

1 Uncertainty and expected utility

2 Expected value

Consider a lottery (gamble) which pays y_1 with probability p_1 , and y_2 with probability p_2 .

y_1 and y_2 are state contingent commodities.

y_1 is realized if state 1 is realized and y_2 is realized when state 2 is realized.

Note that $p_2 = 1 - p_1$.

The expected value (mean value) of the lottery is:

$$Ey = p_1y_1 + p_2y_2$$
$$Var(y) = p_1(y_1 - Ey)^2 + p_2(y_2 - Ey)^2$$

Consider two alternative lotteries with the same expected value:

	A	B
p_1	0.99	0.5
y_1	0	99
p_2	0.01	0.5
y_2	10,000	101
Ey	100	100
$Var(y)$	990,000	2

If you don't like risk, holding the expected value of the lottery constant, you will choose the lottery with the smaller variance.

Or you will be willing to pay less for the lottery with the higher variance.

If you own the lottery tickets, you can sell it as a whole.

Or you can sell the state 1 payoff separately from the state 2 payoff.

Consider selling the state 1 payoff of ticket B. The buyer of state 1 payoff will get 99 if state 1 occurs and 0 if state 2 occurs.

Consider selling state 2 payoff. The buyer of state 2 payoff will get 0 if state 1 occurs and get 101 state 2 occurs.

Do individuals sell part of a lottery? Yes, insurance with deductibles. If you buy car insurance where there is a \$1000. deductible, then you are only buying insurance for states with large losses and not for states with small losses. Why do you do that? Because you pay less for the insurance.

3 Expected utility

Consider a person with a utility function $U(y)$ over income y . What do you think this function should look like? Why? Draw three pictures (risk neutral, averse, preferred)

He has to evaluate the two lotteries above. Consider the two lotteries as jobs with a weekly payoff at the end of the week. Which job will he pick at the beginning of the week? Note that he has to choose at the beginning of the week.

Since he is not going to get paid until the end of the week, he will have to choose today based on how he think he will feel tomorrow when the lotteries are realized. So he may think that his utility tomorrow is $U(y_1^i)$ if y_1^i is realized for lottery i . His utility is $U(y_2^i)$ if y_2^i is realized in lottery i .

But what else? How do you compare?

Expected utility hypothesis: he should evaluate the lotteries based on the expected utility of the outcomes.

$$EU(y^i) = p_1^i U(y_1^i) + p_2^i U(y_2^i)$$
$$\max_{A,B} \{EU(y^A), EU(y^B)\}$$

Risk neutral:

$$U(y) = a + by$$
$$EU(y) = a + bEy$$

So you will pick the lottery with the highest expected value.

Risk averse

A risk averse person, when faced with a fair gamble, will always want to take the expected value of the gamble.

$$U(y), U_y > 0, U_{yy} < 0$$

$$EU(y) = p_1U(y_1) + p_2U(y_2) < U(Ey)$$

avoid gambles. Show in picture.

E.g. let

$$\begin{aligned}U(y) &= \sqrt{y} \\U^A(y) &= 0.99 * 0 + 0.01 * \sqrt{10000} \\&= 0.01 * 100 = 1\end{aligned}$$

Note that $U(1) = \sqrt{1} = 1$

$$\begin{aligned}U^B(y) &= 0.5 * \sqrt{99} + 0.5\sqrt{101} \\&= 9.9999\end{aligned}$$

So B is a lot better than A for a person with \sqrt{y} utility function.

Risk preferred:

$$U(y), U_y > 0, U_{yy} > 0$$
$$EU(y) > U(Ey)$$

Certainty equivalence:

With \sqrt{y} utility function, he will sell lottery A for y_A (certainty equivalent) if his utility from the sale is at least as high as taking the gamble:

$$\sqrt{y_A} = 1 = U^A(y)$$
$$y_A = 1$$

The certainty equivalent, y_B , for lottery B is:

$$\sqrt{y_B} = 9.9999 = U^B(y)$$
$$y_B = 9.9999^2 = 99.998$$

4 Demand for fair insurance

You start with income y_0 . In the bad state, you lose l with probability π_1 . You lose nothing with probability $\pi_2 = 1 - \pi_1$. So without insurance, your expected utility is:

$$U^{NI} = \pi_1 U(y_0 - l) + \pi_2 U(y_0)$$

Now an insurance company is willing to sell you insurance. If you buy x units of insurance at a insurance premium (price) p per unit, the insurance company will pay you x if the bad state is realized and zero otherwise.

Your consumption in the good state is

$$c_2 = y_0 - px$$

Your consumption in the bad state is

$$\begin{aligned} c_1 &= y_0 - px - l + x \\ &= y_0 - l + (1 - p)x \end{aligned}$$

Substituting out x in the second equation gives:

$$c_1 = y_0 - l + (1 - p) \frac{y_0 - c_2}{p}$$
$$c_1 + \frac{(1 - p)}{p} c_2 = y_0 + \frac{(1 - p)}{p} y_0 - l$$
$$p_1 c_1 + p_2 c_2 = y$$

How much insurance should you buy?

So you solve

$$\max_{c_1, c_2} V(c_1, c_2) = \pi_1 U(c_1) + \pi_2 U(c_2)$$

subject to

$$p_1 c_1 + p_2 c_2 = y$$

The above problem is a standard consumer choice problem. So at the optimum,

$$\frac{V_{c_1}}{p_1} = \frac{V_{c_2}}{p_2}$$

$$\frac{\pi_1 U_{c_1}(c_1)}{p_1} = \frac{\pi_2 U_{c_2}(c_2)}{p_2}$$
$$\pi_1 U_{c_1}(c_1) = \frac{\pi_2 p U_{c_2}(c_2)}{(1 - p)}$$

Now what is p ? Insurance company provides fair insurance which means that it breaks even and is risk neutral. In this case,

$$p = \pi_1 + (1 - \pi_1)0$$
$$= \pi_1$$

In this case

$$\pi_1 U_{c_1}(c_1) = \frac{\pi_2 \pi_1 U_{c_2}(c_2)}{(1 - \pi_1)}$$

$$U_{c_1}(c_1) = U_{c_2}(c_2)$$

$$c_1 = c_2$$

$$y_0 - l + (1 - p) \frac{y_0 - c_2}{p} = c_2$$

$$c_2 = c_1 = p \left(y_0 + \frac{(1 - p)}{p} y_0 - l \right)$$

$$= \pi_1 \left(y_0 + \frac{(1 - \pi_1)}{\pi_1} y_0 - l \right)$$

$$= y_0 - \pi_1 l$$

Finally,

$$y_0 - \pi_1 l = y_0 - px$$

$$x = l$$

Full insurance.