Chapter 7

Differential Fecundity III: Class, gender & marriage in 18th century Quebec[1]

Previous chapters show that under particular marital institutions, differential fecundity can generate some commonly observed gender roles. However the quantitative significance of these effects are unclear. The objective of this chapter is to see if differential fecundity can provide a quantitative accounting of gender differences and other empirical regularities in an actual marriage market, 18th century Quebec.

The data comes from a reconstituted family data set from 17th and 18th century New France gathered painstakingly by demographers at the University of Montreal. The data set consists of linked information from all of the birth, marriage, and death parish registers in the Quebec region. It provides the vital life histories of everyone known to have been born in the colony.

The quantitative model emphasizes three features that are universally important in marriage markets–search frictions, socioeconomic status, and differential fecundity.

The chapter proceeds in two parts. The first part estimates a *just identified* reduced form model. The estimates of the reduced form model show that a simple random matching model of the marriage market, in which women may leave the marriage market at a higher rate than men, can match

[1]This chapter reports results from joint work with Gillian Hamilton, an economic historian at the University of Toronto.

the aggregate marital experiences of 18th century Quebec.[2] The second part estimates a structural model that considers agents' behavior in the marriage market. Given the simplicity of the model, it is simply illustrative of the sort of results that can be generated with this class of model. The structural estimates indicate that 18th century Quebec women fared marginally better than men in terms of lifetime discounted consumption (holding social status constant).

Two caveats are in order. First, the chapter ignores other factors that may have a quantitatively significant impact on the marriage market. For example, age is not a state variable in the model. While inaccurate at the individual level, this is a convenient abstraction for studying aggregate behavior.

7.1 Data

The data consist of reconstituted family data elicited from New France's birth, marriage, and death Catholic parish records, linked by demographers at the University of Montreal.

Linking all of these registers produces a record of each person's vital life history: for those whose birth and death was recorded in the colony, we know whether they married, and if so, the dates of each marriage (if there was more than one), known vital information about the spouse, and the number and sequence of children produced from each marriage. If the spouse was born outside the colony sometimes less is known about their birth date and place (and hence their life span).

Two sets of individuals that likely were wealthy have been identified through information in the parish and notary public's records. The first is members and offspring of the nobility (Gadoury, 1991). French royalty conferred noble title, which was inherited through the male line. The nobility were afforded privileges not enjoyed by the typical resident.[3] The

[2] Our companion paper, Hamilton and Siow (1999), studies the individual data underlying the aggregate data presented here.

[3] For example, the King of France offered some nobility large land grants, called seigneuries. Not all seigneurs were member of the nobility (Harris, 1966). The aristocracy also qualified for pensions and some received fur-trade licences. In addition, a substantial portion of those that did not inherit title served as officers in the military and were given (lucrative) officers' commissions. For more information on the nobility, see Gadoury (1991), Dechêne (1992), or Greer (1997: 51).

second is members and offspring of the 'bourgeois' class (Noguera, 1994). 'Bourgeois' was often a self-appointed title taken by men with relatively high status occupations, such as large-scale merchants or crown appointed officials (a complete list appears in the data appendix). Women were bourgeois only if they married a bourgeois. High status or wealthy is defined as individuals from either a noble or bourgeois family. The majority of the remaining population farmed for a living.

The sample employed here, the NB sample, includes individuals born before 1700 with known life span.[5] We also restrict the sample to those individuals that lived until at least age 10, since we are studying marriage market participation. These restrictions affect the sample size as follows: a birth record exists for 19,580 individuals (born before 1700), life span is known for 15,334 of them. Almost 4,000 died before age 10, leaving 11,865 in the sample. The NB sample assumes that the entire province of Quebec was one marriage market. To better approximate a relevant marriage market, we also constructed a sample which consists of individuals born in Quebec City, the NBQ sample.

Column (1) of Table 1 presents summary statistics for the NB sample. 5.9% of the population were high status (row 1). They had shorter life spans than low status individuals (rows 2-3). Men, and high status individuals, had relatively low marriage rates (rows 5-8). For example, 21.2 percent of men and 15.5 percent of women never married. In addition, 40.1 percent of high status individuals never married, compared to 16.7 percent of low status people. Rows 12 to 14 measure the degree of assortative matching by social class. For example, row 14 measures the ratio of the number of high-high to the number of low-low marriages, $\frac{\pi_{Hh}}{\pi_{Ll}}$ (π_{Ss} is the number of marriages involving females of status S and males of status s). As these rows illustrate, a low incidence of within-class marriages, as well as a very low mixed-class marriage rate, contributed to the low marriage rate among the high status.[6]

[5]Hence individuals must have experienced both birth and death in the colony (to derive life span). Because the PRDH have not completed the data reconstitution for parish records written after 1800, we restrict the sample to those born before 1700. This ensures that the data reconstitution was complete for everyone in our sample.

