4 The CS Model

Becker proposed the transferable utility model of marriage in 1973. That model is the current benchmark theoretical model of the marriage market. As discussed in Chapter 2, its implications have been tested and supported. However the parameters of that model has seldom been estimated.

Two problems have to be solved before the model can be estimated. First, equilibrium transfers in modern marriages are seldom observed. Second, there are many different types of men and women in the marriage market. Individuals may differ by age, religion, education, wealth, ethnicity, personality and so on. Different individuals of the same type may marry different types of spouse. Different types of individuals may not agree on the ranking of individuals of the opposite gender as spouses.

To understand what can be learned from an empirical model, consider a society with I types of men and J types of women in the marriage market. A type of man (woman) is defined by his (her) age, religion, ethnicity and so on. Each individual chooses who to marry or to remaing single. For each type of man (woman), there are potentially J (I) preference parameters to characterize his (her) utility from each type of spouse and remaining single. There are potentially a minimum of $2 * I * J$ preference parameters to be estimated.

What is observable to a researcher? In principle, the researcher observes the quantity of each type of man in the marriage market (I), the quantity of each type of women (J), and the quantity of type i men married to type j women $(I * J)$. So the total number of observables are $I + J + I * J$.

For $I, J > 2$, the number of observables is less than the number of parameters to be estimated. There is no way to identify $2 * I * J$ independent preference parameters. So every empirical researcher has to make sufficient identifying assumptions to reduce the number of unknown parameters to be estimated.

The objective of this chapter is to describe the Choo Siow (hereafter CS) model which is an empirical transferable utility model of the marriage market.

Demographers have done the most work on estimating empirical marriage matching models. In general, demographers do not explicitly model how marriage markets operate.[1] Instead they directly propose and estimate marriage matching functions which are reduced form models of the marriage market. As will be shown below, the CS model generates a specific marriage matching function which alleviates well known deficiencies in existing marriage matching functions.

4.1 Marriage matching functions

A marriage matching function summarizes who marries whom at a point in time. It is defined as follows. Let M be the vector of available men by types $i = 1, .., I$ at that time. m_i, the i'th element of M, is the number of available type i men at that point in time. Let F be the vector of available women by types, $j = 1, ..., J$. f_j is the j'th element of F. Usually, researchers associate types of individuals with their ages. Let Π be a vector of parameters. A marriage matching function is an $I \times J$ matrix $\mu(M, F; \Pi)$ such that an element μ_{ij} is the number of type i men married to type j women at that time. Denote the number of unmarried men of type i as μ_{i0} and the number of unmarried women of type j as μ_{0j}. $\mu(M, F; \Pi)$ must satisfy:

$$\mu_{0j} + \sum_{i=1}^{I} \mu_{ij} = f_j \ \forall\, j \tag{4.1.1}$$

$$\mu_{i0} + \sum_{j=1}^{J} \mu_{ij} = m_i \ \forall\, i \tag{4.1.2}$$

$$\mu_{0j}, \mu_{i0}, \mu_{ij} \geq 0 \ \forall\, i, j \tag{4.1.3}$$

(4.1.1), (4.1.2) and (4.1.3) are accounting constraints. (4.1.1) says that the total number of men who marry j type women and the number of unmarried j type women must be equal to the number of available j type women for all j. Similarly (4.1.2) says that the total number of women who marry i type men and the number of unmarried i type men must be equal to the number of available i type men for all i. (4.1.3)

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[1] An important exception is Dagsvik 2000 which we will say more about later.

holds because the number of unmarrieds of any type and gender, and the number of marriages between type i men and type j women must be non-negative.

Demographers usually work with matching functions with a zero spillover matching rule (Pollak 1990):

$$\mu_{ij}(M,F) = \mu_{ij}(m_i, f_j)$$

That is, the number of i,j matches only depends on m_i and f_j. Schoen's harmonic mean mating rule, the current workhorse in demography,

$$\mu_{ij}(M,F) = \frac{\alpha_{ij} m_i f_j}{m_i + f_j} \tag{4.1.4}$$

where $\alpha_{ij} > 0$, $\sum_j \alpha_{ij} \leq 1$, and $\sum_j \alpha_{ij} \leq 1$, is a zero spillover matching rule. This matching function will satisfy all the accounting constraints, (4.1.1), (4.1.2) and (4.1.3). Since μ_{ij} and $\frac{m_i f_j}{m_i + f_j}$ are observable, α_{ij} can be estimated. The marriage matching function is non-parametric in the sense that it can fit any cross section marriage distribution. Given estimates of α_{ij}, equation (4.1.4) can be used to make predictions about marital behavior given new population vectors M' and F'.

While zero spillover marriage matching functions are easy to estimate and use, the zero spillover assumption is restrictive. Holding the parameters of the marriage matching function, $\alpha_{ij}'s$, constant, changes in $m_{i'}$ and $f_{j'}$ where $i' \neq i$ or $j' \neq j$ do not affect μ_{ij}. For example, the number of nineteen year old males married to nineteen year old females is unaffected by variation in the supplies of twenty year old males or females.

Demographers have of course recognized the importance of spillover (substitution) effects in marriage matching function (McFarland 1972; Pollard 1997). The problem is to specify marriage matching functions which include substitution effects and yet remain statistically identified. Pollard and Höhn 1993/94 provide the most sophisticated matching function of this kind:

$$\mu_{ij}(M,F) = \frac{m_i f_j a_i b_j}{\frac{1}{2}(\sum_k m_k a_k h_{kj} + f_k b_k h_{ik}))}$$

where a_i, b_j, and h_{ij} are weight functions that are specified by the analyst. When i and j refer to the ages of the participants, the types of individuals are ordered by age. Using this natural ordering, Pollard and Höhn suggested some plausible weight functions. But if types are also defined by ethnicity, religion and other attributes that are not naturally ordered, then it is difficult to apriori specify the weight functions. But without apriori restriction on the weights, the model is not identified.

