1 Why is monopoly output not a duopoly equilibrium?

- Consider a duopoly with linear demand.

- Demand is

$$p = a - b(q_1 + q_2)$$

- Let both firms have the same constant marginal cost c.

- Monopoly solves:

$$\begin{aligned} \pi_m &= \max_q \{pq - cq\} \\ &= \max_q \{aq - bq^2 - cq\} \\ &= \end{aligned}$$

- $$\begin{aligned} q_m &= \frac{a-c}{2b} \\ \pi_m &= (a-c)\frac{a-c}{2b} - b(\frac{a-c}{2b})^2 \\ &= \frac{(a-c)^2}{4b} \end{aligned}$$

- Is the pair of strategies, $\{\frac{q_m}{2}, \frac{q_m}{2}\}$, a NE?

- $\{\frac{q_m}{2}, \frac{q_m}{2}\}$ is a NE if:

  1. Given that firm 1 produces $\frac{q_m}{2}$, firm 2's best response is to produce $\frac{q_m}{2}$.

  2. Given that firm 2 produces $\frac{q_m}{2}$, firm 1's best response is to produce $\frac{q_m}{2}$.

- So assume firm 1 produces $\frac{q_m}{2}$. What is firm 2's best response?

- Given q_j, firm i solves:

$$\begin{aligned}\max_{q_i} \pi(q_j) &= pq_i - cq_i \\ &= (a - b(q_i + q_j))q_i - cq_i\end{aligned}$$

$$\begin{aligned}MR(q_j, q_i) &= a - bq_j - 2bq_i \\ MC(q_i) &= c\end{aligned}$$

At the optimum,

$$\begin{aligned}MR(q_j, \widehat{q_i}) &= MC(\widehat{q_i}) \\ \widehat{q_i} &= \frac{a - bq_j - c}{2b}\end{aligned}$$

- So if firm 1 produces $\frac{q_m}{2}$, firm 2's best response is:

$$\begin{aligned}
\widehat{q_2} &= \frac{a - b\frac{q_m}{2} - c}{2b} \\
&= \frac{a - b\frac{a-c}{4b} - c}{2b} \\
&= \frac{3(a-c)}{8b} > \frac{a-c}{4b} = \frac{q_m}{2}
\end{aligned}$$

- So $\{\frac{q_m}{2}, \frac{q_m}{2}\}$ is not a NE.

- This the problem of all cartels.

- You can ask the government to enforce the price floor. Do not allow any firm to charge less than monopoly price.

- Airline fare regulation. Raise specter of consumer safety.

2 Cournot equilibrium

- Finding the NE of a duopoly with linear demand and firms choose quantities.

- Consider firm 1 with marginal cost c_1 and firm 2 with marginal cost c_2.

- Demand is

$$p = a - b(q_1 + q_2)$$

- Let $\{\widehat{q_1}, \widehat{q_2}\}$ be a pair of equilibrium strategies.

- Then given $\widehat{q_j}$, firm i solves:

$$\begin{aligned} \max_{q_i} \pi(\widehat{q_j}) &= pq_i - c_i q_i \\ &= (a - b(q_i + \widehat{q_j}))q_i - c_i q_i \end{aligned}$$

$$MR(\widehat{q_j}, q_i) = a - b\widehat{q_j} - 2bq_i$$
$$MC(q_i) = c_i$$

At the optimum,

$$MR(\widehat{q_j}, \widehat{q_i}) = MC(\widehat{q_i})$$
$$\widehat{q_i} = \frac{a - b\widehat{q_j} - c_i}{2b}$$

Similarly,

$$\widehat{q_j} = \frac{a - b\widehat{q_i} - c_j}{2b}$$

Solving the two equations,

$$\widehat{q_1} = \frac{a + c_2 - 2c_1}{3b}$$
$$\widehat{q_2} = \frac{a + c_1 - 2c_2}{3b}$$

If $c_1 = c_2 = c$

$$\widehat{q_1} = \widehat{q_2} = \frac{a-c}{3b} > \frac{a-c}{4b} = \frac{q_m}{2}$$

So duopoly equilibrium is inefficient relative to monopoly. Consumers benefit.

As the number of firms increase, oligopoly output is even lower.

3 Committment and entry deterrence

- An industry has two technologies.

- The cheap technology costs \$25 to build and will produce 10 units at zero marginal cost.

- The expensive technology costs \$60 to build and will produce 20 units at zero marginal cost.

- Industry demand is

$$P = 10 - \frac{Q}{4}$$

- Consider an incumbent monopolist. Profits from cheap technology is:

$$\pi_c = (10 - \frac{10}{4})10 - 25$$
$$= 75 - 25 = 50$$

- Profits from expensive technology is:

$$\pi_e = (10 - \frac{20}{4})20 - 60$$
$$= 100 - 60 = 40$$

- So monopolist chooses cheap technology.

- What if there is a potential entrant? If monopolist chooses cheap technology and entrant chooses cheap technology, show that $q_m = 10$ and $q_{en} = 10$ is NE output.

- Entrant's profit with cheap technology is

$$\rho_c = (10 - \frac{20}{4})10 - 25$$
$$= 50 - 25 = 25$$

- Monopolist's profit is also 25.

- If monopolist chooses cheap technology and entrant chooses expensive technology, show that $q_m = 10$ and $q_{en} = 15$ is NE output.

- Entrant's profit with expensive technology is

$$\begin{aligned}\rho_{en} &= (10 - \frac{25}{4})15 - 60 \\ &= -\frac{15}{4}\end{aligned}$$

- So entrant will enter with cheap technology if monopolist chooses cheap technology.

- If monopolist chooses expensive technology, and entrant chooses cheap technology, show that $q_m = 15$ and $q_{en} = 10$ is NE output.

- Entrant's profit with cheap technology is

$$\begin{aligned}\rho_{en} &= (10 - \frac{25}{4})10 - 25 \\ &= \frac{25}{2}\end{aligned}$$

Note that monopoly profits in this case is $-\frac{15}{4}$.

If monopolist chooses expensive technology, and entrant chooses expensive technology, show that $q_m = \frac{40}{3}$ and $q_{en} = \frac{40}{3}$ is NE output.

Entrant's profit with expensive technology is:

$$\rho_{en} = (10 - \frac{80}{4*3})\frac{40}{3} - 60 \quad (1)$$

$$= -\frac{140}{9} \quad (2)$$

So entrant will enter with cheap technology and monopolist with expensive technology will lose money.

- The subgame perfect equilibrium is one where the monopolist will choose a cheap technology, the entrant will enter with a cheap technology and both parties produce 10 each.

- What if the monopolist chooses the expensive technology and promises to produce 20 if there is entry. The best response of a cheap entrant if the monopolist produces 20 is 10. In this case, entrant's profit is:

$$\rho_{en} = (10 - \frac{30}{4})10 - 25 \tag{3}$$
$$= 0 \tag{4}$$

- The best response of an expensive entrant if the monopolist produces 20 is also 10. In this case, entrant's profit is:

$$\rho_{en} = (10 - \frac{30}{4})10 - 60 \tag{5}$$
$$= -35 \tag{6}$$

- So entrant can at best make 0 with cheap technology. May as well not enter if monopolist threat is credible.

- But we have shown that monopolist threat to produce 20 is not credible.

- Question: What should the potential entrant infer if the monopolist builds an expensive technology?

- The value of reputation.