Most births occurred soon before 1700. While the first birth in this sample was recorded in 1620, 84 percent of individuals in this sample were born between 1670 and 1699, and 61 percent were born after 1679.

[6]To calculate the number of marriages by status, we include only those observations where both spouses were born in the province. This reduces a possible source of sample

If there was non-assortative matching, the expected ratio of mixed status to low status matches would be 0.1128/0.8836 or 0.128, rather than the observed ratio of 0.045 ($\frac{\pi_{Hl}}{\pi_{Ll}} + \frac{\pi_{Lh}}{\pi_{Ll}}$).[7] In other words, there were fewer mixed status matches than would be predicted by non-assortative matching. In addition, high status individuals also tended to marry late (rows 15-18). Men also married later than women. Row 21 indicates that there were more women than men in the sample. Part of this imbalance stems from a higher death rate for young males.

Apart from differences in occupation and privilege, there was also some geographic distinction between 'high' and 'low' status individuals. Most of the upper echelon lived in the cities. One consequence of this segregation is that high status people lived relatively unhealthy lives, because the cities were notoriously unsanitary.[8] Life was already fairly short at this time: 24.7 percent of boys and 19.6 percent of girls died before they reached the age of 10. Average life span for those who lived past age 10 was 56.5.[9] The wealthy class had higher infant and childhood mortality rates, and their average life span, conditional on living past age 10, was lower than that found in the general population.[10]

Before turning to the models, it is instructive to further explore the social context and the individual behavior of men and women to illustrate the importance of socioeconomic status and differential fecundity in this society's marriage market.

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selection bias because status is determined through parental attributes and parents were systematically underidentified for individuals that were not born in the province. In addition, for computational convenience in estimating the standard errors we do not count all marriages for these statistics, but only those observations in which the individual is male (and spouse is female). Whether we instead define males as spouses, or count all unique marriages, makes almost no difference. There are slight differences in these statistics because the individual is restricted to being in their first marriage, but their spouse is not similarly restricted.

[7] The proportion of high status people in the sample was 6 percent, hence the likelihood of a mixed status match was $(0.06 \times 0.94 + 0.94 \times 0.06)$ and the likelihood of a low status match was (0.94×0.94).

[8] Unsanitary conditions was not the only factor affecting the life span of the higher class. They also tended to wet-nurse their children, which may have contributed to their higher infant and child mortality rates.

[9] In addition, the maternal mortality rate was 9 percent. In spite of the risks, families were large. Adults in their first marriage had an average of 8.2 children.

[10] Today, in contrast, life expectancy is positively correlated with wealth.

7.1.1 Class

Under New France's property laws, husbands and wives held equal claim to most marital assets. Hence high status parents had incentive to deter their children from marrying down because their family's wealth would have been diluted. In addition, female children of noble families stood to lose their 'status' if they married a non-noble male.

The colony's inheritance rules, on the other hand, did not appear to deter marriage. For most of the New French, all children (male and female) claimed an equal share of their parent's estate and dowries were rare (Dechêne (1992)).[12] Thus one presumes that most elite children could have attracted mates. In contrast, a number of European countries practiced primogeniture, hence subsequent sons were less attractive than first-born sons, and dowries were prohibitively large (e.g., Hufton, 1995). These rules appear to have contributed to the low marriage rates among European aristocratic families (e.g., Hurwich, 1998).

A variety of evidence suggests that parents in New France attempted to orchestrate their children's marriage prospects and alliances. First, the surviving written record, although sparse, indicates as much.[13] Second, the rate of consanguine marriages appears to have been relatively high among the noble population, which is consistent with these families exhibiting concern about wealth and status maintenance.[14] Third, not all elite children entered the marriage market because noble families supplied priests and nuns to the church. This phenomenon may have reflected piety alone, but it also conveniently limited a family's chances of marrying down.[15]

The colonial (French) government was also firmly against its military offi-

[12]The small number of families that owned seigneuries, however, could (legally) practice a watered-down form of primogeniture. Half of the estate, including the house, passed to the first son, while the remainder was divided equally among the other children. The objective was to discourage partitioning of the seigneuries. For more information on seigneuries see, for example, Greer (1997) or Harris (1966).

[13]In one case a mother was incensed with her son's (an army officer) prospective bride because she thought his fiancé was not wealthy. Source: Letter from Madame Vassal de Monviel, cited in Gadoury (1991: 94)].

[14]Gadoury estimates that 10 percent of noble marriages were within-family, compared to 4 percent for the population as a whole. (Gadoury (1991: 108)). Among the aristocracy in France 'arranged' marriages were the norm, and it was common to marry distant, or not so distant, relatives. (Gadoury (1991: 93)).

[15]Second sons typically entered the priesthood. Hamilton and Siow (1999) find evidence of such a second birth effect among the noble population.

cers (noble men) marrying common girls.[16] The reasons for the government's preoccupation with advantageous alliances are unclear, except to the extent that social standing mattered.[17]

7.1.2 Gender

As Table 1 illustrates, there were substantial differences in the marriage market experiences of men and women. As noted, men were less likely to marry than women. In addition, men were more likely to remarry than women and they tended to marry at a later age than women.