The CS marriage matching function is defined as:

$$\frac{\mu_{ij}}{\sqrt{(m_i - \sum_k \mu_{ik})(f_j - \sum_k \mu_{kj})}} = \frac{\mu_{ij}}{\sqrt{\mu_{i0}\mu_{0j}}} = \Pi_{ij} \tag{4.1.5}$$

Given μ_{ij}, μ_{i0} and μ_{0j}, Π_{ij} can be estimated. Substitution (spillover) effects are present in the matching function. Holding the numbers of all other types of marriages and population vectors M and F constant, if females of type j marry more males of type i, the estimates of $\Pi_{i'j'}$ for all i' and j' will change. Put differently, holding population vectors M and F, and also $\Pi_{i'j'}$ for all $i' \neq i$ and $j' \neq j$ constant, a change in Π_{ij} will change the numbers of all types of marriages. These effects are absent in zero spillover marriage matching functions.

A major advantage of the CS matching function is the analyst does not have to apriori specify weighting functions for substitution effects that the analyst may have little prior information about. Moreover the CS matching function does not require, within a gender, individuals of different types to have the same ranking of individuals of the opposite gender as spouses. The model is non-parametric in the sense that it will fit any observed cross-section marriage distribution.

Although present, the substitution patterns embodied in CS are restrictive. At this point, the substitions effects are not well understood. Whether the substitution patterns are empirically reasonable remains to be determined.

4.2 The CS model

4.2.1 Motivation

As discussed in the introduction, there are more model parameters than observables in the marriage market. Thus any empirical model will have to make identifying restrictions to reduce the number of unknown parameters to be estimated.

To motivate our identifying assumption, consider a marriage market with I types of men and J types of women. Let the marital output of an i type male and a j type female only depends in i and j. Then there are $I \times J$ number of these marital outputs plus $I + J$ outputs of the types being single. If the behavior of the marriage market is characterized by these outputs alone, then we may be able to estimate all the parameters which are necessary to determine marital behavior.[2] In particular, we do not have to estimate separate male and female preferences for spouses.

Chapter 2 showed that a transferable utility model of the marriage market maximizes the sum of marital output in the society. Thus behavior in a transferable utility model can be characterized by knowledge about marital output alone, and knowledge about male and female preferences separately is not necessary.

The second benefit of using a transferable utility model of the marriage market to derive a marriage matching function is that by virtue of marriage market clearing, all the accounting identities of a marriage matching function, (4.1.1) to (4.1.3), are automatically satisfied.

Since we are focusing on transferable utility models of the marriage market, the next objective is to find convenient functional forms for the demand and supply of spouses when individuals are heterogenous. The choice of a spouse is a discrete choice problem. It turns out that the well know McFadden's 1974 random utility model serves this purpose well.

4.3 The model

Consider a marriage market with I types of men and J types of women. Let M be the population vector of men whose i'th element is m_i, and F be the population vector of women whose j'th element is f_j.

Individuals can live alone or choose someone to marry. Both parties to a marriage must agree before it occurs. For a type i man to marry a type j woman, he must transfer τ_{ij} amount of income to her. There are $I \times J$ sub-marriage markets. The marriage market clears when given equilibrium transfers, τ^*_{ij}, the demand by men of type i to marry type j women, μ^d_{ij} is equal to the supply of type j women to marry type i men, μ^s_{ij}, for all i, j. There is no apriori restriction on the sign of $\overset{*}{\tau}_{ij}$ for any i, j.

To implement the above framework empirically, we adopt the extreme value random utility model of McFadden 1974 to generate market demands for partners. At a point in time, each type of individual considers matching with each type of the opposite gender. Given transfer τ_{ij}, let the utility of male g of type i who marries a female of type j be:

$$V^g_{ij} = \widetilde{\alpha}_{ij} - \tau_{ij} + \varepsilon^g_{ij} \tag{4.3.1}$$

$\widetilde{\alpha}_{ij}$: Systematic gross return to male of type i married to female of

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[2] In the language of econometrics, marital outputs are sufficient statistics for modelling behavior in the marriage market.

type j.

ε_{ij}^{g} : realization of i.i.d. random variable with type I extreme value distribution.[3]

Equation (4.3.1) says that the payoff to person g married to a female of type j consists of two components, a systematic and an idiosyncratic component. The systematic component, $\widetilde{\alpha}_{ij}-\tau_{ij}$, is common to all males of type i married to type j females. The systematic return is reduced when τ_{ij}, the transfer, is increased.

The idiosyncratic component, ε_{ij}^{g}, measures the departure of his individual specific match payoff, V_{ij}^{g}, from the systematic component. We assume that the distribution of ε_{ij}^{g} does not which particular female of type j that he is matched with. This is a strong assumption and is relaxed by Dagsvik as discussed in Section 4.6. The payoff to g from remaining alone, denoted by $j = 0$, is:

$$V_{i0}^{g} = \widetilde{\alpha}_{i0} + \varepsilon_{i0}^{g} \tag{4.3.2}$$

where ε_{i0}^{g} is also the realization of an i.i.d. random variable with type I extreme value distribution. ε_{ij}^{g} and ε_{i0}^{g} are uncorrelated for all i, j.

Male g has $J+1$ living arrangements to choose from. He will choose the living arrangement which maximizes his utility and thereby will receive utility:

$$V_{i}^{g} = \max[V_{i0}^{g}, V_{i1}^{g}.., V_{ij}^{g}, .., V_{iJ}^{g}] \tag{4.3.3}$$

We assume that the numbers of men and women of each type is large. Let μ_{ij}^{d} be the number of i, j marriages demanded by i type men and μ_{i0}^{d} be the number of type i type men who choose to remain single. Then following McFadden 1974, the appendix shows that:

$$\begin{aligned} \ln \mu_{ij}^{d} - \ln \mu_{i0}^{d} &= \widetilde{\alpha}_{ij} - \widetilde{\alpha}_{i0} - \tau_{ij} \\ &= \alpha_{ij} - \tau_{ij} \end{aligned} \tag{4.3.4}$$

The term $\alpha_{ij} = \widetilde{\alpha}_{ij} - \widetilde{\alpha}_{i0}$, is the systematic gross return to a i type male in an i, j marriage relative to being single. The above is a quasi-demand equation by type i men for marriage to type j women.[4] Unlike the usual demand equation, the transfers for non-type j women appear nominally absent in Equation (4.3.4). But they are not absent as these other transfers are all embodied in $\ln \mu_{i0}$.

