

1. Since there is only one type of marriage, let μ be the number of marriages. Let M and F be the numbers of men and women respectively. The CS marriage matching function is:

$$\frac{\mu}{\sqrt{(M - \mu)(F - \mu)}} = \pi$$

$$\begin{aligned} \mu &= \pi \sqrt{(M - \mu)(F - \mu)} \\ \frac{\partial \mu}{\partial F} &= \pi \frac{M - \mu}{2\sqrt{(M - \mu)(F - \mu)} + \pi(F - \mu) + \pi(M - \mu)} > 0 \\ \frac{\partial \mu}{\partial \pi} &= 2 \frac{(M - \mu)(F - \mu)}{2\sqrt{(M - \mu)(F - \mu)} + \pi(F - \mu) + \pi(M - \mu)} > 0 \end{aligned}$$

2. Now the quasi demand curve for men becomes:

$$\ln \mu = \ln(M - \mu) + \alpha - b\tau$$

The quasi supply curve of women is:

$$\ln \mu = \ln(F - \mu) + \gamma + \tau$$

Solving out τ in the above two equations to get:

$$\begin{aligned} \ln \mu &= \ln(M - \mu) + \alpha - b(\ln \mu - \ln(F - \mu) - \gamma) \\ (1 + b) \ln \mu - \ln(M - \mu) - b \ln(F - \mu) &= \pi' \end{aligned} \quad (1)$$

where $\pi' = \alpha + b\gamma$.

Even if we observe μ , M and F , there are still two unknowns in (1), b and π' . So all we can identify is the locus of b and π' which satisfy (1).

(ii) From (1):

$$\frac{\partial \pi'}{\partial b} = \ln \frac{\mu}{F - \mu}$$

Given $\ln \frac{\mu}{F - \mu}$, $\frac{\partial \pi'}{\partial b}$ is constant. So letting b range from $1 - z$ to $1 + z$, you will be able to find the range of π' which are consistent with that. Since $\frac{\partial \pi'}{\partial b}$ is constant, for each b , there will be a unique π' .

(iii) b measures the marginal cost of resources for men relative to its marginal value to women. Its hard to think about whether b should be 1 or not because we dont know what else to compare it to. Put another way, if b increase, then we know that there will be less marriages. But perhaps π' , the gains to marriage is low. This shows the limit of what we can say even in a simple model of marriage.