A number of factors indicate that differential fecundity played a role in the varied marriage market experiences of men and women. For example, men procreated at an age at which women of their own cohort could no longer bear children, something they accomplished by marrying younger women. This is illustrated in a plot of the distribution of fathers and mothers' ages at the time their children were born (Figure 1).

In addition, individuals appear to have considered spouses' prospective fertility when remarrying. We would expect remarriage to be less common among widows compared to widowers (because they may have experienced menopause), and widowers to prefer fertile over infertile women if fertility mattered in this marriage market. Women who remarried are not expected to exhibit the same preference for young men, since male fertility is not correlated with age. Overall, the remarriage rate for men is almost twice that of women (50 percent versus 33 percent, Table 1, column (1)). Part of this difference could simply reflect the relative availability of potential spouses, because there were more widows than widowers in the population. As Figure 2 illustrates, however, the age difference between men and women was even larger for men who were remarrying as compared to men in their first marriage. In contrast, women entering their second marriage tended to be closer to their spouse's age than women entering their first marriage.

[16] In one case, a letter document that a 'poor' marriage between an army officer (and nephew of a colonial governor) and a non-aristocratic woman was an affront to the groom's family as well as the local governors. [Source: "Délibération du Conseil de la Marine" January 1721, Archives des colonies, Series C11A, vol. 43, folio 131. Cited in Gadoury (1991: 93)].

[17] On one occasion the colonial governor wrote to the government minister in France, assuring him that "I will insist...that in the future officers will make marriages that are both suitable and profitable." Letter dated October 20, 1691, cited in Dechêne (1992: 235).

To examine this behavior more carefully we estimate the probability of remarriage as a function of gender, the presence of children from the previous marriage, and age at widowhood. We also include status, year, and urban marriage controls in each regression. Age at widowhood and previous children are expected to act as proxies for future female fertility. Age-at-widowhood is predicted to be a deterrent to female remarriage and a first marriage that did not produce children may have signalled infertility, hurting the survivor's chances of remarriage.

Table 7.2 presents the probit estimates. The sample consists of individuals known to have been widowed from their first marriage (subsequent remarriage behavior is ignored). We examine women and men together in column (1), and their behavior separately in columns (2) and (3), respectively. Column (1) indicates that the probability of a man remarrying was 0.314 higher than that of a woman (a larger difference than found in the raw means).[19] Comparing the age-at-widowhood coefficients for women and men across columns (2) and (3) reveals that age hurt both men's and women's chances of remarriage. Column (2) also indicates that a widow without a child from her first marriage had a 0.15 lower probability of remarriage. This infertility penalty fell as the age of widowhood increased (it disappears by age 30). Potential spouses did not, however, value children from the first marriage per se—the remarriage rate fell as the number of (previous) children increased. On the other hand, an absence of children from the first marriage did not affect the remarriage probability of widowers (column (3)). These results provide evidence of the importance of differential fecundity in the marriage market.[20]

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[19] This result is well known and holds for many countries and across time. For example, in a current study Chamie and Nsuly (1981) show that divorced men were more likely to remarry than divorced women in all 47 countries they examined. For some historic evidence, see Dupâquier et al. (1981).

[20] Research on historic remarriage behavior argues fertility as well as other factors were important deterrents to female remarriage. For example, Hufton (1995: 218-22) states that post-menopausal women did not tend to remarry. She goes on to argue that women faced more social pressure than men to remain in their widowed state. The minimum acceptable mourning period was much longer for women (at least a year, compared to 3-6 months for men) and men's honor required them to replace their wives quickly because engaging in menial tasks like cooking, cleaning, and child rearing was degrading. Hufton also cites contemporary correspondence that illustrates various church's views on widowhood. In short, they tended to believe that because widows had acquired 'carnal knowledge' the best antidote to this unfortunate situation was chastity.

7.2 A statistical framework

7.2.1 Model

We begin with a statistical model of the marriage market that takes account of the possible effects of class distinctions and differential fecundity on marriage market outcomes. This model can reproduce the observed aggregate behavior of our data set. We consider the steady state of a non-linear Markov model. Figure 3 provides a flow chart. In every period, new individuals enter the marriage market. Individuals may leave the market in three ways. First, two single individuals who meet in the marriage market leave temporarily if they marry. Second, individuals may die. Third, women exogenously leave the market at a higher rate than men. Married individuals may return to the marriage market when their spouses die. Widows may return to the marriage market at a lower rate than widowers.

To address heterogeneity within the marriage market, we assume that there are two types of individuals: high status (h) and low status (l). Equal numbers of males and females are born each period. We normalize the number of new entrants of each gender to 1, a_h fraction of these adults, equally divided between men and women, are high status and the rest are low status.