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[3] The random variable $\varepsilon \sim EV(0,1)$, with the cumulative distribution given by $F(\varepsilon) = e^{-e^{-\varepsilon}}$. The expected value of ε is the Euler's constant, $c \approx 0.577$.

[4] It is not a demand curve because $a_{i0} = m_i - \sum_l \sum_j l_{ij}$.

The appendix shows that:

$$V_i = \mathrm{E}V_i^g = \varsigma + \widetilde{\alpha}_{i0} + \ln(\frac{m_i}{\mu_{i0}^d}) \tag{4.3.5}$$

ς: Euler's constant

E is the expectations operator. V_i is the expected utility of a male of type i before he sees his realizations of his idiosyncratic payoffs for all j. It is his expected benefit from being able to participate in the marriage market. Equation (4.3.5) shows that it is proportional to the log of the ratio of the number of available type i men relative to the number of type i men who choose to remain single.

$\mathrm{E}V_{i0} = \varsigma + \widetilde{\alpha}_{i0}$ is the expected payoff of being single is the only option. $\ln(\frac{m_i}{\mu_{i0}^d})$ measures the expected gain of a type i male from being able to participate in the marriage market where non participation means only choosing to be single.

The random utility function for women is similar to that for men except that in a marriage with a type i men, a type j women receives a transfer, τ_{ij}. Let $\widetilde{\gamma}_{ij}$ denote the systematic gross gain that j type women get from marriage with i type men, and $\widetilde{\gamma}_{0j}$ be the systematic payoff that j type women get from remaining single. The term $\gamma_{ij} = \widetilde{\gamma}_{ij} - \widetilde{\gamma}_{0j}$, is the systematic gross gain that j type women get in marriage with i type men relative to being single. Let μ_{ij}^s be the number of j type women who wants to marry i type men and μ_{0j}^s be the number of single j type women.

The quasi-supply equation of type j women to i, j marriages is be given by:

$$\ln \mu_{ij}^s - \ln \mu_{0j}^s = \gamma_{ij} + \tau_{ij} \tag{4.3.6}$$

Again, the transfers for all other relevant marriages are embodied in $\ln \mu_{0j}$.

Following (4.3.5), the expected utility of a type j female from being able to participate in the marriage market relative to only being single is:

$$\ln(\frac{f_j}{\mu_{0j}^s})$$

There are $I \times J$ sub-marriage markets for every combination of types of men and women. The marriage market clears when given equilibrium transfers, τ_{ij}^*, the demand by men of type i for i,j marriages, μ_{ij}^d, is equal to the supply of type j women for i,j marriages, μ_{ij}^s, for all i,j.

When the markets for all i,j marriages clear,

$$\mu_{ij}^d = \mu_{ij}^s = \mu_{ij} \; \forall \; i,j \tag{4.3.7}$$

Substituting (4.3.7) into Equations (4.3.4) and (4.3.6) to get:

$$\ln \mu_{ij} - \ln \mu_{i0} = \alpha_{ij} - \tau_{ij}^* \tag{4.3.8}$$

$$\ln \mu_{ij} - \ln \mu_{0j} = \gamma_{ij} + \tau_{ij}^* \tag{4.3.9}$$

The left hand sides of (4.3.8) and (4.3.9) are observable. $\ln \mu_{ij} - \ln \mu_{i0}$ measures the systematic net gains to a type i male in an i,j marriage. $\ln \mu_{ij} - \ln \mu_{0j}$ measures the systematic net gains to a type j female in an i,j marriage. It is important to remember that the systematic net gain is the systematic payoff from marriage minus the systematic payoff from remaining single.

In principle, (4.3.8) and (4.3.9) allows us to compare systematic net gains for each type of individual, male or female, in any feasible marriage match. However we cannot compare the systematic payoffs to different marriages for different types of individuals because the systematic payoff to remaining single is not identified.

Add (4.3.8) and (4.3.9) to get:

$$\ln \mu_{ij} - \frac{\ln \mu_{i0} + \ln \mu_{0j}}{2} = \frac{\alpha_{ij} + \gamma_{ij}}{2} = \pi_{ij} \tag{4.3.10}$$

Equation (4.3.10) has an intuitive interpretation. The left hand side of (4.3.11) is the log of the ratio of the number of i,j marriages to the geometric average of those types who remain single. The right hand side, π_{ij}, measures the systematic payoff per partner for **any** i,j marriage minus the average systematic payoff from remaining single. We will call π_{ij} the systematic gains to an i,j marriage.

One expects the systematic gains to i,j marriages to be large if one observes many i,j marriages. There are two problems with this interpretation. First, there may be many i,j marriages because the available population is big. Second, what is the systematic gains large relative to? If we scale the numerator by a function of the relevant populations, m_i and f_j, we can fix the first problem.[5] It still does not address the

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[5] An example is Schoen's harmonic mean mating rule, (4.1.4).

second problem, what other alternatives the individuals could have chosen. Scaling μ_{ij} by the geometric average of the numbers of singles of those types makes it clear that we are measuring the systematic gains to i, j marriages relative to the alternative of remaining single.[6]

Since $(\ln \mu_{ij} - \frac{\ln \mu_{i0} + \ln \mu_{0j}}{2})$ is observable, the systematic gains to an i, j marriage is also observable.

Let $\Pi_{ij} = \exp(\pi_{ij})$. Rewrite equation (4.3.10) as:

$$\begin{aligned} \Pi_{ij} &= \frac{\mu_{ij}}{\sqrt{\mu_{i0}\mu_{0j}}} \\ &= \frac{\mu_{ij}}{\sqrt{(m_i - \sum_k \mu_{ik})(f_j - \sum_k \mu_{kj})}} \end{aligned} \tag{4.3.11}$$

Taking Π_{ij} as given, equation (4.3.11) is the CS marriage matching function. The CS marriage matching function is non-parametric. That is, it will fit any cross section marriage distribution. It is also fully saturated in the sense that the current assumptions underlying the model can only be relaxed by imposing other restrictions.[7]

If Π_{ij} stays fixed, doubling M and F will result in a doubling of μ_{ij}. Thus the CS marriage matching function is homogenous of degree one in population vectors. Equivalently, the CS marriage matching function has constant returns to scale in population vectors.

