1 Children as Public Goods

Parents care about the welfare of their children. From the point of view of the parents, the care of children has a public good component. A public good is a commodity which has the characteristic that one person's consumption of the good does not reduce its availability for another person to consume. For example, both parents want their baby to wear a dry diaper. In this case, the dry diaper is the public good for the two parents. If the father changes the wet diaper, he incurs some effort which is costly to him. His utility will go up because his child now has a dry diaper. His net utility will rise or else he would not have changed the diaper. The mother's utility will also rise because their child has a dry diaper and she did not have to incur the effort of changing the diaper. Since it takes only one parent to change a diaper but both parents benefit from the act, how often will parents change their baby's diapers? Will the frequency of changes be efficient?

When there are public goods, spouses may be tempted to contribute less than the efficient contributions. They may want to free ride off the contribution of the other spouse. After considering the efficient provision of public goods, we will consider the potential for free riding. We will also show that the free riding can be mitigated by the repeated interactions of spouses with each other.

1.1 Children as Public Goods

Within marriage, the utility that a parent derive from a child often does not lower the utility that the other parent can get from the child. Thus children are often viewed as public goods by the two parents.

This section shows the benefit of marriage relative to raising children as a single parent. It also shows the efficient allocation of resources within a marriage. Essentially, children obtain more consumption within a marriage than from a single parent.

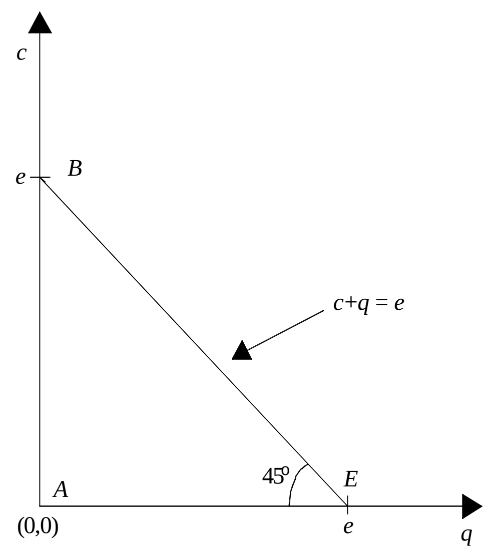

Figure 1.1.: Mother's budget constraint

First, consider the case of a single parent. Consider a mother who has an endowment of e. The endowment is to be divided between her and her child. Let her child's consumption be q. Then her consumption is:

$$c = e - q \tag{1.1.1}$$

(1.1.1) is the budget constraint of the mother. A graphical representation of the budget constraint is in Figure (1.1). c is measured along the vertical axis and q is measured along the horizontal axis. An allocation is a point on the positive quadrant because, in this case, negative quantities are infeasible. The budget constraint, (1.1.1), is the line BE. Points in the triangle ABE are feasible for the mother. Points to the northeast the budget constraint BE are not feasible. They will demand a larger endowment than what the mother has.

Let her utility function be:

$$U(c, q) \tag{1.1.2}$$

where $U(.,.)$ is increasing in both arguments and concave.

Figure (1.2) shows an indifference curve U^1 which consists of a locus of points in which the mother's utility is equal to U^1. If c increases along an indifference curve, then q must fall to hold utility constant. The points on the indifference curve U^* provide a lower level of utility for the mother. Consider point D on indifference curve U^* with point F on indifference curve U^1. Both points have the same level of c but point D has less q than point F. Holding c the same, the mother values more child consumption rather than less. So she will prefer F to D. Her utility from point D is lower than from point F. Thus we have shown $U^* < U^1$.

We can now show graphically how the mother will make her optimal allocation. Her budget constraint BE is also in Figure (1.2). The budget line BE has a slope of -1 because if she reduces her own consumption by x units, she can at most increase her child's consumption by x units. As discussed earlier, all points in ABE are feasible. She will want to choose a point which is feasible and also gives her the highest possible level of utility. In the figure, she will choose point G where her own consumption is q^* and her child's consumption is c^*. She will obtain a utility of U^*. She will of course prefer any point on the indifference curve U^1. However those points are outside ABE and are infeasible.