Individuals are potentially infinitely lived. Each h individual who is alive in the current period will live in the next period with probability p_h. Each l individual who is alive in the current period will live in the next period with probability p_l.[21] A person may re-enter the marriage market when his or her spouse dies.

Consistent with differential fecundity, we assume that women "leave" the marriage market at a faster rate than men. This faster exit rate can be interpreted as the arrival rate of menopause, with the proviso that infertile women do not enter the marriage market.[22] In contrast, men are fecund every period of their lives. This is the only distinction between men and women. Specifically, each woman in a current period will remain in the marriage market in the next period with probability η. This applies to both

[21] ¿From a purely descriptive view point, it is unnecessary to interpret these probabilities exclusively as survival probabilities. p_h and p_l are the per period probabilities that individuals from different social classes will remain in the marriage market.

[22] In theory, death from childbirth also would have contributed to the early exit of women from the marriage market. The average life span of men and women were quite similar, however, since older men died at a faster rate than older women. Since age is not in the model, we do not take childbirth deaths into account.

single and married women. A married woman who "leaves" the market (experiences menopause) does not leave her marriage, but she will not re-enter the marriage market if she is widowed. Fecund widows re-enter the marriage market.

We assume that there is random matching in the marriage market. Each individual may meet at most one other individual of the opposite sex per period. When an eligible man of status s and a woman of status S meet, they will fail to marry with probability $\lambda(S,s)$.[24] With probability $\overline{\lambda(S,s)}$, an Ss pair that meets will marry (for any variable α, $\overline{\alpha}$ is $1-\alpha$). When they do not marry, they may return to the marriage market in the next period.

Following the custom in 18th century Quebec, we assume that there is no divorce. A marriage ends when a spouse dies. Individuals who remain in the market may remarry after the death of a spouse. Participants in the marriage market do not distinguish between never married individuals and reentrants.

The statistical marriage market model is determined by the eight parameters: a_h, p_h, p_l, η, $\lambda(H,h)$, $\lambda(H,l)$, $\lambda(L,h)$ and $\lambda(L,l)$. We first construct the steady state quantities generated by these parameters. Because we do not observe all of these quantities we go on to define other observable variables that allow us to recover the reduced form parameters.

Let n_s be the steady state stock of eligible s type males. Let N_S be the steady state stock of eligible S type females. Since females exit the market at a higher rate then men, assume $n = n_h + n_l > N = N_l + N_h$. We assume a simple matching rule where every eligible woman is able to find an eligible male. With random matching in the marriage market, the probability of a woman meeting a man of type s, and the probability of a man meeting a women of type S, is (respectively):

$$Q(s) = \frac{n_s}{n} \tag{7.1}$$
$$q(S) = \frac{N_S}{n}$$

Let π_{Ss} be the equilibrium number of married Ss couples. In steady state,

$$\pi_{Ss} = p_S p_s\{\pi_{Ss} + n_s q(S)(1-\lambda(S,s))\} \tag{7.2}$$

[24]We use upper case letters for women, small case letters for men. Note, however, that there is no theoretic distinction between men and women of a given status (apart from the higher exit rate for women, which is not status dependent).

The first term within brackets is the number of existing Ss marriages in the previous period. The second term within the brackets is the number of new Ss marriages formed in the previous period.

Let Π_{Ss} be the number of fecund married women of type S in an Ss marriage. In steady state,

$$\Pi_{Ss} = \eta p_S p_s \{\Pi_{Ss} + N_S Q(s)(1 - \lambda(S, s))\} \tag{7.3}$$

The steady state number of eligible men of status s in each period is determined by:

$$n_s = a_s + \sum_{S'} p_s \overline{p}_{S'} \{\pi_{S's} + n_s q(S')(1 - \lambda(S', s))\} + p_s n_s \{1 - \sum_{S'} q(S')(1 - \lambda(S', s))\} \tag{7.4}$$

The first term in this expression is the number of new entrants (a_s is the fraction of type s born in each period). The second term is the entry from new widowers. The third term is the contribution of unsuccessful searchers from the previous period. Similarly, after adjusting for the extra attrition of women from the market, the steady state number of eligible women of status S in each period is:

$$N_S = a_S + \sum_{s'} \eta p_S \overline{p}_{s'} \{\Pi_{Ss'} + N_S Q(s')(1 - \lambda(S, s'))\} + \eta p_S N_S \{1 - \sum_{s'} Q(s')(1 - \lambda(S, s'))\} \tag{7.5}$$

Equations (7.1) to (7.5) provide a complete description of the steady state of this marriage market.

7.2.2 Empirical methodology

In estimating the statistical (reduced form) model, we do not observe all the quantities in equations (7.1) to (7.5), such as the match probabilities, $q(S)$ and $Q(s)$. We observe π_{Ss} (the number of marriages) and other statistics which are functions of the above quantities, such as the probability of not marrying throughout an individual's lifetime and the average age at marriage. We use these statistics to recover the reduced form parameters ($\lambda(S, s)$, a_h, p_h, p_l, and η). The variables we can observe are described below.

