

## 1 Chapter 1: Answers

1. We take “true” to mean what would happen to the child if he grew up with his biological father and mother. There are two reasons why a child grows up with one parent rather than two. First, one of the parent is absent due to exogenous factors, such as death. Second, one of the parent is absent due to endogenous factors. That is, the father and the mother decide not to form a household to bring up the child. The “true” benefit of having two parents rather than one is to compare families with two parents with one parent families where a parent is absent exogenously. It is likely that the benefit of having two biological parents rather than one in the one parent families where one parent is absent endogenously may be quite small if not negative.

Observed one parent families contain families where parents are missing both exogenously and endogenously. So a comparison with two parents households with one parent households will over estimate the “true” gains to two parents household.

2. We can solve this in two ways.

(a) Let  $c^*$  and  $q_m^*$  denote the optimal quantities. We know from the optimality solution for the single parent problem that:

$$U_c(c^*, q_m^*) = U_{q_m}(c^*, q_m^*) \quad (1)$$

Since  $U(c, q_m) = cq_m$ ,

$$\begin{aligned} U_c(c^*, q_m^*) &= q_m^* \\ U_{q_m}(c^*, q_m^*) &= c^* \end{aligned}$$

(1) implies:

$$c^* = q_m^* \quad (2)$$

From the budget constraint,  $c^* = y_m - q_m^*$ . So (2) becomes:

$$\begin{aligned} y_m - q_m^* &= q_m^* \\ \frac{y_m}{2} &= q_m^* = c_m^* \end{aligned}$$

(b) Alternative solution method.

Substituting the budget constraint into the mother utility function, the mother will solve:

$$\begin{aligned} &\max_{q_m} (y_m - q_m)q_m \\ &= \max_{q_m} y_m q_m - q_m^2 \end{aligned}$$

The above is just a simple quadratic objective function in  $q_m$ . At the optimum, we set the derivative of the objective function to zero:

$$\begin{aligned} y_m - 2q_m^* &= 0 \\ \frac{y_m}{2} &= q_m^* = c_m^* \end{aligned}$$

which give the same solution as method (a).

3. Let the Nash Equilibrium quantities be  $\hat{c}_m$  and  $\hat{c}_f$  for the mother's contribution to the child and the father's contribution to the child respectively.

Then we know that given  $\hat{c}_m$ , the father will solve:

$$\begin{aligned} \max_{c_f} (y_f - c_f)(\hat{c}_m + c_f) \\ = \max_{c_f} y_f \hat{c}_m + y_f c_f - \hat{c}_m c_f - c_f^2 \end{aligned}$$

At the optimum, his choice of  $c_f$ ,  $\hat{c}_f$ , will set the derivative of the above objective function to zero:

$$y_f - \hat{c}_m - 2\hat{c}_f = 0 \quad (3)$$

Similarly, given  $\hat{c}_f$ , the mother will solve

$$\max_{c_m} (y_m - c_m)(c_m + \hat{c}_f)$$

At the optimum, her choice of  $c_m$ ,  $\hat{c}_m$ , will set the derivative of the above objective function to zero:

$$y_m - \hat{c}_f - 2\hat{c}_m = 0 \quad (4)$$

Solving (3) and (4) for  $\hat{c}_m$  and  $\hat{c}_f$  gives:

$$\begin{aligned} \hat{c}_m &= \frac{2y_m - y_f}{3} \\ \hat{c}_f &= \frac{2y_f - y_m}{3} \\ \hat{c}_m + \hat{c}_f &= \frac{y_f + y_m}{3} \end{aligned}$$

So the child will get in equilibrium  $\hat{c}_m + \hat{c}_f = \frac{y_f + y_m}{3}$ .

Note that for  $y_m = y_f$ ,

$$\hat{c} = \hat{c}_m + \hat{c}_f = \frac{2y_m}{3}$$

which is more than what the child would get from a single mother.

What happens if  $y_f = 0$ . Note that in this case:

$$\hat{c}_m = \frac{2y_m}{3}$$

$$\hat{c}_f = \frac{-y_m}{3}$$

which means that the mother is contributing more to the child than as a single mother. The father makes a negative contribution to the child and his own consumption is  $\hat{q}_m = \frac{y_m}{3} = -\hat{c}_f$ . That is, he is consuming part of the mother's contribution to the child and certainly more than his own income.