The mother's optimal allocation may also be determined analytically. Due to her budget constraint, (1.1.1), by choosing q, her consumption is also determined. Substituting $e - q$ for c in the utility function, she will want to choose q to solve:

$$\max_q H(q) = U(e - q, q) \tag{1.1.3}$$

Let q^* be her optimal choice of child's consumption. We can find q^* by taking the derivative of the above objective function, $H(q)$, with respect to q and setting the derivative to zero:

$$H_q(q^*) = -U_c(e - q^*, q^*) + U_q(e - q^*, q^*) = 0 \tag{1.1.4}$$
$$U_c(e - q^*, q^*) = U_q(e - q^*, q^*)$$

U_i is the marginal utility of consuming good i, $i = c, q$. It quantifies the increase in utility that she derives from increasing her consumption of good i by a marginal (small) amount. (1.1.4), which is the solution to the mother's optimization problem, says that she will want to choose q such that she is equating her marginal utility of own consumption with her marginal utility of child's consumption. The economic interpretation is as follows. If she increases q from q^* by Δ marginal units more, her increase in utility from the increase in child's consumption is approxi-

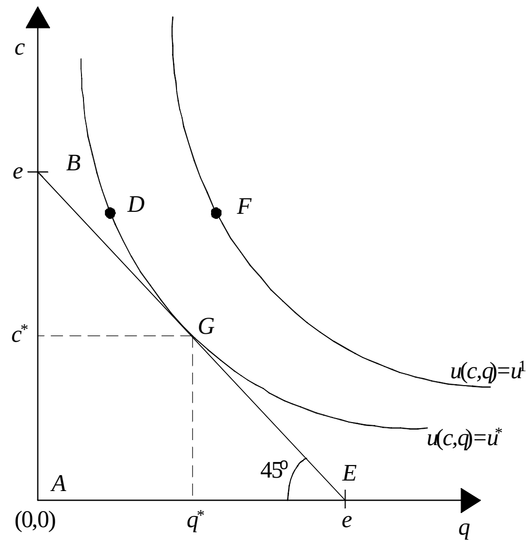

Figure 1.2.: Single parent's optimal allocation.

mately $U_q(e - q^*, q^*)\Delta$. But when she increases q by Δ more marginal units, she is substracting Δ marginal units from her own consumption. Her decrease in utility from the decrease in her own consumption is $U_c(e - q^*, q^*)\Delta$. (1.1.4) says that when $q = q^*$, the gain in utility from increasing her child's consumption by Δ marginal units is exactly equal to the loss in utility from decreasing her own consumption by Δ marginal units. Put another way, she cannot increase her utility by marginally deviating from q^*.[1]

Her optimal amount of own consumption is $c^* = e - q^*$ and the level of utility that she can achieve is $U^* = U(c^*, q^*)$.

Now consider the mother being married to a father. We will assume that neither parent inherently cares about the welfare of the other parent. Thus the only gain from marriage for a parent is that the parent can either increase his or her own consumption or their child's consumption relative to being single.

[1] The concavity of the utility function also implies that she cannot gain from non-marginal deviations.

Let the father's endowment be h. Let his utility function be:

$$V(g, q)$$

where $V(.,.)$ is concave and increasing in g, his consumption, and q, his child's consumption. The public good aspect of children within marriage is captured by q, his child's consumption which is also the mother's child's consumption.

In this environment, a marriage is an allocation of parental and child consumption that are feasible when the parents' endowment are pooled.

In general, there are gains to marriage for both parties. Let g^s and q^s be the optimal allocation of consumption for the father and his child when he is single. Under marriage, consider an allocation where the mother consumes c^*, the father consumes g^s, and the child consumes $q^s + q^*$. When parental endowments are pooled, this allocation is feasible. Both parents will obtain higher utility than if they were to remain single because child consumption is higher than what was feasible for each of them as a single.

To study optimal allocations under marriage, let the father offer the mother a reservation utility of U^r for marrying him. U^r must be greater than or equal to U^* or she will not be willing to marry him. In general, $U^r > U^*$ because her best alternative to marrying him is not to remain single, but to marry someone else. Then the father's objective is to solve:

$$\max_{g,q} V(g, q)$$

subject to:

$$U(e + h - g - q, q) \geq U^r$$

Figure (1.3) shows the solution to the father's optimization problem graphically. g is measured along the vertical axis and q is measured along the horizontal axis. Given g and q, the wife's utility is determined by $U(e + h - g - q, q)$. Since he has to give her a reservation utility of U^r, the line HI shows the reservation utility constraint that the father faces. Points inside HGI are feasible. Points to the northeast of HI are not feasible. HI is not a straight line with a slope of -1. In general, it is bowed (concave) towards the origin and the slope is less steep than -1. The for the slope is as follows. Consider a point $K = \{g^k, q^k\}$ on HI. At this point, the mother obtains a utility of U^r. Let the father increases his child's consumption by Δ marginal units. The mother's utility is increased and therefore he can reduce the mother's consumption