Let z_s be the probability that an eligible man will not marry in the current period. Let Z_S be the probability that an eligible woman will not marry in the current period. Then:

$$z_s = 1 - \sum_{S'} q(S') + \sum_{S'} q(S')\lambda(S', s)$$
$$Z_S = \sum_{s'} Q(s')\lambda(S, s')$$

Thus the likelihood that a man (woman) of type s (S) will never marry, y_s (Y_S), is:

$$y_s = z_s - \frac{p_s z_s \overline{z_s}}{1 - p_s z_s} \tag{7.6}$$
$$Y_S = Z_S - \frac{\eta p_S Z_S \overline{Z_S}}{1 - \eta p_S Z_S}$$

After entering the marriage market, a type s individual will live for an average of l_s periods:

$$l_s = \frac{1}{1 - p_s}$$

The average ages of first marriage of type s males and females are (in terms of periods):

$$ma_s = \frac{1}{(1 - p_s z_s)}$$
$$MA_S = \frac{1}{(1 - \eta p_S Z_S)}$$

Let all individuals enter the marriage market at age t_0. Let one period in the model be of length δ years. Hence to express MA_S (or l_s) in years, the following conversion must be made: $(t_0 + \delta MA_S)$. δ is estimated along with the rest of the model.

Given t_0 and data on π_{Hl}/π_{Ll}, y_h, y_l, Y_h, Y_l, l_h, l_l, MA_l and a_h, we can estimate the parameters $\lambda(H, h)$, $\lambda(H, l)$, $\lambda(L, h)$, $\lambda(L, l)$, a_h, p_h, p_l, η, and δ. The model is *just identified*. To illustrate the effectiveness of the model we also report estimates for the other marriage ages (those for men and high status women) and the ratios of the number of other social combinations of marriages relative to the number of low-low marriages (e.g., π_{Hh}/π_{Ll} and π_{Lh}/π_{Ll}).

7.2.3 Reduced form estimates

We set t_0, the entry age to the marriage market, at 16. The estimated parameters are sensitive to the assumed date of entry.[26] We chose age 16 because very few girls and no boy married at earlier ages.

Table 7.1, column (3) presents point estimates and standard errors of the reduced form model. In general, the parameters are estimated quite precisely.[27] As shown in row 1, the point estimate for η is 0.988 with a standard error of 0.0008. That is, women had an additional 1.2% chance per period (1.178 years) of leaving the marriage market. This means that a single woman at age 40 would remain in the marriage market with probability 0.782. This difference in exit rates between men and women explains the difference in marriage rates between the two sexes. The estimates for p_h and p_l are 0.968 and 0.971 respectively. The survival rate calculations simply reflect the fact that high status individuals had shorter lives than low status individuals.

Row 4 shows that the calculated value of δ is 1.178 with a standard error of 0.070. Hence one period in the model was 1.178 years or an eligible woman received a match every 1.178 years.[28] This estimate seems reasonable for the duration of a trial match between a man and a woman in the marriage market.[29]

The values for probabilities of meeting a high status female and a low status female are calculated to be 0.055 and 0.478 respectively (rows 5-6). Thus an eligible male may not have met any eligible female with probability 0.467, whereas an eligible female always met an eligible male (rows 7-8). This difference in the meeting probabilities (that arises because of the gender-specific exit rates) explains the difference in marriage rates between the sexes.

The estimated rejection probabilities appear in rows 9 to 12. The estimate for $\lambda(H, h)$ is 0.695. Relative to the rejection probabilities for mixed status matches, high status individuals were less likely to reject other high status individuals when they met. There were few of these high status pair

[26] The results are fairly robust to small changes in the start age (they are not very different for age 15, for example). We present one set of estimates for brevity.

[27] Standard errors are calculated via bootstrap and the delta method.

[28] Since the length of a period is estimated and not fixed a priori, one may interpret it as the expected length of time between meetings for women. As discussed earlier, it should be interpreted as conditional on the age of entry into the marriage market (in this case, 16 years of age).

[29] The estimated length of a period is 2.18 years for entry at age 15 and 0.17 years for entry at age 17.

marriages because high status individuals were so unlikely to meet each other. The estimated rejection rates for mixed status matches, $\lambda(H, l)$ and $\lambda(L, h)$, are very high, each roughly 0.95. These high rejection rates are needed to fit the low number of observed mixed status marriages because a high status individual was quite likely to meet a low status individual. The estimated rejection rate for low status pairings is 0.787, higher than that for Hh matches. The marriage rate of low status individuals was nonetheless high because low status people were so prevalent in this society.

