

Economics 332F
Midterm test

Name: _____

Student number: _____

Answer all questions. Explain your answers.

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1. Jack and Mary each have to spend effort to help their daughter Kim in her home work. If Jack puts out effort e_j , he incurs a cost of e_j^2 . If Mary puts out effort e_m , she incurs a cost of $2e_m^2$. If total cost of e is ce^2 , the marginal cost of effort is $2ce$.

Jack's benefit from his and her effort is:

$$B(e_m, e_j) = e_m + e_j$$

His marginal benefit of her effort is

$$B_m = 1$$

His marginal benefit of his own effort is

$$B_j = 1$$

Given e_m and e_j , Jack's utility is:

$$U(e_m, e_j) = e_m + e_j - e_j^2$$

Mary's benefit function is the same as Jack. So, given his and her efforts, Mary's utility is:

$$V(e_m, e_j) = e_m + e_j - 2e_m^2$$

Both parents are interested in maximizing their own utility. (1) Find the Nash Equilibrium allocation of effort of both parents. (2) Find the social planner's solution if the social planner wants to maximize the sum of Jack's and Mary's utility.

(1) Let e_m^* and e_j^* be the NE allocations. Given e_j^* , Mary will solve:

$$\max_{e_m} \{V(e_m, e_j^*) = e_m + e_j^* - 2e_m^2\}$$

The solution is

$$\begin{aligned} 1 - 4e_m^* &= 0 \\ e_m^* &= \frac{1}{4} \end{aligned}$$

Given e_m^* , Jack will solve

$$\max_{e_j} \{U(e_m^*, e_j) = e_m^* + e_j - e_j^2\}$$

His solution is

$$e_j^* = \frac{1}{2}$$

(2) Planner will solve:

$$\max_{e_m, e_j} \{2(e_m + e_j) - e_j^2 - 2e_m^2\}$$

The planner's solution is:

$$\hat{e}_m = \frac{1}{2}$$

$$\hat{e}_j = 1$$

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2. In order to test whether parents bargain over resources within the family, researchers have run the following regression on a population of families at a point in time:

$$c_i = \alpha_0 + \alpha_1 y_i + \alpha_2 r_i + u_i$$

where c_i is the ratio of expenditure on children clothing to adult clothing in family i , y_i is total family income of family i , r_i is the fraction of family income earned by the mother, u_i is an error term in the regression. Researchers interpret an estimate of α_2 different from zero as evidence in favor of bargaining between parents within the household. (a) Explain why the researchers may make such a claim. (b) Explain Lundberg and Pollak's (LP) criticism of this methodology. (c) What did LP do to avoid the criticism? (d) What do their results on children clothing with respect to one child versus more than one children families say about the bargaining hypothesis?

(a) If treat income as endowment, if α_2 is different from zero, then it says that the children's consumption relative to their parents depend on the distribution of parental income. If the utility functions of both parents do not change when the the distribution of parental income changes, then a cooperative solution to the household allocation problem should not depend on the distribution of parental income. If the parents act non-cooperatively, in general the distribution of children's consumption will depend on the distribution of parent income.

(b) The distribution of parental income may depend on the allocation of clothing. E.g. mothers who work (earn more of the household income) may also need to spend more on clothing. In this case, there will be a non-zero relationship between r_i and c_i which may have nothing to say about whether parents are bargaining or not.

(c) Instead of running a cross-section regression, i.e. across families, they look at a change in the distribution of parental income due to a change in government subsidies to families with children. Since this change occurred for all families, we do not have to worry about this change in the parental distribution of income selecting different families.

(d) Their results on one child families do not support the bargaining hypothesis where their results on more than one children families do support the bargaining hypothesis.

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3. Consider a special case of the CS model where $I = 1$ and $J = 1$. In this case, there is only one type of men and one type of women in the marriage market. Let $M = 9$ and $F = 14$. Let there be 5 married couples. (a) What is the systematic gains to marriage in this society? (b) Suppose that you also observe equilibrium transfers in this society, $\tau = 2$. What can you learn about α and γ , the systematic gains to marriage for men and women respectively?

(a)

$$\Pi = \frac{5}{\sqrt{(9-5)(14-5)}} = \frac{5}{6}$$
$$\pi = \ln \frac{5}{6}$$

(b) Looking at the men's demand for spouses

$$\ln \mu = \ln \mu_0^m + \alpha - \tau$$
$$\ln 5 = \ln 4 + \alpha - 2$$
$$\alpha = 2 + \ln \frac{5}{4}$$

looking at the females' demand for spouses

$$\ln \mu = \ln \mu_0^f + \gamma + \tau$$
$$\ln 5 = \ln 9 + \gamma + 2$$
$$\gamma = -2 + \ln \frac{5}{9}$$

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4. (a) The marriage hazard (rate) for females of age j is defined as the number of females who marry at age j divided by the number of females who are unmarried at the beginning of age j . Explain why the observed marriage hazard for females as documented in Figure 4.2 in the lectures increases and then decreases with age. Use search frictions in the marriage market as part of your answer.