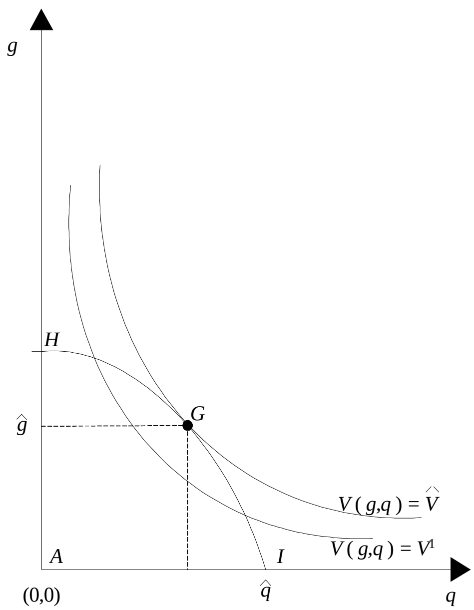

Figure 1.3.
: Father's optimal choice subject to mother's reservation utility.

by $\frac{U_q(c^k,q^k)}{U_c(c^k,q^k)}\Delta$ to keep her utility at U^r. Since he can consume this reduction in mother's consumption, he will not have to reduce his own consumption by Δ marginal units to increase his child's consumption by Δ marginal units. So the slope of HI is less steep than -1. In fact this lesser slope is the gain to marriage due to the public good nature of child's consumption. The HI line is bowed towards the origin because if the child has a lot of consumption already (a point on HI to the right of K), as the father transfers another Δ marginal units to the child, the mother is willing to give up less than $\frac{U_c(c^k,q^k)}{U_q(c^k,q^k)}\Delta$ of her own consumption in order to keep her utility the same.[2] Therefore the father has to give up increasingly more of his own resources to transfer Δ marginal units more to the child if the child has a lot of consumption already.

The father's indifference curves, V^1 and V^*, are drawn in the figure.

[2] The concavity of the mother's utility function means that we are assuming the mother has diminishing marginal utilities of own and child's consumption.

Points on V^1 provide less utility than points on V^*. The father can maximize his utility by choosing point G. He consumes $\widehat{g}$ and his child consumes $\widehat{q}$.

We gain further insight by studying the father's optimization problem analytically. Write the father's constrained optimization problem as a Lagrangian problem:[3]

$$L(g, q, \lambda) = V(g, q) + \lambda(U(e + h - g - q, q) - U^r)$$

where λ is the lagrange mutiplier for the problem. Let $\widehat{x}$ denote the optimal choice of x and for a function $Z(x_1, .., x_n)$, $\widehat{Z}$ denotes the value of $Z(\widehat{x}_1, .., \widehat{x}_n)$. For an interior maximum, take the partial derivatives of $L(g, q, \lambda)$ with respect to g, q and λ and equate them to zero. Assuming an interior solution,[4] the first order conditions are:

$$\widehat{L}_g = \widehat{V}_g - \lambda \widehat{U}_c = 0 \tag{1.1.5}$$

$$\widehat{L}_q = \widehat{V}_q - \lambda \widehat{U}_c + \lambda \widehat{U}_q = 0 \tag{1.1.6}$$

$$\widehat{L}_\lambda = \widehat{U} - U^r = 0 \tag{1.1.7}$$

(1.1.7) says that the mother will get her reservation utility.

(1.1.5) and (1.1.6) may be combined to get:

$$\widehat{V}_q + \frac{\widehat{U}_q}{\widehat{U}_c} \widehat{V}_g = \widehat{V}_g \tag{1.1.8}$$

where V_i is the father's marginal utility of consuming good i, $i = g, q$. The interpretation of (1.1.8) is as follows. Consider increasing $\widehat{q}$ by Δ marginal units and decreasing his own consumption by Δ marginal units. The increase in utility that the father gets from the increase in child consumption is $\widehat{V}_q \Delta$. The mother also gets an increase in utility of $\widehat{U}_q \Delta$. Since he is not interested in increasing her utility, he may lower her consumption by $\frac{\widehat{U}_q}{\widehat{U}_c} \Delta$ and leave her as well off as before. His increase in utility from consuming the mother's decreased consumption is $\left(\frac{\widehat{U}_q}{\widehat{U}_c} \Delta \right) \widehat{V}_g$. $\left(\frac{\widehat{U}_q}{\widehat{U}_c} \Delta \right) \widehat{V}_g$ is the gain from marriage. Since his wife is better off when he increases child's consumption, she is willing to compensate him for doing so. This compensation is missing if he is a single

[3] Constrained optimization problems of this kind are discussed in intermediate microeconomics textbooks which use calculus.

[4] $\widehat{q} \neq 0$ and $\widehat{g} \neq 0$.

parent. So under marriage, his total change in utility from increasing q by Δ marginal units is the left hand side of (1.1.8) mutiplied by Δ. On the other hand, when he increases q by Δ marginal units, he decreases his own consumption by Δ. His fall in utility from the drop in own consumption is $\widehat{V}_g\Delta$. At the optimum, represented by (1.1.8), he is indifferent between an additional transfer of Δ marginal units from his own consumption to his child.