The estimated model generates additional moments that may be matched against the data. The model predicts approximately the same fraction of $\frac{\pi_{Lh}}{\pi_{Ll}}$ matches as that observed in the data (row 13). On the other hand, the predicted value for $\frac{\pi_{Hh}}{\pi_{Ll}}$ is a bit lower than the actual mean (0.014 versus 0.017–row 14). Regarding age at first marriage, the predicted mean age of first marriage (MAFM) for high status males is higher than for all other types, which corresponds to the pattern observed in the data (rows 15-17). The model also correctly predicts that the age of first marriage for both high status males and females is higher than that for their lower status counterparts. The predicted ages of first marriage for low and high status males are also roughly consistent with the data, but the predicted age of first marriage for high status women is much higher than observed in the data.

With the NB sample we assume that New France was a single marriage market. It is likely that the relevant marriage markets for participants were smaller. Also, the low marriage rate of high status individuals in the NB sample may be due to the fact that high status individuals in rural Quebec had difficulty meeting other high status individuals. To address these concerns, we employ the NBQ sample that includes only those individuals who were born in Quebec City.[31] While this sample has geographic advantages, it has several important disadvantages. First, it is more likely that immigrants constituted a non-trivial portion of marriage market participants. Because

[31] Quebec City was the largest city in the colony at this time, but most people lived and farmed along the St. Lawrence River near the urban centres. In 1688, 14 percent of the colony lived in Quebec City. In comparison, 12 percent of births that occurred after 1680 were registered in the city. Prior to this, when the population was smaller, a higher proportion of people lived close to Quebec City and registered their children's births there: 35 percent of births before 1680 were registered in Quebec City, and between 13 and 37 percent of the 1666 population lived in this city (the 1666 census excluded roughly one thousand Royal troops from the census; 13 and 37 percent correspond to the proportion if none or all of the troops lived in the city).

immigrants are not fully captured in these data the model may less accurately capture marriage market conditions in the city. Second, the sample size is small, especially for variables involving high status individuals.

In the NBQ sample, 10.8% of the population were high status (first row in column 2). High status individuals tended to live in urban areas. Average life spans were lower in the city than for the entire province. As noted, the unsanitary conditions of colonial cities made them relatively unhealthy places to live. The marriage rates were lower in the city than in the colony as a whole. This was especially true for men, where just over half of high-status men, and 24 percent of low-status men, never married (compared with 43 and 20 percent, respectively, for the colony as a whole). In contrast, the marriage rates for women varied little between Quebec City and the rest of the colony. A preponderance of male immigrants may have contributed to these differences.

In order to fit the lower marriage rates the model estimated η as 0.974 (with a standard error of 0.401). While the estimated value of η is statistically significant, for our purposes it is imprecise given that values both greater and less than one are possible.[32] On the other hand, the low estimated value of η (compared to the NB sample) is consistent with a heightened scarcity of women. It implies that a 40 year old single woman would remain in the marriage market with probability 0.558. In addition, the hazard of meeting a woman (0.354) was substantially smaller than that found in the NB sample. The estimate of δ implies that one period was 1.08 years (hence meetings were a bit more frequent in the city).

The rejection rate was 0.615 for Hh matches, and 0.735 for Ll matches (rows 9 and 12). The rejection rates for mixed status matches were very high (roughly 0.97) as found in the NB sample. These estimates are generally statistically significant ($\lambda(H, h)$ is an exception), although they are less precise than the NB estimates.

The estimated model underpredicts $\frac{\pi_{Lh}}{\pi_{Ll}}$ (0.007 as compared to 0.022 in the data) and overpredicts $\frac{\pi_{Hh}}{\pi_{Ll}}$ (0.047 versus 0.034). The ranking of average ages of first marriage is consistent with the data, but the model overpredicts the ages of first marriage for high status individuals.

In summary, the two samples provide slightly different but economically plausible estimates of the reduced form parameters. Since the model is just

[32]The statistical accuracy of this estimate is sensitive to the entry age. For age 15, the estimate of η is 0.952 with a standard error of 0.011.

identified, there is no way to discriminate between the two sets of estimates, although the NBQ estimates are generally less precise. The estimated models also generated predictions for other moments that are largely consistent with the data. A number of features are particularly notable. First, given the small standard errors we cannot reject the hypothesis that the value of η was less than one. This is the first evidence that a larger exit rate for women from the marriage market is significant in explaining aggregate gender differences in the marriage market. Given that this was the only gender difference in the model, it is worth stressing. Without it, the model would have predicted no differences in male and female marriage market behavior. Second, given a match, the estimates of the failure probabilities are generally larger than one-half. That is, the modal match is rejected. These estimated failure probabilities show that search friction was a significant factor in affecting marriage market behavior. Third, the estimated models also provide rankings of the average ages of first marriages for the different groups that are largely consistent with the data.

Four caveats are in order. First, the model predict declining marriage hazards for single men and women (when we do not control for status). Controlling for social status, it predicts a constant hazard for men and a declining hazard for women due to unobserved exit of single women from the marriage market. As shown in Figure 4, the empirical marriage hazards are not consistent with the theoretical hazards. How these models should be extended to incorporate more realistic marriage hazards is a topic for future research.