Given M, F and μ, whose i, j element is μ_{ij}, we can apply (4.3.11) to estimate Π, whose i, j element is Π_{ij}. Given an estimate of Π, we can it to generate predictions of new marriage distributions μ' given new population vectors, M' and F'. The predicted distribution μ' is obtained by solving the system of quadratic equations defined in (4.3.11).[8] While a system of quadratic equations have multiple solutions, CS shows that the predicted distribution is locally unique. In otherwords, there will not be two predicted distributions which are close to each other.

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[6] The term $2\pi_{ij}$ is not the expected total gain to marriage for an i, j couple that chooses to marry each other. Observed i, j married couples get in total $2\pi_{ij}$ plus the idiosyncratic payoffs of each spouse which is the result of optimizing behavior. Since they could have married other types or not marry, the average total payoff of i, j couples who married each other relative to not marrying is weakly larger than $2\pi_{ij}$.

[7] The well known limitations of McFadden's multinomial logit demand model apply here.

[8] Stata code for estimating Π and generating μ' is available at...

4.3.1 Substitution effects

Using (4.3.11),

$$\frac{\partial \mu_{ij}}{\partial m_r} = \frac{\mu_{ij}(\Lambda(i,r) - \frac{\partial \mu_{i0}}{\partial m_r})}{2\mu_{i0}} - \frac{\mu_{ij}\frac{\partial \mu_{0j}}{\partial m_r}}{2\mu_{0j}} \tag{4.3.12}$$

$$\frac{\partial \mu_{ij}}{\partial f_{r'}} = \frac{\mu_{ij}(\Lambda(r',j) - \frac{\partial \mu_{0j}}{\partial f_{r'}})}{2\mu_{0j}} - \frac{\mu_{ij}\frac{\partial \mu_{i0}}{\partial f_{r'}}}{2\mu_{i0}} \tag{4.3.13}$$

where $\Lambda(a,b) = 1$ if $a = b$ and 0 otherwise, $i, r = 1, .., I$, $j, r' = 1, .., J$.

Let $\mathcal{E}_x^{ij} = \frac{x}{\mu_{ij}}\frac{\partial \mu_{ij}}{\partial x}$, the elasticity of μ_{ij} with respect to x where x is the population supply of type x. Then for $x \neq i$ or j, the spillover or substitution effects are:

$$\mathcal{E}_x^{ij} = -\frac{\mathcal{E}_x^{i0} + \mathcal{E}_x^{0j}}{2} \tag{4.3.14}$$

The elasticity of μ_{ij} with respect to x is equal to the negative of the average of the elasticities of unmarried type i males and type j females with respect to x. So if we can calculate $\mathcal{E}_x^{i0}$ and $\mathcal{E}_x^{0j}$, we can compute $\mathcal{E}_x^{ij}$. This simplification restricts feasible substitution effects. Restricted substitution effects are not surprising since McFadden's random utility model, on which the CS model is based on, is known to have restrictive substitution effects.

$\mathcal{E}_x^{i0}$ and $\mathcal{E}_x^{0j}$ may be obtained as follows. Rewrite (4.3.11) as:

$$\mu_{ij} = \Pi_{ij}\sqrt{\mu_{i0}\mu_{0j}}$$

Sum over j to get:

$$m_i - \mu_{i0} = \sum_j \Pi_{ij}\sqrt{\mu_{i0}\mu_{0j}} \tag{4.3.15}$$

Likewise,

$$f_j - \mu_{0j} = \sum_i \Pi_{ij}\sqrt{\mu_{i0}\mu_{0j}} \tag{4.3.16}$$

Using (4.3.15) and (4.3.16) to get:

$$2\Lambda(i,r)\frac{m_r}{\mu_{i0}} - 2\mathcal{E}^{i0}_{m_r} = \sum_j (\frac{\mu_{ij}}{\mu_{i0}})\mathcal{E}^{i0}_{m_r} + (\frac{\mu_{ij}}{\mu_{i0}})\mathcal{E}^{0j}_{m_r} \tag{4.3.17}$$

$$-2\mathcal{E}^{0j}_{m_r} = \sum_k (\frac{\mu_{kj}}{\mu_{k0}})\mathcal{E}^{k0}_{m_r} + (\frac{\mu_{kj}}{\mu_{k0}})\mathcal{E}^{0j}_{m_r} \tag{4.3.18}$$

The above linear $I + J$ equations may be solved to get $\mathcal{E}^{0j}_{m_r}$ and $\mathcal{E}^{i0}_{m_r}$ for all i and j. Likewise we can also get $\mathcal{E}^{0j}_{f_{r'}}$ and $\mathcal{E}^{i0}_{f_{r'}}$. At this point, the properties of these elasticities are not known.

4.4 Estimating the systematic gains to marriage in 1970 and 1980

Population vectors in 1970 and 1980 are obtained from the respective *US Census.* The number of marriages in 1970 and 1980 are obtained from *Vital Statistics.* A state has to report the number of marriages to *Vital Statistics* in 1970 and 1980 to be in the sample. This requirement eliminated 12 states.[9] In our sample of states, there were 16.8 million and 20.9 million available men and women respectively between the ages of 16 to 75 in 1970. From this population, there were 1,625,789 marriages in 1970. There were 24.2 million and 28.6 million available men and women respectively in 1980. Although the available population increased by more than 35% over the decade, there were only 1,698,579 marriages in 1980, an increase of 4.5%. A summary of the data set is in Table 4.1 below.

Figure 4.1 (a) and (b) show the bivariate age distribution of the marrieds in 1970 and 1980 respectively. In both years, most marriages occur between young adults and there is strong positive assortative matching by age. Figures 4.1 (a) and (b) cannot fully capture the magnitudes of the changes in the marriage market and population vectors between the decades.