Consider the special case of transferable utility where parental utility functions are linear in own consumption:

$$U(c, q) = \alpha c + \pi(q)$$
$$V(g, q) = \beta g + \rho(q)$$

where α and β are positive, $\pi(.)$ and $\rho(.)$ are increasing concave functions.

When each parent is a single parent, from (1.1.4), $\pi_q(q^*) = \alpha$ and similarly, $\rho_q(q^s) = \beta$ where q^s is the optimal choice of the father. In both cases, optimal child consumption are independent of parental endowment. The reason is that with constant marginal utility of own consumption, a parent will want to provide child's consumption up to the point where the marginal utility of child's consumption is equal to the marginal utility of own consumption. Any additional endowment is used for own consumption because the marginal utility of own consumption does not fall with additional own consumption whereas the marginal utility of child's consumption will fall with additional consumption.

When both parents care for one child, from (1.1.8),

$$\frac{\pi_q(\widehat{q})}{\alpha} + \frac{\rho_q(\widehat{q})}{\beta} = 1 \tag{1.1.9}$$

(1.1.9) implies that both $\frac{\pi_q(\widehat{q})}{\alpha}$ and $\frac{\rho_q(\widehat{q})}{\beta}$ are less than one. Since $\frac{\pi_q(q^*)}{\alpha} = \frac{\rho_q(q^s)}{\beta} = 1$, and π_q and ρ_q are both decreasing in q, (1.1.9) implies that $\widehat{q} > \max(q^*, q^s)$. The child's consumption under marriage is larger than with either single parent.

1.2 Free Riding within a Household

In our analysis of the public goods problem discussed above, we assumed that the parents will allocate resources efficiently within a marriage. Since parents repeately interact in a marriage, this assumption of efficient allocation may be reasonable for many married households. However there is evidence that some parents do not allocate resources efficiently within a household. In fact the availability of a public good provides an opportunity to allocate resources inefficiently within marriage.

Although it is better for a child to be supported by two parents rather than a single parent, this section shows how resources are inefficiently allocated when parents act non-cooperatively. In general, non-cooperative parents allocate too little consumption to their children relative to cooperating parents.

Consider a marriage with the same parents and endowments as above. Instead of pooling their endowments, let each parent choose his or her own allocation between own consumption and child's consumption. Thus the mother will choose c and leave $q^m = e - c$ for the child. The father will choose g and leave $q^f = h - g$ for the child. Total child's consumption will be $q = q^m + q^f = e - c + h - g$. If both parents act purely in their own self interest, what are the equilibrium allocations? We use the term equilibrium in a predictive sense. That is, the equilibrium allocations are what is most likely to be observed in this environment. We will use the concept of Nash Equilibrium from game theory to find the equilibrium allocation.[5]

Let q^{mn} be the equilibrium allocation of the mother to the child and q^{fn} be equilibrium allocation of the father to the same child. If the pair of equilibrium allocations $\{q^{mn}, q^{fn}\}$ constitutes a Nash Equilibrium, then $\{q^{mn}, q^{fn}\}$ must simultaneously satisfy

(1) $U(e - q^{mn}, q^{mn} + q^{fn}) \geq U(e - q^m, q^m + q^{fn})$ for all feasible q^m.

(2) $V(h - q^{fn}, q^{mn} + q^{fn}) \geq V(h - q^f, q^{mn} + q^f)$ for all feasible q^f.

Condition (1) says that if q^{fn} is the equilibrium allocation of the father to the child, then the mother's best response is to choose q^{mn} as her allocation. Any other choice by her will not increase her utility.

Condition (2) says that if q^{mn} is the equilibrium allocation of the mother to the child, then the father's best response is to choose q^{fn} as his allocation. Any other choice by him will not increase his utility.

When (1) and (2) are satisfied simultaneously, $\{q^{mn}, q^{fn}\}$ is a pair of equilibrium allocations of this parental allocation game.

A justification for focussing on Nash Equilibrium for the parental allocation game is as follows. If the mother chooses q^{mn}, then the father can do no better than choosing q^{fn} and vice versa. So if the parents choose $\{q^{mn}, q^{fn}\}$, there is no incentive for either of them to deviate from their equilibrium allocation. Thus we should not be surprized to observe parents choosing $\{q^{mn}, q^{fn}\}$. On the other hand, consider an

[5] Nash Equilibrium is discussed in most modern intermediate economics textbooks.

alternative pair of allocations $\{q^{m'}, q^{f'}\}$ where condition (1) is not satisfied. In that case, we can find feasible allocations for the mother, q^m, where:

$$U(e - q^{m'}, q^{m'} + q^{f'}) < U(e - q^m, q^m + q^{f'})$$

The mother can obtain higher utility from doing something else other than choosing $q^{m'}$. Thus we should be surprized to observe the parents choosing $\{q^{m'}, q^{f'}\}$. So from a predictive point of view, we expect to see parents choosing $\{q^{mn}, q^{fn}\}$ rather than other allocations. It is from this predictive perspective that we want to find the Nash Equilibrium of the parental allocation game.