Second, the model predicts non-assortative matching in husbands' and wives' ages, which is counterfactual. There are many ways to generate positive assortative matching in spouses' ages. For example, age dependent survival probabilities will induce a demand for younger spouses which will lead in equilibrium to positive assortative matching in spouses' ages.

Third, although the model is able to predict the high average age of first marriage for high status males, it assumes that these males entered the marriage market at age 15. Thus most married men had long searches before they succeeded in marrying. An alternative interpretation of the data is that males do not enter the marriage market until much later. In the meantime, they tried to accumulate wealth. Those who were successful married and those who were not did not marry (Siow (1998)). Such an approach also may be able to account for the general under prediction of the average age of first marriage of low status men. This data set is not suitable for estimating such a model because it does not have any measure of the changes in individuals' wealth status. Of course, a model in which the only gender difference was the fact that men entered the marriage market later than women would yield a number of counterfactual predictions, such as a higher rate of marriage among men relative to women. In addition, younger women would be no more desirable than older women.

Fourth, we model the difference in the marriage rates and age at marriage as arising from women's higher exit rate from the marriage market. One also may be able to generate these results with a model that allowed for a differential entry rate, with men entering at a higher rate than women. The disproportionate entry of male immigrants into the colony suggests that such an interpretation may have some relevance. As noted, however, immigrants are believed to have been of minor importance in the colony after the mid-1670s. Given that the vast majority of births and certainly marriages occur after 1680, immigrant flows are not expected to introduce significant bias into the sample.[34] Working in the other direction, there were more women than men that survived to age 10. At the least, it would have taken more than 800 male immigrants and no female migrants (a substantial portion of the roughly 10,000 inhabitants in 1690) to begin swinging the gender balance in favor of males.

7.3 A behavioral framework

7.3.1 A Behavioral Model

As noted above, the statistical model is determined by eight parameters: a, p_h, p_l, η, $\lambda(H,h)$, $\lambda(H,l)$, $\lambda(L,h)$ and $\lambda(L,l)$. From an economic point of view, p_h and p_l are survival probabilities and η represents differential fecundity. These parameters may be regarded as exogenous to the marriage market. On the other hand, the failure probabilities $\lambda(H,h)$, $\lambda(H,l)$, $\lambda(L,h)$ and $\lambda(L,l)$ are endogenous variables. In what follows, we provide a simple behavioral model for determining these endogenous failure probabilities. We are highly constrained in constructing this model. It cannot have more than four structural parameters because there are only four endogenous failure probabilities. So we view it as illustrative rather than the "right" structural model of the marriage market in 18th century Quebec.

[34] Nevertheless, it may help to explain the excessive scarcity of women in Quebec City.

When an eligible man of status s and a woman of status S meet, they draw an idiosyncratic match value w from the cumulative distribution $F(w)$. Let $2w\gamma(S,s)$ be the total per period marital output to be divided by the husband and his fecund wife if they marry. $\gamma(S,s)$ is the systematic component of output that depends on the statuses of the couple. Let the man's expected per period return in marriage if his wife is fecund be $wg(S,s) + b(S,s)$. Let $wG(S,s) + B(S,s)$ be the woman's per period expected return in the marriage if she is fecund. Since total output is shared between the man and the woman if they marry, $wg(S,s) + b(S,s) + wG(S,s) + B(S,s) = 2w\gamma(S,s)$. We assume the parties involved in a match can credibly commit to transferring resources to each other after marrying. That is, if there are gains from marriage, the two individuals involved can divide the expected gains to facilitate the marriage. So all matches that result in a gain for the two parties combined will occur. For the case of Quebec, Hamilton (1999) provides micro evidence from nineteenth-century Quebec marital contracts that illustrates that potential spouses transferred resources to each other to facilitate marriages.[35] We employ a Nash bargaining solution to divide the marital output (Manser and Brown (1980) and McElroy and Horney (1981)). The appendix derives these shares explicitly.

Both individuals must agree for the marriage to occur. For analytic convenience, we assume that if the wife is menopausal each spouse receives the return that he or she derives as a single person.[37]

Let $u(s)$ be the value that a man of status s obtains from entering the marriage market. Let him meet an eligible woman of status S. Let them

[35]In the early 19th century, roughly one-quarter of couples that married in Montreal signed prenuptial contracts (around 1700 such contracts were almost universal, at least in urban areas). In these contracts couples altered the marital property and inheritance rights that accorded to a husband and wife in the absence of a contract. To illustrate, marital property laws in Quebec proscribed equal division of assets accumulated during the marriage, with the exception of real inherited assets, which each spouse held separately. A surviving spouse was entitled to half of the joint assets, but the deceased spouse's separate assets returned to his or her family (the other half of the joint assets were acquired by the children of the marriage). With a marriage contract the couple could, for example, declare all of their assets (inherited or not) part of the jointly held marital community or declare that all of their assets would be held separately. Such provisions altered the property rights of the individuals during marriage and especially after the death of a spouse.