Figure 4.2 (a) shows the 1970 age distributions of the population vectors and the marrieds.[10] For each gender, the area under the married

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[9] The states of Arizona, Arkansas, Nevada, New Mexico, North Dakota, Oklahoma, Texas, and Washington never reported. Compared with 1970, three more states reported in 1980: Minnesota, South Carolina, Colorado, while Iowa ceased to report.

[10] The average age of available men and women were 30.7 and 39.4 respectively. This gender difference reflected the larger fraction of available older women. The average age of the married men and women were 27.1 and 24.6 respectively, reflecting

Summary Statistics

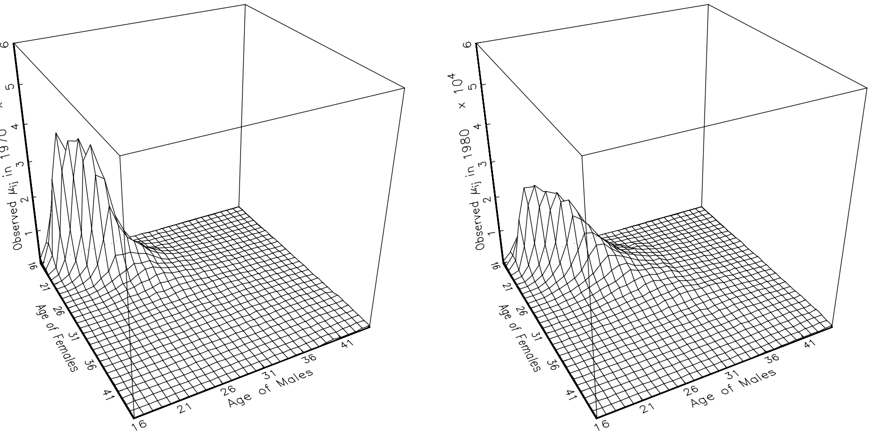
	1970	1980	Δ
Number of Males (M^t)	16.8 mil	24.2 mil	44%
Number of Females (F^t)	20.9 mil	28.6 mil	37%
Number of Marrieds (μ^t)	1.63 mil	1.70 mil	4.5%
Average age of Males	30.7	29.8	
Average age of Females	39.4	37.4	
Average age Married Males	27.1	28.7	
Average age Married Females	24.6	26.1	

Table
4.1.

distribution is almost equal to the height of the available individuals at 16, reflecting the fact that most individuals will eventually marry. The figure also shows that at each age, most available individuals do not marry.

Figure 4.2 (b) shows the age distributions of the population vectors and the marrieds in 1980. Comparing figures (a) and (b), we see that the baby boom generation came of marriageble age between the decade, substantially increasing the population of the availables in 1980. The average age of available men fell from 30.7 in 1970 to 29.8 in 1980 and that of available women fell from 39.4 in 1970 to 37.4 in 1980. However, the number of young marrieds in 1980 barely increased. Marriage rates in 1980 were much lower than that in 1970. Put another way, the systematic gains to marriage for young adults fell substantially over the decade.

the usual gender difference in ages of marriage.



a) Surface of observed μ_{ij} for 1970

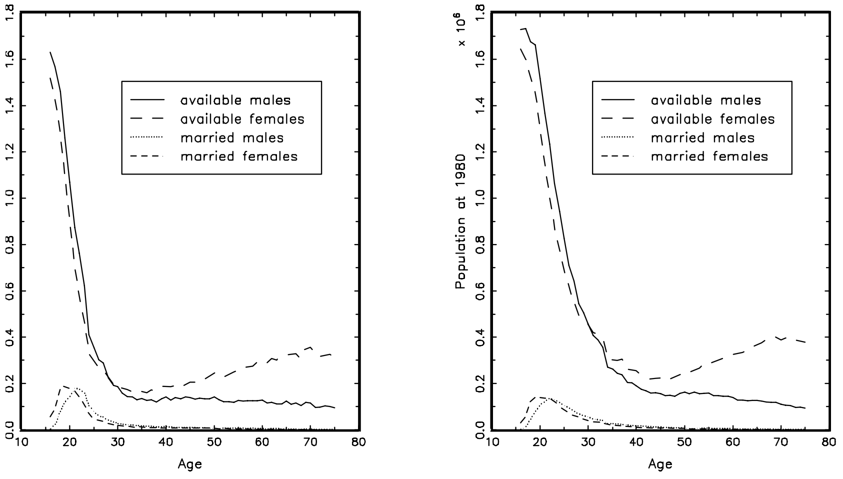
b) Surface of observed μ_{ij} for 1980

Figure 4.1. Bivariate distributions of marriages by age.

4.4.1 Estimating the net gains to marriage by gender

Our model allows us to estimate the systematic "net gain" relative to not marrying, for each party in any i, j marriage. The 1970 estimates for type i males, given by $\widehat{n^{70}_{ij}} = \frac{\mu^{70}_{ij}}{\mu^{70}_{i0}}$, and j type females, given by $\widehat{N^{70}_{ij}} = \frac{\mu^{70}_{ij}}{\mu^{70}_{0j}}$ are compared in Figure 4.3 (a).[11] Figure 4.3 (a) plots $\widehat{n^{70}_{ij}}$ and $\widehat{N^{70}_{ij}}$ for 20 and 30 year old males and females by the ages of their spouses and Figures 4.3 (b) plots them for 40 and 50 year old males and females. In Figure 4.3 (a), the distribution of $\widehat{N^{70}_{i,20}}$ is right skewed, with the 20 years old female receiving the largest systematic net gain when she marries a slightly older male. In contrast, the distribution of $\widehat{n^{70}_{20,j}}$ is more symmetric and concentrated, with the 20 years old male receiving the largest systematic net gain when he marries a slightly younger female.

[11] We plot $\frac{\mu_{ij}}{\mu_{i0}}$ and $\frac{\mu_{ij}}{\mu_{0j}}$ rather than their log counterparts because μ_{ij} is zero for some types of marriages.



a) Availables and Marrieds at 1970 b) Availables and Marrieds at 1980

Figure 4.2.