In order to analytically find the equilibrium allocation, let q^{mn} be the mother's allocation. Then the father will solve

$$\max_{q^f} V(h - q^f, q^{mn} + q^f) \tag{1.2.1}$$

subject to:

$$q^f \geq 0 \tag{1.2.2}$$

(1.2.2) says that he cannot make a negative contribution to his child's consumption to increase his own consumption. According to condition (2), q^{fn} must be the solution to the father's optimization problem above if it is part of the equilibrium allocation.

A graphical representation of the father's problem is in Figure (1.4). The father's contribution to his child, q^f, is measured along the vertical axis. The mother's contribution to her child, q^m, is measured along the horizontal axis. Since we will not allow negative contributions, we only have to focus on allocations on the positive quadrant. A point on the quadrant will determine both the utility of the father and the utility of the mother. The points A, B and C are on a locus of points which give the father the same utility level $V^{'}$. The curvature of the indifference locus is explained as follows. At point C, the mother is contributing more than at point B. Although the mother's contribution has increased, his own contribution is sufficiently reduced such that total child's consumption is reduced and he feels no increase in utility. At point A, he is contributing more than at point B. The mother is also contributing more which keeps his utility the same. In general, he will get a higher level of utility if his contribution stays the same, but the mother's contribution increases. Therefore the locus of points which gives him utility V^n gives him higher utility than $V^{'}$.

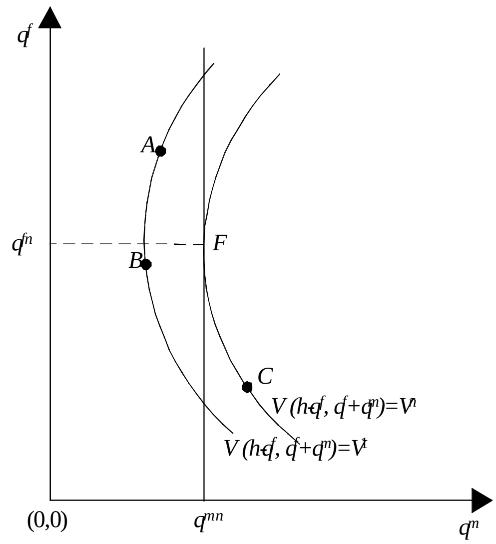

Figure 1.4.: Father's reaction to mother's choice of q^{mn}

Now let the mother make a contribution q^{mn} to the child. Given the mother's choice, his contribution will determine a point on the line DE. The father will want to pick a contribution which maximizes his utility. As shown in the figure, he will pick point F, by contributing q^{fn}, which will give him a utility of V^n. He will prefer points to the east of the locus V^n but those points are infeasible.

Analytically, if his optimal allocation to the child (best response), q^{fn} is strictly larger than zero, it will satisfy the first order condition of the maximization problem (1.2.1):

$$V_g(h - q^{fn}, q^{fn} + q^{mn}) = V_q(h - q^{fn}, q^{fn} + q^{mn}) \tag{1.2.3}$$

Given q^{mn}, the father's optimal choice will equate his marginal utility of own consumption with his marginal utility of child's consumption. In this case, he cannot increase his utility by switching some of his own consumption to his child and vice versa.

If $q^{fn} = 0$, then:

$$V_g(h, q^{mn}) > V_q(h, q^{mn}) \tag{1.2.4}$$

If his marginal utility of own consumption is higher than his marginal utility of child's consumption when he makes no contribution towards his child's consumption, he will consume all his endowment.

Likewise, when the father contributes q^{fn} to the child, the mother's optimal choice (best response), q^{mn}, if strictly larger than zero will satisfy:

$$U_c(e - q^{mn}, q^{mn} + q^{fn}) = U_q(e - q^{mn}, q^{mn} + q^{fn}) \qquad (1.2.5)$$

Given q^{fn}, the mother's optimal choice will equate her marginal utility of own consumption with her marginal utility of child's consumption. In this case, she cannot increase her utility by switching some of her own consumption to her child and vice versa.

If $q^{mn} = 0$,

$$U_c(e, q^{fn}) > U_q(e, q^{fn}) \qquad (1.2.6)$$

If q^{mn} and q^{fn} are strictly larger than zero, then the equilibrium allocation is obtained by solving (1.2.3) and (1.2.5) for q^{mn} and q^{fn}.

If $q^{fn} = 0$ and $q^{mn} > 0$, then the equilibrium allocation satisfies (1.2.4) and (1.2.5) with $q^{fn} = 0$.

If $q^{mn} = 0$ and $q^{fn} > 0$, then the equilibrium allocation satisfies (1.2.6) and (1.2.3) with $q^{mn} = 0$.

