Answers:
1. $\frac{V_s}{2}$ is her reservation match value.

$$\begin{aligned} V_s &= s + E\max(s,\theta) \\ &= s + sF(s) + (1-F(s))E(\theta|\theta \geq s) \end{aligned}$$

For θ, a uniformly distributed random variable,

$$\begin{aligned} V_s &= s + s^2 + (1-s)\frac{1+s}{2} \\ &= s + \frac{1+s^2}{2} \end{aligned}$$

When $s = \frac{1}{2}$, the reservation match value, $\frac{V_s}{2} = \frac{9}{16}$.
2. The reservation match value for remaining married is

$$\Omega(\theta_1) = V_d - \lambda - \theta_1$$

Given θ is uniformly distributed between 0 and 1, $V_d = \frac{1+s^2}{2}$, and since $s = \frac{1}{2}$ and $\lambda = 0$,

$$\Omega(\theta_1) = \frac{5}{8} - \theta_1$$

Her utility from marriage in the first period is:

$$\begin{aligned} V_m(\theta_1) &= \theta_1 + \theta_1 + \overline{G(\Omega(\theta_1))}\Gamma(\omega,\theta_1) + G(\Omega(\theta_1))\Omega(\theta_1) \\ &= 2\theta_1 + (1 - \frac{1}{2} - \Omega(\theta_1))(\frac{\Omega(\theta_1)+\frac{1}{2}}{2}) + (\frac{1}{2} + \Omega(\theta_1))\Omega(\theta_1) \\ &= 2\theta_1 + \frac{\Omega(\theta_1)+\frac{1}{2}}{2} + (\frac{1}{2} + \Omega(\theta_1))\frac{\Omega(\theta_1)}{2} - \frac{\frac{1}{2}+\Omega(\theta_1)}{4} \\ &= 2\theta_1 + \frac{(\Omega(\theta_1)+\frac{1}{2})^2}{2} \\ &= 2\theta_1 + \frac{(\frac{5}{8} - \theta_1 + \frac{1}{2})^2}{2} \end{aligned}$$

When $s = \frac{1}{2}$, her utility from not marrying in the first period is:

$$V_s = s + \frac{1+s^2}{2} = \frac{9}{8}$$

The reservation match value of θ_1, θ_r, for marrying in the first period is

$$\begin{aligned} V_m(\theta_r) &= V_s \\ 2\theta_r + \frac{(\frac{5}{8} - \theta_r + \frac{1}{2})^2}{2} &= \frac{9}{8} \end{aligned}$$

The only root of θ_r between 0 and 1 is 0.448. Note that $\theta_r < \frac{1}{2}$.

3. Corresponding to (3.17) in the chapter, we have:

$$\frac{1 - G(V_d - \theta_1 - 2e_i^n)}{4} = e_i^n$$

The only difference from (3.17) is that now we have 4 in the denominator rather than 2. This difference correspond to the fact that the cost of effort per individual is $2e_i^2$ rather than e_i^2 as in the text. The marginal cost of effort is $4e_i$ rather than $2e_i$. So we have

$$\frac{1 - (\frac{1}{2} + V_d - \theta_1 - 2e_i^n)}{4} = e_i^n$$

$$e_i^n = \frac{\frac{1}{2} - V_d + \theta_1}{2}$$

e_i^n, effort is increasing in θ_1. The reason is that as the first period match is better, holding effort constant, the probability of divorce falls. But as the probability of divorce falls, the couple is more likely to remain in the married state which increases the value of investment in marriage specific capital. So the spouses increase their equilibrium investments as θ_1 increases.