The area below these plots is proportional to the type specific marriage rates.[12] The smaller area under $\widehat{n^{70}_{20,j}}$ relative to that of $\widehat{N^{70}_{i,20}}$ suggests that the marriage rate of 20 year old females is larger than that of 20 year old males. Comparing the distribution of systematic net gain for a 30 years old female, $\widehat{N^{70}_{i,30}}$, with her 20 years old counterpart, we find the distribution for a 30 years old female to be more dispersed and the marriage rate to also be significantly lower. Again she receives the largest net gain when she marries someone slightly older. If we consider the distribution for 30 year old males, $\widehat{n^{70}_{30,i}}$, we also find the distribution to be more dispersed than for his 20 year old counterpart. Again his largest net gain is to marrying someone slightly younger. Comparing the areas under the respective distributions, his marriage rate is higher than his 30 year old female counterpart.

Figure 4.3 (b) compares the systematic net gains to marriage for 40 and 50 year old males and females. The difference in the scale of the

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[12] $\sum_j \frac{\mu_{ij}}{\mu_{i0}} = \frac{m_i-\mu_{i0}}{\mu_{i0}}$ and $\sum_i \frac{\mu_{ij}}{\mu_{0j}} = \frac{f_i-\mu_{0j}}{\mu_{0j}}$

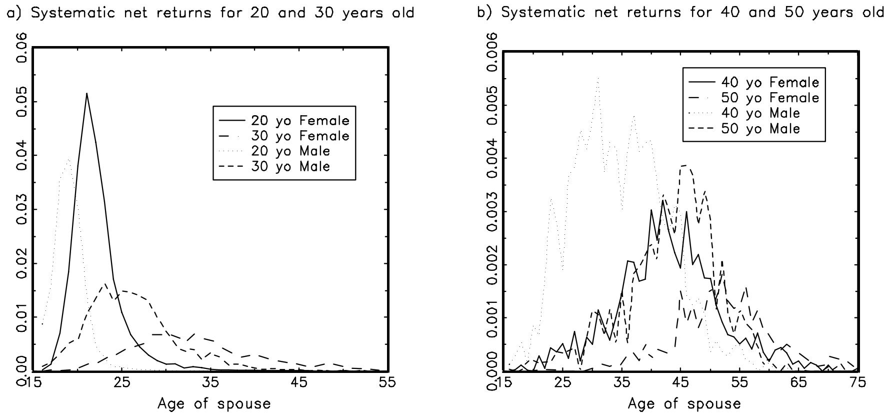


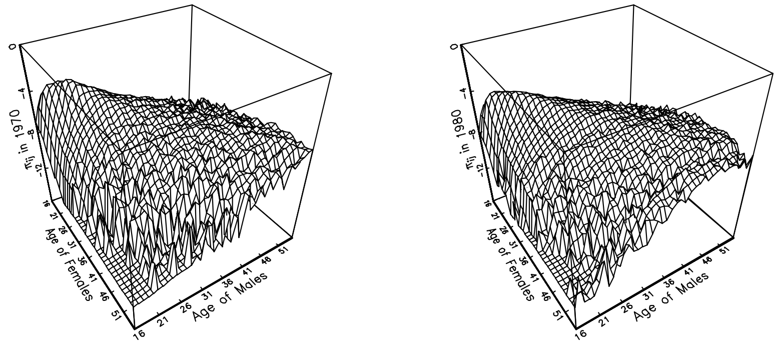
Figure 4.3. Systematic net gains

vertical axis in Figures 4.3 (b) and (a) reflects the fact that the net gains to marriage fell substantially by age. The net gains to marriage for 40 year old males were higher than for 40 year old females and the marriage rate is also higher. The distribution of net gains to marriage for 40 year old females is similar to that of 50 year old males! Put another way, the age distribution of spouses of 40 year old females in 1970 is similar to the age distribution of spouses of 50 year old males. Finally, the net gains to marriage for 50 year old females were lower than the other groups.

Most of the features of the empirical distributions in Figures 4.3 (a) and (b) are expected; What is new is that our model provides a normative interpretation of these empirical distributions. It is important to remember that our estimates of net gains reflect both preferences and equilibrium transfers.

4.4.2 Estimating π

Figure 4.4 (a) presents a plot of the distribution of $\widehat{\pi}^{70}$. All the values in $\widehat{\pi}^{70}$ are negative reflecting the fact that the systematic gains to marriage is smaller than not marrying. The parametric specification of our



a) Observed π_{ij} at 1970 b) Observed π_{ij} at 1980

Figure 4.4. Systematic total gains to marriage

model predicts a match to occur only if the idiosyncratic draw is large. The negative values should also not be surprising since at any age, most available individuals do not marry (as suggested by Figure 4.2 (a). The estimated systematic gains to marriage fell as individuals aged. It also systematically fall away from the diagonal (where $i = j$). Thus there are large gains to positive assortative matching by age. Although somewhat more difficult to see, there is also a ridge along the diagonal. The systematic gains fell faster in the southwest direction away from the ridge than in the northeast direction. In other words, the systematic gains to marriage between older women and younger men were less than that between older men and younger women.

Comparing Figure4.4 (a) which plots $\widehat{\pi}^{70}$ with Figure 4.1 (a) which plots μ^{70}, illustrates the importance of scaling the number of marriages to obtain estimates of the systematic gains to marriage. While the estimate of the systematic gains to marriage is highest between young adults, the decline for older adults and between young and old adults is relatively gradual. On the other hand, Figure 4.1 (a) shows that the number of marriages is largest between young adults and falls rapidly elsewhere. Thus one would not want to use the distribution of μ to estimate the systematic gains to marriages.

Figure 4.4 (b) plots $\widehat{\pi}^{80}$. The shape of $\widehat{\pi}^{80}$ is similar to that of $\widehat{\pi}^{70}$.