(b) Consider the model of marriage where there is investment in marriage specific capital, arrival of new information and divorce as in Section 3.3 in the lectures. What will happen to the first period marriage rate if the government increases divorce cost?

(a) Increasing marriage hazard with age is due to the end date effect of marital search. As there are less years left to enjoy marriage, reservation match values will fall. When reservation match values fall, the marriage hazard will rise. One explanation of a eventual fall in the marriage hazard is that there may be two kinds of individuals. One kind of individuals want to marry and search as describe in the lectures. The second kind of individuals do not want to marry and do not search. As time progresses, the individuals who want to marry leave the unmarried pool. So the pool ends up with disproportionately more and more of the individuals who dont want to marry. So the marriage rate eventually falls with time.

(b) When parents can make marriage specific investments that are public goods, parents will under invest in these marriage specific investment if they invest non-cooperatively. Their investments will rise as divorce cost increases because as divorce cost increases, they are more likely to remain married in the second period. Since there is under investment, an increase in investment due to an exogenous increase in divorce cost may or may not increase the gains to marriage in the first period. If the gains to marriage in the first period is increased, then individuals may become less selective and the marriage rate increases. If the gains to marriage in the first period, then individuals may become more selective and the marriage rate falls.

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5. Consider a society with M men where M is large. All men have the same utility from being single, $\frac{1}{2}$. There are two types of men with respect to marriage, h and l types. $\frac{1}{2}$ of the men are h types and the other half are l types. There are $\frac{3}{4}M$ number of women in this society. All women have the same utility from being single, $\frac{1}{3}$. There are also two types of women, h and l types, equally divided among the female population.

The total marital output generated by a m type male with a f type female is Z_{mf} . Let $Z_{ll} = 1$, $Z_{hl} = 2$, $Z_{lh} = 3$, $Z_{hh} = 4$.

For a marriage between type m male and type f female, the male will transfer τ_{mf} to the female as her share of the marital output. He will keep $Z_{mf} - \tau_{mf}$ for himself where Z_{mf} is the total marital output produced by that pair.

The marriage market clears when given τ_{mf} for every type of marriage, every individual can find a spouse of his or her choice if he or she wants to.

(a) Since women are scarce in this society, some men will not be married. Which type of men is likely to be these unmarried men? If some type m men are married and others are not in equilibrium, what must the equilibrium transfers in marriage be to make these men indifferent between being single versus being married?

(b) Find the equilibrium number of marriages of each type and transfers which clears the marriage market.

(a) l type men are likely to be unmarried. They get a payoff of $\frac{1}{2}$ if single. If l men marry l type women, then the men must get the same payoff as being single:

$$\begin{aligned} 1 - \tau_{ll} &= \frac{1}{2} \\ \tau_{ll} &= \frac{1}{2} \end{aligned}$$

(b) Assume h type men marry h type women and l type women. So l type women must be indifferent between h type men and l type men:

$$\begin{aligned} \tau_{ll} &= \tau_{hl} \\ \tau_{hl} &= \frac{1}{2} \end{aligned}$$

Also h type men must be indifferent between h type women and l type women:

$$\begin{aligned} 4 - \tau_{hh} &= 2 - \tau_{hl} \\ \tau_{hh} &= 2 + \frac{1}{2} = \frac{5}{2} \end{aligned}$$

h type men get $\frac{3}{2}$ in marriage. So they get a scarcity rent from being high type. l type men get $\frac{1}{2}$ in marriage which is also what they get from being

single. l type men obtain no rent from marriage in this society. Women obtain rents from marriage.

If a l type man marries an h type woman, he will need at least:

$$\begin{aligned} 3 - \tau_{lh} &\geq \frac{1}{2} \\ \tau_{lh} &\leq \frac{5}{2} \end{aligned}$$

If $\tau_{lh} = \frac{5}{2}$, then some l type men may marry h type women. Note that if $\tau_{lh} = \frac{5}{2}$, h type women are happy to marry these l type men.

In equilibrium, l type men are indifferent between marrying h type women or l type women or remaining single. But all h type men want to marry although they are indifferent between h and l type women. Women are also indifferent between the different types of men. So the only quantity that is uniquely determined is that $\frac{1}{2}$ of the l type men are single. We cannot predict the exact proportions of the marital matches among the marrieds.