1 Problems

1. Consider a simplification of the statistical model presented above. Assume that there is no high type individual in the society. That is, there is no class distinction. Let the number of new individuals of each gender enter the marriage market each period be 1. Let $\lambda = 0$. I.e. all matches are accepted. So this model has two unknown parameters, p and η.

(a) What are the analogs to equations (**??**) to (**??**)? Solve for n, N, π, Π as functions of p and η.

(b) Let n, the number of unmarried males in the marriage market, be 7. Let π, the number of marriages, be 13. What is your estimate of p and η?

(a)

$$
\begin{aligned}
N &= 1 + \eta p(1-p)(\Pi + N) \\
\Pi &= \eta p^2(\Pi + N) \\
\pi &= p^2(\pi + N) \\
n &= 1 + p(1-p)(\pi + N) + p(n - N)
\end{aligned}
$$

$$
\begin{aligned}
\Pi &= \frac{\eta p^2}{1-\eta p} \\
N &= \frac{1-\eta p^2}{1-\eta p} \\
n &= \frac{1}{1-p} - \frac{p^2(1-\eta p^2)}{(1-p^2)(1-\eta p)} \\
\pi &= \frac{p^2(1-\eta p^2)}{(1-p^2)(1-\eta p)}
\end{aligned}
$$

(b)

$$
\begin{aligned}
p &= 0.95 \\
\eta &= 0.93681
\end{aligned}
$$

2. Following question 1, further, assume that θ fraction of males and θ fraction of females in every birth cohort never enter the marriage market at all. These individuals die at the same rates as individuals who enter the marriage market. So from our perspective, we cannot distinguish between an individual who never entered the marriage market from another who entered the marriage market but died before marrying.

(a) Now n_o, the number of observed unmarried males, consists of both males who are in the marriage market, n, and males who are not in the marriage market, n_θ. What are the analogs to equations (**??**) to (**??**)? What are n_θ and n_o in terms of p, η and θ?

(b) Let $\theta = 0.1$. Let $n_o = 7$ and $\pi = 13$. What is your new estimate of p and η?

(c) Since question 1 essentially sets θ to zero, in which direction are the estimates of the parameters of p and η biased if you do not know what θ is?

Explain the direction of the bias.

(d) What data from Table 1 will be useful for you to estimate θ as well?

(a)

$$\begin{aligned} N &= 1 - \theta + \eta p(1-p)(\Pi + N) \\ \Pi &= \eta p^2(\Pi + N) \\ \pi &= p^2(\pi + N) \\ n &= 1 - \theta + p(1-p)(\pi + N) + p(n - N) \end{aligned}$$

$$\begin{aligned} \Pi &= \frac{(1-\theta)\eta p^2}{1-\eta p} \\ N &= \frac{(1-\eta p^2)(1-\theta)}{1-\eta p} \\ n &= \left(\frac{1}{1-p} - \frac{p^2(1-\eta p^2)}{(1-p^2)(1-\eta p)}\right)(1-\theta) \\ \pi &= \frac{(1-\theta)p^2(1-\eta p^2)}{(1-p^2)(1-\eta p)} \end{aligned}$$

$$\begin{aligned} n_\theta &= \theta + p n_\theta \\ &= \frac{\theta}{1-p} \\ n_o &= n_\theta + n \\ &= \frac{\theta}{1-p} + \left(\frac{1}{1-p} - \frac{p^2(1-\eta p^2)}{(1-p^2)(1-\eta p)}\right)(1-\theta) \\ &= \frac{1}{1-p} - \frac{(1-\theta)p^2(1-\eta p^2)}{(1-p^2)(1-\eta p)} \end{aligned}$$

$$\begin{aligned} p &= 0.95 \\ \eta &= 0.96642 \end{aligned}$$

(c) The estimate of p is unbiased. The estimate of η is biased down if θ is set to zero. When θ is positive, some of the observed unmarried men are not interested in marriage. If we assume that they are all interested in marriage, then we over estimate the number of men who want to marry. This over estimate is rationalize by a "low" estimate of η, the retention rate of fecund females.

(d) Don't know.