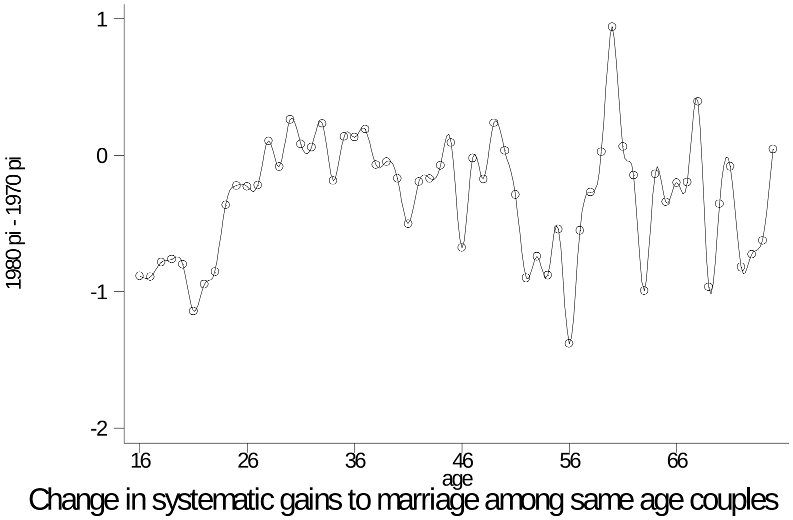


Figure 4.5. $\pi_{ii}^{80} - \pi_{ii}^{70}$

The peak of $\widehat{\pi}^{80}$ is lower than that of $\widehat{\pi}^{70}$ which simply reflects the fact that the systematic gains to marriage fell over the decade.

The drop in the systematic gains to marriage is easier to see in Figure 4.5 which plots $\widehat{\pi}_{ii}^{80} - \widehat{\pi}_{ii}^{70}$, the change in $\pi' s$ for same aged couples. The drop is particularly significant for marriages between same aged young adults.

4.5 Testing model mispecification

A strong implication of our model, as given in Equation (4.3.11), is that π^t only reflect preference parameters and is independent of population vectors. To the extent that dynamic considerations and scale effects are important, π^t will be a function of the population vectors at time t. Table 2 presents some regressions of $\widehat{\pi}_{ij}^{70}$ on demographics and 1970

Table 2

	OLS	MED	IV
Dependent var.	$\widehat{\pi}_{ij}^{70}$	$\widehat{\pi}_{ij}^{70}$	$\widehat{\pi}_{ij}^{70}$
$\ln(M_i^{70})$	2.1778	1.7970	7.3138
	(0.3707)	(0.0565)	(1.3917)
$\ln(F_j^{70})$	2.3693	2.4146	3.6382
	(0.3247)	(0.0579)	(0.8229)
Age polynomials	Y	Y	Y
Observations	2456	2456	2456
R^2	0.73		0.52
Instruments			$\ln(M_i^{80}),\ \ln(F_j^{80})$

Robust standard errors in parentheses.

population vectors.[13]

We present results from OLS, median and IV regressions. IV regression was carried out because $\widehat{\pi}_{ij}^{70}$ is constructed with 1970 population vectors. So if there is measurement error in our measure of the population vectors, this may induce a correlation between $\widehat{\pi}_{ij}^{70}$ and the population vectors even when there is no true relationship. We use the 1980 population vectors as instruments for the 1970 population vectors. In all regressions, we also include cubic polynomials of ages, i and j.

All three methods of estimation give the same results. Even after controlling for demographics (using a cubic polynomial in ages), the 1970 population vectors can still explain variations in $\widehat{\pi}_{ij}^{70}$.[14] The estimated

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[13] We do not use the observations where the actual number of marriage in those i, j cells is zero because our smoothed estimates of $\widehat{\pi}_{ij}^{70}$ are relatively large negative numbers in those cases and including them change some point estimates substantially. But the qualitative conclusion remains unchanged.

[14] The 1970 population vectors remain statistically significant when we used fifth

coefficients for the population vectors are statistically significant at the 1% level in all cases. This result strongly suggests that π_{ij}^{70} is correlated with the population vectors in 1970, a violation of our model. This violation suggests that dynamic considerations, scale effects in the marriage market and/or other forms of misspecifications are important.

4.6 Bibilographic notes

Dagsvik (2000) anticipated our methodology of using an explicit model of the marriage market to construct marriage matching functions.[15] The marriage matching function in Dagsvik is defined by

$$\theta_{ij} = \frac{\mu_{ij}}{\mu_{i0}\mu_{0j}} \tag{4.6.1}$$

where θ_{ij} are unrestricted. The term θ_{ij} has a similar normative interpretation as our Π_{ij}. His model is also non-parametric and will fit any observed marriage distribution. Thus given data from a single cross section, we cannot differentiate between his model and ours in terms of fit of the data.

Empirically the two marriage matching functions differ in that Dagsvik's model has scale effects. For the simple case of one type of male and one type of female, it is easy to check that Dagsvik's model satisfies increasing returns to scale in the population vectors. The two models also employ different specification of payoffs to marriage. In his model, the payoff that male g of type i gets from marriage to female k of type j is defined by:

$$V'_{ijgk} = \widetilde{\alpha}'_{ij} + \varepsilon_{ijgk}.$$

$\widetilde{\alpha}'_{ij}$ denotes the systematic return to i type male from an i, j match. ε_{ijgk} denotes his idiosyncratic returns from a match between him and the j type female individual k.[16] So if he is matched with another female k' of type j, he will get a different payoff. Likewise for the payoffs of the females when they choose between different males. Since individuals in Dagsvik's model value every potential spouse differently, he cannot use price taking behavior (equilibrium transfers) to clear the marriage market. Instead, he uses the deferred acceptance algorithm and stable

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order polynomials in age.

[15] Also see Johansen and Dagsvik 1999; Dagsvik, et. al. 2001.

[16] The random variable ε_{ijgk} is also assumed to have type I extreme value distribution.

matching as an equilibrating device. Stability per se is not the difference between his model and ours because our equilibrium is also stable.

