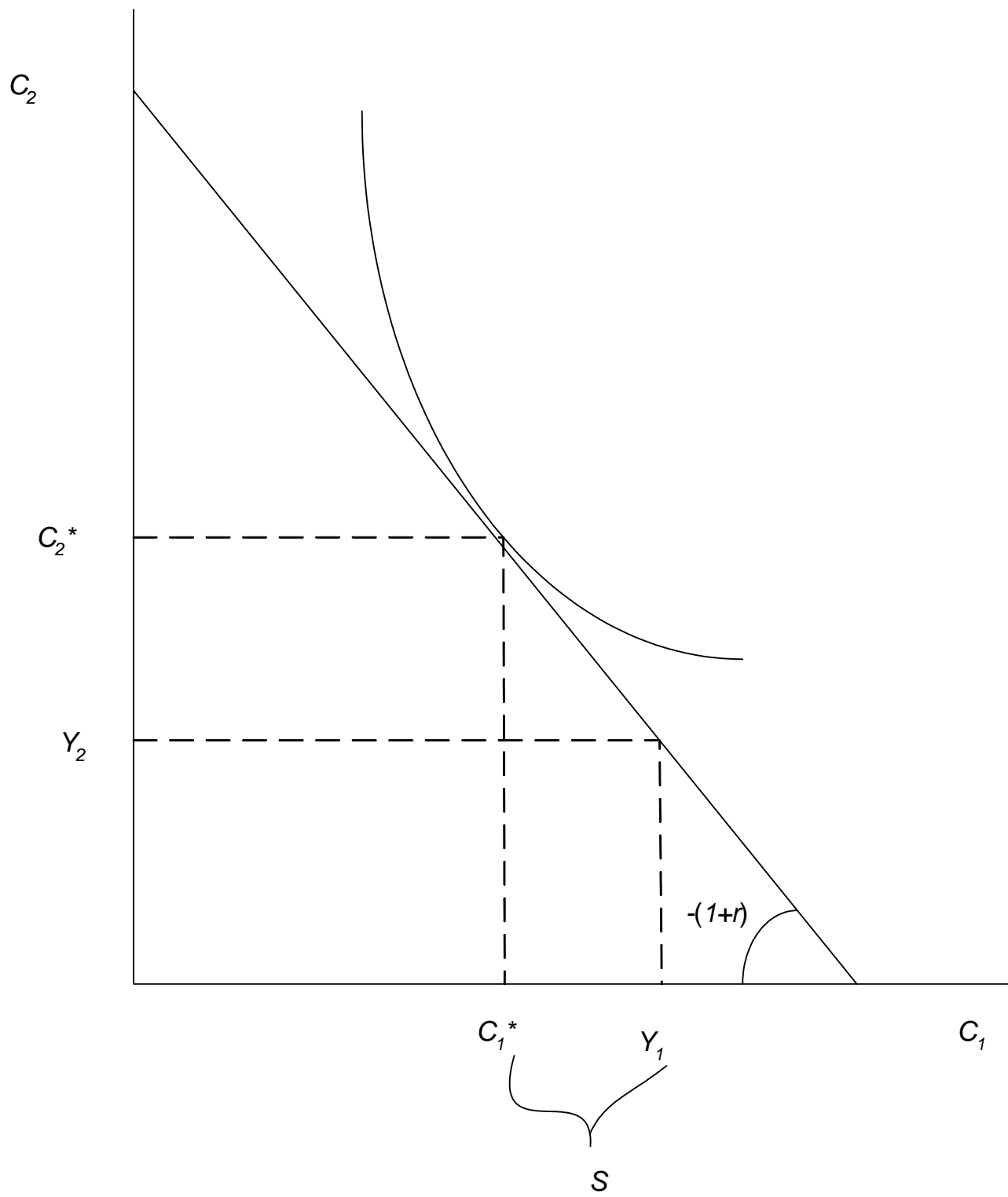
This is an endowment economy - price of consumption and income changes as relative prices change.

i.e. when  $r$  falls,  $\beta$  rises, the price of  $c_2$  rises but also value of  $y_2$  rises.