At this level of generality, there may be more than one Nash Equilibrium for this game.[6]

A graphical representation of a Nash Equilibrium (point F) where $q^{mn} > 0$ and $q^{fn} > 0$ is in Figure (1.5). In the figure, we have superimposed the mother's best response of q^{mn} given the father's contribution of q^{fn}. The mother cannot obtain a higher level of utility by choosing another contribution. Similarly, given the mother's contribution q^{mn}, the father best response is to choose q^{fn}.

In general, the Nash Equilibrium is inefficient. Consider for example point G where the father contributes $q^{f'}$ and the mother contributes $q^{m'}$. Both parents will achieve higher levels of utility that from point F. However point G is not an equilibrium allocation. Consider for the moment that the mother is willing to contribute $q^{m'}$. In this case, the father considers all points on LN as feasible and he will pick point K. Thus $q^{f'}$ is not his best response to $q^{m'}$ and therefore point G cannot be supported as a Nash Equilibrium.

The inefficiency of a Nash equilibrium of this game may also be demonstrated analytically. Without loss of generality, assume that $q^{fn} > 0$.

[6] For many types of games, multiple Nash Equilibria are not uncommon.

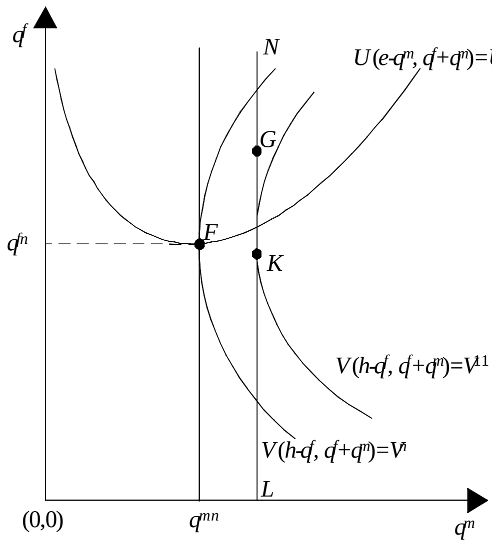

Figure 1.5.: Nash Equilibrium

Let the father consider allocating an additional Δ marginal units of consumption to the child. By (1.2.3), his welfare will not change. His loss in utility from the fall in own consumption is matched by the gain in utility from the rise in his child's consumption. But note that the mother's utility will rise by $U_q(e - q^{mn}, q^{mn} + q^{fn})\Delta$ if he makes the transfer. At no cost to himself, he can make another own consumption allocation marginally different from q^{fn} that will make the mother better off. Similarly, the mother can choose another allocation that is marginally different from q^{mn} that will make the father better off without making herself worse off. Thus the equilibrium allocation is inefficient. The inefficiency derives from the fact that each parent does not value the impact of his or her contribution to their child's consumption on the welfare of the other parent.

Although inefficient, equilibrium welfare under marriage is still better than or equal to being a single parent. The reason is that, even without assuming that both parents simultaneously make positive contributions to the child, the worst outcome under marriage is when the other parent contributes nothing to child's consumption. In this case, the parent who contributes to child's consumption is as well off as a single parent.

Otherwise, the parent is better off under marriage.

We will now apply the above considerations to the case of transferable parental utility functions:

$$U(c, q) = \alpha c + \pi(q)$$
$$V(g, q) = \beta g + \rho(q)$$

In this case, (1.2.3) and (1.2.5) cannot both simultaneously hold in equilibrium. So one parent will not make any contribution towards the child's consumption in equilibrium. Let q^{mn} be such that $\beta > \rho_q(q^{mn})$ and $\alpha = \pi_q(q^{mn})$. The father's marginal utility of own consumption is larger than his marginal utility of child's consumption evaluated at q^{mn}. Then $q^{fn} = 0$ and $\{0, q^{mn}\}$ is the Nash equilibrium of the contribution game.

1.3 Efficient Allocations Again

Are there mechanisms that may lead parents to act efficiently within the household? One mechanism that promotes efficiency is that parents interact repeatedly. Thus one may expect that if both parents decide to invest in their children efficiently in one period, they do so in the expectation that this cooperation will be rewarded by further cooperation in the future. If a parent reduces his or her current investment to gain some additional current payoff, the other parent may punish the lack of cooperation by also reducing his or her investment in the future. Then both parties are worse off in the future. This expectation of future punishment may deter either party from deviating from cooperative behavior. The role of discounting will be key. If the discount rate is small, i.e. individuals value future payoffs relatively highly, cooperative behavior can be supported by repeated interactions.

Our discussion of supporting cooperation by repeated interactions as a Nash equilibrium will be informal, at the level of intermediate microeconomics.