In contrast, our model assumes that for any type j, there are sufficient number of females of that type such that male g is indifferent between them. Likewise for any type i males, female k has enough males of that type to choose from such that she is indifferent between them. So f_j does not directly affect the idiosyncratic payoff that male g gets from choosing to marry a female of type j. Likewise for female k. Given these indifference assumptions about within type spouses, we can use types specific transfers to clear the marriage market. Thus we have a transferable utilities model of the marriage market whereas Dagsvik (2000) has a non-transferable utilities model.

4.7 Problems

1. Consider a special case of the CS model where $I = 1$ and $J = 1$. In this case, there is only one type of men and one type of women in the marriage market. (i) Show that the equilibrium number of marriages, μ, will increase as F increases. (ii) Show that the equilibrium number of marriages, μ, will decrease as π falls.

2. Continuing with the special case discussed in problem 1, let the utility that male g gets from marrying be:

$$V^g = \widetilde{\alpha} - b\tau + \varepsilon^g$$

Unlike the case discussed in the text, b need not be equal to 1. All the other assumptions are the same as in the text. A woman k's utility from marriage is:

$$U^k = \widetilde{\gamma} + \tau + \varepsilon^k$$

where ε^k is drawn from the extreme value distribution. The difference between the specification of preferences in this problem and the text is that in this case, the male's valuation of the transfer is not the same as the woman's valuation. Put another way, the CS model assumes that males and females have the same marginal utility of income.

(i) What is identified in this case from observing μ, M and F?

(ii) If you are willing to put a bound on b, i.e. $1 - z < b < 1 + z$, what can you learn?

(iii) Is setting $b = 1$ as in CS reasonable? Why or why not?

0.1 Derivation of (4.3.4)

(4.3.1) may be rewritten as

$$V_{ijg} = \widetilde{\alpha}_{ij} - \tau_{ij} + \varepsilon_{ijg} = \eta_{ij} + \varepsilon_{ijg}$$

As specified by (4.3.3), g solves

$$V_{ig} = Max_j[V_{i0g}, .., V_{ijg}, .., V_{iJg}]$$

The probability that a type j woman is chosen is:

$$\Pr\{V_{ig} = V_{ijg}|\eta\} = E\{\Pi_{k\neq j} F(\varepsilon_{ijg} + \eta_{ij} - \eta_{ik})\} \tag{.1}$$

$$= \int_{-\infty}^{\infty} \exp\{-\sum_{k\neq j} e^{-\varepsilon-\eta_{ij}+\eta_{ik}}\} e^{-\varepsilon-e^{-\varepsilon}} d\varepsilon$$

The index k runs from 0 to J.
Let

$$c = 1 + \sum_{k\neq j} e^{-\eta_{ij}+\eta_{ik}}$$

Also note that

$$\int e^{-\varepsilon-ce^{-\varepsilon}} d\varepsilon = \frac{e^{-ce^{-\varepsilon}}}{c} \tag{.2}$$

Then (.1) becomes:

$$\Pr\{V_{ig} = V_{ijg}|\eta\} = \int_{-\infty}^{\infty} \exp\{-\varepsilon - c\exp(-\varepsilon)\} d\varepsilon$$
$$= \left[\frac{e^{-ce^{-\varepsilon}}}{c}\right]_{-\infty}^{\infty}$$
$$= \frac{\exp \eta_{ij}}{\sum_k \exp \eta_{ik}}$$

So

$$\frac{\Pr\{V_{ig}=V_{ijg}|\eta\}}{\Pr\{V_{ig}=V_{i0g}|\eta\}}=\exp(\eta_{ij}-\eta_{i0})=\exp(\widetilde{\alpha}_{ij}-\alpha_{i0}-\tau_{ij}) \qquad (.3)$$

When there are many men of each type, we may approximate $\Pr\{V_{ig} = V_{ijg}|\eta\}$ with $\frac{\mu_{ij}}{m_i}$. Then (4.3.4) follows from (.3).

0.2 Derivation of (4.3.5)

The index k runs from 0 to J. Observing male g of type i choose choice j, the expected utility of that individual is:

$$EV_{ijg}=\eta_{ij}+E(\varepsilon_{ijg}|\varepsilon_{ijg}+\eta_{ij}>\eta_{ik}+\varepsilon_{ikg}\ \forall k\neq j)$$

$$E(\varepsilon_{ijg}|\varepsilon_{ijg}+\eta_{ij}>\eta_{ik}+\varepsilon_{ikg}\ \forall k\neq j) \qquad (.4)$$
$$=\frac{\int_{-\infty}^{\infty}\varepsilon\exp\{-\sum_{k\neq j}e^{-\varepsilon-\eta_{ij}+\eta_{ik}}\}e^{-\varepsilon-e^{-\varepsilon}}d\varepsilon}{\Pr\{V_{ig}=V_{ijg}|\eta\}}$$

Using (.2) and the fact

$$\int_{-\infty}^{\infty}xe^{x}\exp(-\phi e^{x})dx=-\frac{\Gamma+\ln\phi}{\phi}$$

where Γ is Euler's constant, $\simeq 0.577215$, (.4) may be expressed as

$$E(\varepsilon_{ijg}|\varepsilon_{ijg}+\eta_{ij}>\eta_{ik}+\varepsilon_{ikg}\ \forall k\neq j)=\Gamma+\ln(\sum_k\exp\eta_{ik})-\eta_{ij}$$

Thus

$$EV_{ijg}=\eta_{ij}+E(\varepsilon_{ijg}|\varepsilon_{ijg}+\eta_{ij}>\eta_{ik}+\varepsilon_{ikg}\ \forall k\neq j)=\Gamma+\ln(\sum_k\exp\eta_{ik}) \qquad (.5)$$

which is independent of j. Since knowing the optimal choice of the individual is not informative about his expected payoff, $EV_{ig} = EV_{ijg}$. Then (.5) and (4.3.4) imply:

$$EV_{ig}=\Gamma+\ln(\sum_k\exp(\widetilde{\alpha}_{ik}-\tau_{ik}))=\Gamma+\widetilde{\alpha}_{i0}+\ln m_i-\ln\mu_{i0} \qquad (.6)$$

which is (4.3.5).