If income was all second period income, the person may consume more  $c_1$  and  $c_2$  as  $\beta$  increases.

Savings:

$$s = y_1 - c_1$$



What should happen to savings as  $\beta$  increases ( $r$  falls to  $r_1$ ) ?

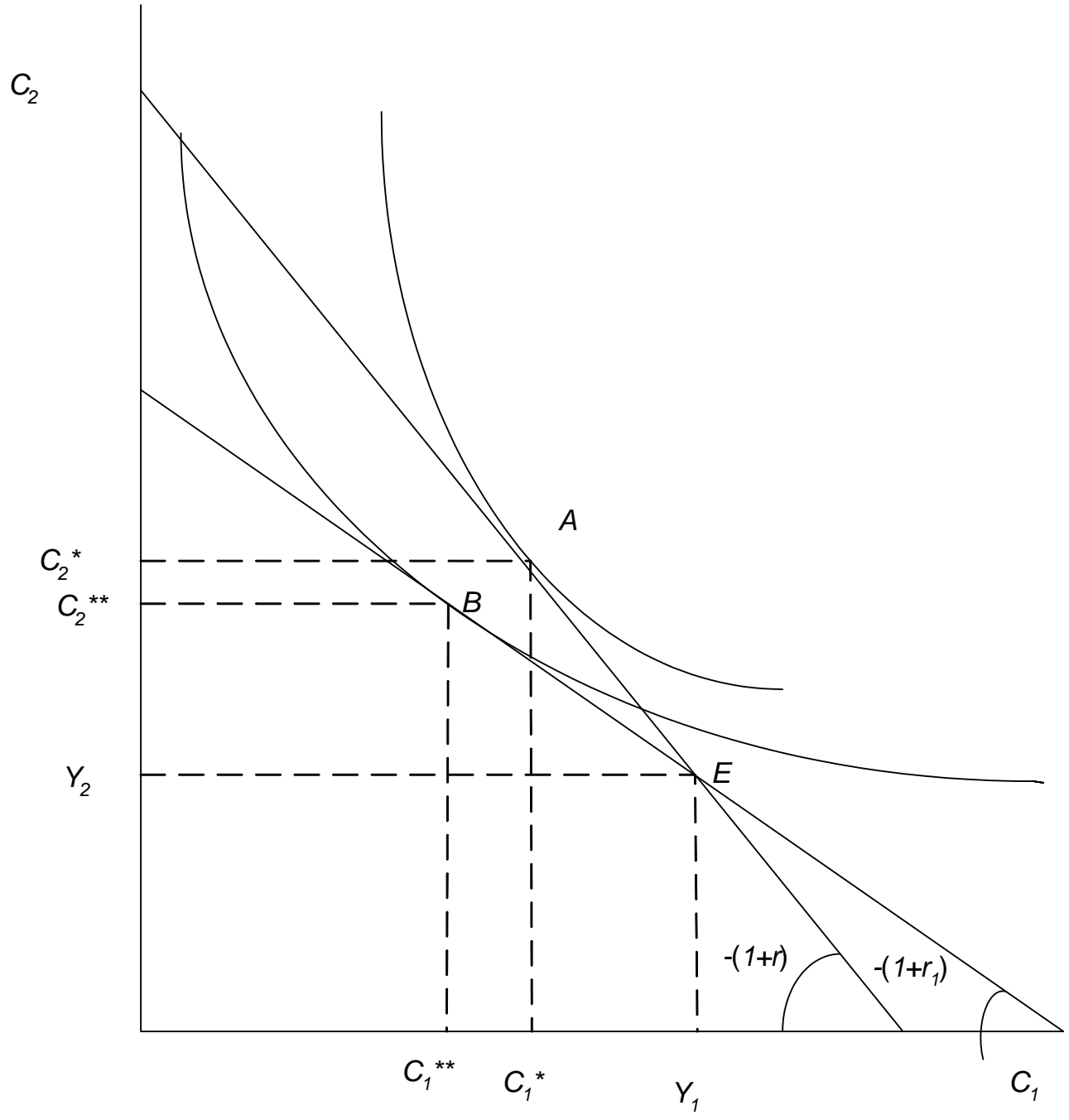
$c_2$  has gotten more expensive.  $c_1$  is cheaper so should substitute towards  $c_1$ .