Consider a repeated environment in which in every period, parents have to make non-cooperative allocations to their child's consumption. In the transferable utility example above, the father contributes nothing towards his child's consumption in the per period Nash equilibrium, then an efficient outcome cannot be reached because the father cannot credibly "punish" the mother for acting inefficiently. He cannot "punish" be contributing less since he is already contributing zero. So the inefficient Nash equilibrium for the transferable utility example studied above cannot be improved on using repeated interactions.

Now consider another transferable utility example where $e = h$, the per period utility of the mother is $U = c + \ln(2e - c - g)$ and the per period utility of the father is $V = g + \ln(2e - c - g)$. In this case, by solving (1.2.3) and (1.2.5), there are multiple per period Nash equilibria all of which satisfy:

$$q^{fn} + q^{mn} = 1$$

Consider the static Nash equilibrium with symmetric contributions, $q^{fn} = q^{mn} = \frac{1}{2}$. If each parent treats each period as separate and chooses the static Nash Equilibrium contribution in each period, the per period equilibrium level of utilities are:

$$U^n = e - \frac{1}{2} \tag{1.3.1}$$

$$V^n = e - \frac{1}{2} \tag{1.3.2}$$

The efficient level of child consumption, from (1.1.9), is $\widehat{q} = 2$. Assuming symmetric contributions, per period utility under the efficient contribution level is:

$$\widehat{V} = \widehat{U} = e - 1 + \ln 2 > e - \frac{1}{2} \tag{1.3.3}$$

So in each period, both parents are better off if they make the efficient level of contributions rather than the static Nash Equilibrium contribution.

Let both parents have infinite horizons and let the discount factor be δ, $0 < \delta < 1$. In this case, the present value of utility for each parent from the efficient action is:

$$\sum_{i=0}^{\infty} \delta^i \widehat{U} = \frac{\widehat{U}}{1-\delta} = \frac{e - 1 + \ln 2}{1 - \delta}$$

If the father deviates from the efficient action in a period, let the mother punish the father by choosing q^{mn} thereafter and vice versa. If the mother chooses q^{mn} every period, the best response for the father is also to choose q^{fn} and vice versa. In otherwords, if one parent deviates from the efficient action in a period, both parents will revert to the per period Nash Equilibrium behavior thereafter. If the mother chooses the efficient level and the father chooses to optimally deviate for a period, his return in that period is e which is larger than $e - 1 + \ln 2$, his return from also acting efficiently. But both parties will revert to static Nash Equilibrium behavior thereafter. So the present value of utility for the husband from deviating in a single period is:

$$\begin{aligned} V^d &= e + \delta \sum_{i=0}^{\infty} \delta^i (e - \frac{1}{2}) \\ &= e + \frac{\delta(e - \frac{1}{2})}{1 - \delta} \end{aligned} \tag{1.3.4}$$

e is the current period return from deviating. $\sum_{i=0}^{\infty} \delta^i (e - \frac{1}{2})$ is the present value of payoffs starting from the next period, where both parties make the static Nash Equilibrium contributions in every period.

The father will not choose to cheat if:

$$\frac{e - 1 + \ln 2}{1 - \delta} > e + \frac{\delta(e - \frac{1}{2})}{1 - \delta}$$

That is, if $\delta > 2(1 - \ln 2)$, neither spouse will find it profitable to deviate from the efficient investment level in each period. In other words, if the discount factor is large enough, the parents will find it in their own self interest to cooperate and not free ride. It should be clear that the cooperative equilibrium is supported by the expectation of future cooperation. Economists have also shown that one can also generate cooperative behavior in finite horizon models as long as the horizons are long enough. In the family context, if parents anticipate an end to their relationship as in a divorce, we expect efficient, cooperative, behavior to break down near the end of the relationship.

In the above example, we studied a symmetric equilibrium where both parents receive the same equilibrium payoffs from cooperating or non-cooperating. It is likely that parents may receive different equilibrium payoffs from non-cooperating. For example, if non-cooperation leads to divorce, the payoffs from divorce may be different for the husband and the wife. In this case, we will expect that the equilibrium payoffs from cooperation, if it can be sustained as a Nash Equilibrium, will be different for the husband and the wife. Put another way, the above example is misleading in the sense that we focussed on an equilibrium with symmetric payoffs. Efficiency does not imply symmetric payoffs.

1.4 Empirical evidence

What do marriages do for children? There are numerous studies which compare the socioeconomic outcomes of children who grow up in families with both parents to children who grow up in families with one parent. The outcomes include measures of schooling achievement, teenage pregnancy, criminal behavior, drug use, etc.. Waite (1995) summarizes many of these studies. These studies overwhelmingly conclude that children who grow up with two parents fare better.