This is a real life problem in some cases when one divorced parent claims that the other divorced parent is consuming some of the child support payments that the first parent makes.

4. The parents will jointly solve

$$\max_{c, q_m, q_f} cq_m + cq_f$$

subject to

$$c + q_m + q_f = 2y_m$$

Use the budget constraint to solve for  $c$  and substitute into the parents objective function to get:

$$\max_{q_m, q_f} Z = (2y_m - (q_m + q_f))(q_m + q_f)$$

$$\max_{q_m, q_f} Z = 2y_m(q_m + q_f) - (q_m + q_f)^2$$

To solve for the optimal choice of  $q_m$  and  $q_f$ ,  $\tilde{q}_m$  and  $\tilde{q}_f$ , take the partial derivatives of  $Z$  with respect to  $q_m$  and  $q_f$  and set them to zero:

$$\frac{\partial Z}{\partial q_m} = 2y_m - 2(\tilde{q}_m + \tilde{q}_f) = 0$$

$$\frac{\partial Z}{\partial q_f} = 2y_m - 2(\tilde{q}_m + \tilde{q}_f) = 0$$

which implies:

$$\tilde{q}_m + \tilde{q}_f = y_m$$

$$\tilde{q}_m = \tilde{q}_f = \frac{y_m}{2}$$

$$\tilde{c} = y_m$$

if we assume symmetric contributions.

Note that the child consumes twice the amount of each parent.

Ranking the three equilibrium outcomes.

$$\tilde{c} > \hat{c} > c^*$$

Cooperative parents are best for the child. Non-cooperative parents are not as good as cooperative parents but better than a single parent.

5. When both parents cooperative, each parent in each period will receive a utility of

$$\begin{aligned}\tilde{w} &= (y_m - \tilde{c}_m)\tilde{c} \\ &= \frac{y_m^2}{2}\end{aligned}$$

Let the discount factor be  $\delta$ . The lifetime utility from cooperation is

$$\begin{aligned}\tilde{W} &= \frac{\tilde{w}}{1 - \delta} \\ &= \frac{y_m^2}{2(1 - \delta)}\end{aligned}$$

If the parents do not cooperate, the one period payoff for each parent from playing non-cooperatively is:

$$\begin{aligned}\hat{w} &= (y_m - \hat{c}_m)\hat{c} \\ &= \frac{4y_m^2}{9}\end{aligned}$$

The lifetime utility from playing non-cooperatively is

$$\hat{W} = \frac{4y_m^2}{9(1 - \delta)}$$

The 1 period gain from cheating (not cooperating) when the other parent is cooperating is:

$$\begin{aligned}\bar{w} &= \max_{c_m}(y_m - c_m)(\tilde{c}_f + c_m) \\ &= \max_{c_m}(y_m - c_m)\left(\frac{y_m}{2} + c_m\right) \\ &= \frac{9y_m^2}{16}\end{aligned}$$

So the lifetime gain from cheating is

$$\begin{aligned}\bar{W} &= \bar{w} + \delta\hat{W} \\ &= \frac{9y_m^2}{16} + \delta\frac{4y_m^2}{9(1 - \delta)}\end{aligned}$$

So parent will cooperate if  $\widetilde{W} > \overline{W}$  or

$$\begin{aligned} \frac{y_m^2}{2(1-\delta)} &> \frac{9y_m^2}{16} + \delta \frac{4y_m^2}{9(1-\delta)} \\ \frac{1}{2} &> \frac{9(1-\delta)}{16} + \frac{4\delta}{9} \end{aligned}$$

When  $\delta = \frac{3}{4}$ ,  $\widetilde{W} > \overline{W}$ .

6. With full custody, a mother's per period utility remains

$$cq_m$$

With a share of custody, we may model the mother's per period utility as

$$\alpha cq_m$$

where  $0 < \alpha < 1$ . So in this case, there are two effects from non-cooperation. First we have the usual effect of an inefficient reduction in investments. Second, for any level of investments, utility is lower due to the lack of full custody. So divorce makes non-cooperation even more costly. In this case, the cost of defecting in a cooperative marriage is even higher than where there was no divorce. This makes cooperation easier to sustain. So if divorce policies lower  $\alpha$ , it may make more cooperation in marriage. However if a divorce did occur, the divorced parties will be worse off.