But what about income effects ? depends where you are. two possible cases.

If you are borrower, then your income increased and you consume net more  $c_1$  and impact of  $c_2$  unclear.

If you are a lender, income fell and so you consume net less  $c_2$  but the impact on  $c_1$  is ambiguous.

Expect backward bending supply of savings. why ? as  $r$  increases, you move from being a borrower to being a lender and as the interest rate keep rising, the income effect becomes bigger and bigger which you expect soon to dominate the substitution effect. so if  $r = 2$ , you may decide to save less today because you will have so much consumption tomorrow.



### 3 Exhaustible resource

Lets look at exhaustive resource problem. I.e. you have a stock of output today. if you sell it, the stuff is gone.

When should you sell your wine?

Let  $p_t$  be the price of wine at time  $t$ .

Let interest rate be  $r$ .

Assume price increases with time at a decreasing rate.

Hold if

$$p_{t+1} > (1 + r)p_t$$

Why?

Sell if

$$p_{t+1} < (1 + r)p_t$$

So indifferent if

$$p_{t+1} = (1 + r)p_t$$
$$\frac{p_{t+1} - p_t}{p_t} = r$$

I.e. rate of interest is equal to price appreciation rate. So if price path is increasing and concave in time, you sell when slope of path is equal to the rate of interest. so as interest rate rises, you sell wine sooner.

If you sell wine in both periods, what must be true, then price must be rising at the rate of interest.

Think about oil. you have the stuff in the ground. let extraction cost be  $c$  per unit.

then you will be indifferent if

$$p_{t+1} - c = (1 + r)(p_t - c)$$
$$r = \frac{p_{t+1} - p_t}{p_t - c}$$

This implies that  $p_t$  is increasing at an increasing rate.

As price rise at that rate, if demand is stable overtime, consumption falls overtime.

If price rise sufficiently high, demand goes to zero and you move to new technology. E.g. we don't use whale oil anymore. Ethanol (made from sugarcane or wheat) costs about 70 cents a litre to produce. If crude oil prices rises to say \$75 per barrel in the long run, it will be cost efficient to drive with ethanol.

So is running out of oil a long run social issue? No.

Simple model has some flaws. For oil and other exhaustible resources, prices have fallen for long periods of time and consumption increasing. There are two reasons (1) new finds, and (2) cost of extraction falling over time due to technological change.

## 4 Renewable natural resource

When to cut a tree and replant?

Let  $V(t)$  be value of a tree at time  $t$ . Assume that it is increasing and concave in  $t$ .

When you cut it, you start again with a new tree.

Let  $t$  be date to cut tree.

Then for one tree slot you can get

$$\begin{aligned} PV(t) &= \beta^t V(t) + \beta^{2t} V(t) + \dots + \beta^{nt} V(t) + \dots \\ &= \frac{\beta^t V(t)}{1 - \beta^t} \end{aligned}$$

You will want to choose  $t$  to maximize  $PV(t)$ . At the maximum,

$$\begin{aligned}PV(t) &= PV(t + 1) \\ \frac{\beta^t V(t)}{1 - \beta^t} &= \frac{\beta^{t+1} V(t + 1)}{1 - \beta^{t+1}} \\ \frac{V(t)}{1 - \beta^t} &= \frac{\beta V(t + 1)}{1 - \beta^{t+1}} \\ \frac{V(t + 1) - V(t)}{V(t)} &= \frac{r}{1 - \beta^t} > r\end{aligned}$$

What does that mean?

You cut down tree earlier than if tree is nonrenewable.

What is the economics?

## **5 What is the policy issue over deforestation?**

Deforestation is a problem for many parts of the world, including many developing countries.

In India and China, peasants cut trees for firewood.

In Brazil, they cut trees for timber and for agriculture.

So what is the problem?

A problem of externality and property rights.