While consistent with the hypothesis that children with two parents do better, there are other differences between the environments of the two groups of children other than the number of parents present. These other differences also affect the socioeconomic outcomes of children. Put another way, individuals who are ill equipped to be parents may also be less able to marry or remain married when they have children. In that case, children who grow up with married parents are more likely to have parents who are also better equipped to bring up children well. Then if we compare children growing up with one parent or two parents, we are comparing both the difference between household structures (married or otherwise) and parenting abilities. We cannot disentangle the effects of parenting abilities from the effects of household structures.

The ideal empirical experiment would be to compare children growing up in different household structures, holding parenting abilities constant. Corak (2001) uses the death of a parent to approximate this experiment. Consider two groups of otherwise statistically identical children: (i) Children growing up with two parents, and (ii) children growing up families where one parent has died. Let the death of a parent be unrelated to parental ability. Then the socioeconomic difference between children in group (i) and group (ii) captures the change in socioeconomic outcomes due to a change in household structure from two parents to one parent, holding parental ability constant. Corak showed that the death of a parent when a child is young lowered the average annual earnings of that adult child by about 5%. These adult children were also 30% more likely to receive income assistance from the government. Thus Corak's evidence imply, all other things equal, that there is a significant benefit to children growing up in two parents households.

In this chapter, parents may disagree about how much resources to allocate to their children. What is the evidence in support of such disagreement? An alternative hypothesis is that parents largely agree on how much resources to spend on their children. Many studies show that resources allocated to children change when the distribution of labor earnings between husbands and wives change, holding total labor earnings of the family constant. In general, children gain resources when the mother earns relatively more (Strauss and Thomas 1995, Thomas 1994). One criticism of these studies is that when we compared resources of children across different families with different distribution of labor earnings, the different distribution of labor earnings may also be correlated with different parental preferences. Lundberg, Pollak and Wales (1997) deal with this criticism by comparing outcomes to British children when the British government changed a government monetary transfer from the father to the mother, holding total monetary transfer to the family fixed. They showed that children obtained more resources after mothers obtain the transfers rather than fathers.

What about efficient versus non-efficient allocations within the family? The empirical evidence on this question is mixed. There is a growing empirical literature, based on the seminar contributions of Chiappori and his co-authors (E.g Chiappori 1992; Chiappori, et. al. 2002), in estimating the implications of efficient intrahousehold allocations. These authors assume that there are sufficient mechanisms to ensure efficient allocations for most families and they study the empirical implications of that assumption. These studies generally show that the implications of efficient allocations for all households their study are not rejected by the data. A limitation of many of these studies so far is that they focus on families without children. There are other studies which explicitly or implicitly reject the assumption of efficient intrahousehold allocations for all households (E.g. Udry 1996). Some families are likely to act efficiently and others will not. So the existing empirical studies have not sorted out the circumstances which lead to some families to act efficiently and others to act otherwise.

1.5 Bibilographic Notes

The static model of public goods is standard and discussed in intermediate economics texts. The infinite horizon game of cooperation is adapted from a repeated game model of oligopoly behavior, also discussed in modern intermediate economics texts. More formal discussions are found in graduate economic texts or undergraduate game theory texts.

1.6 Problems

1. Using nationally representative datasets, many research papers compare the socioeconomic outcomes of children who grow up in one parent households with children who grow up in two parents house-

holds. After controlling for other observable differences between the two groups, the researchers generally show that children who grow up in one parent households have worse socioeconomic outcomes. Explain why these comparisons do or do not measure the "true" benefit of growing up in one versus two parents households. What do you mean by "true"?

2. Consider a mother with income y_m. She has to divide the income between her own consumption, q_m, and the consumption of her child, c. Let her utility function be $U = cq_m$. Find the optimal level of child's consumption.
3. Add a father with income y_f to the above problem. His own consumption is q_f. The rest is given to the child. The child's consumption is now $c = y_f - q_f + y_m - q_f$. The father's utility function is $u = cq_f$. If the parents act non-cooperatively, find the equilibrium level of c. Is it possible for one parent to consume more than his or her income?
4. Assume $y_m = y_f$. If the parents act cooperatively to maximize $cq_m + cq_f$, the sum of parental utilities, find the equilibrium level of c. Rank the levels of consumption of the child under the three different regimes. Explain the order of the rankings.
5. If both parents contribute equally under cooperation and the discount factor of both parents is 3/4, can the cooperative level of child consumption be sustained in the infinitely repeated game? Assume that the parents defect to the static Nash Equilibrium contributions if cooperation fails.
6. The above repeated game implicitly assumes that the parents remain married even if cooperation breaks down. Consider the alternative that the parents divorce when cooperation breaks down. The utility of a divorced parent gets depends on how much child custody he or she gets. How would you modify the parents' utility functions to allow for different custody arrangements? Will child custody laws in divorce affect the likelihood of cooperation within marriage?