[37]This assumption must be changed if there is divorce. As it stands, all married men would prefer to divorce their menopausal wives. We make this assumption here because a richer specification is not identified since we do not observe any divorce.

draw a match value of w. If they marry, he will get in expected present value $v(S,s,w)$:

$$v(S,s,w) = wg(S,s) + b(S,s) + p_s[p_S(\eta v(S,s,w) + \overline{\eta} u^i(s)) + \overline{p}_S u(s)], \tag{7.7}$$

where $wg(S,s) + b(S,s)$ is his current payoff if he marries. If he survives into the next period, which occurs with probability p_s, there are three mutually exclusive outcomes. His wife also survives and remains fecund. In this case, his value from marriage in the next period will be $v(S,s,w)$. This will occur with probability $p_S\eta$. He will get the expected present value $u^i(s)$ if his marriage survives but his wife becomes menopausal. This will occur with probability $p_s\overline{\eta}$. Finally, he will get $u(s)$ if his wife dies and he returns to the marriage market. This will occur with probability $\overline{p}_S$.

When his wife becomes menopausal, he gets k_s per period while he remains married. He returns to the marriage market when she dies. So:

$$\begin{aligned} u^i(s) &= k_s + p_s(p_S u^i(s) + \overline{p}_S u(s)) \\ &= \frac{k_s + p_s\overline{p}_S u(s)}{1 - p_s p_S} \end{aligned}$$

Thus (7.7) becomes:

$$v(S,s,w) = \frac{wg(S,s) + b(S,s) + p_s\overline{p}_S(1 + \frac{\overline{\eta} p_s p_S}{\overline{p_s p_S}})u(s) + \frac{\overline{\eta} p_s p_S k_s}{\overline{p_s p_S}}}{1 - \eta p_s p_S}$$

If they do not marry, he will get k_s, the per period return from being single, and $p_s u(s)$ the expected return from re-entering the marriage market in the next period.

Assuming that he wants to maximize the expected present value of marital consumption, he will choose:

$$u(S,s,w) = \max[v(S,s,w), k_s + p_s u(s)] \tag{7.8}$$

His reservation match value, $\omega(S,s)$, is defined by:

$$\begin{aligned} v(S,s,\omega(S,s)) &= k_s + p_s u(s) \qquad (7.9) \\ \omega(S,s) &= \frac{k_s(1 - \frac{p_s p_S \overline{\eta p_S p_s}}{\overline{p_s p_S}}) + u(s)\frac{p_s p_S \overline{\eta p_S p_s}\ \overline{p_s}}{\overline{p_s p_S}} - b(S,s)}{g(S,s)} \end{aligned}$$

Let $U(S)$ be the value that a fecund woman of status S obtains from entering the marriage market. Let k_S be her per period gain from being single.

Using the same reasoning as before, when a woman of type S meets a man of type s and draws a match value of w, the woman's gain from marriage is:

$$V(S,s,w) = wG(S,s) + B(S,s) + \eta p_S[p_s V(S,s,w) + \overline{p}_s U(S)] + \frac{\overline{\eta} p_S k_S}{1-p_S}$$

The last term in this expression is her payoff when she becomes menopausal. It includes only her per period return from being single because she will not return to the marriage market as a menopausal widow. To maximize utility, she will choose:

$$U(S,s,w) = \max[V(S,s,w), k_S + p_S(\eta U(S) + \frac{\overline{\eta} k_S}{1-p_S})] \tag{7.10}$$

Her reservation match value, $\Omega(S,s)$, is defined by:

$$V(S,s,\Omega(S,s)) = k_S + p_S(\eta U(S) + \frac{\overline{\eta} k_S}{1-p_S}) \tag{7.11}$$

$$\Omega(S,s) = \frac{k_S(\overline{\eta p_S p_s}\frac{\overline{\eta p_S}}{\overline{p_S}} - \frac{\overline{\eta} p_S}{\overline{p_S}}) + U(S)\overline{\eta p}_S \eta p_S p_s - B(S,s)}{G(S,s)}$$

The binding reservation match value for an Ss match is then:

$$\underline{w(S,s)} = \max[\omega(S,s), \Omega(S,s)] \tag{7.12}$$

The expected value of an Ss match for a man is then:

$$x(S,s) = F(\underline{w(S,s)})(k_s + p_s u(s)) \tag{7.13}$$
$$+ \frac{\int_{\underline{w(S,s)}}(wg(S,s) + b(S,s) + p_s\overline{p}_S(1 + \frac{\overline{\eta} p_s p_S}{\overline{p_s p_S}})u(s) + \frac{\overline{\eta} p_s p_S k_s}{\overline{p_s p_S}})dF(w)}{1-\eta p_s p_S}$$

The expected value of an Ss match for a woman is:

$$X(S,s) = F(\underline{w(S,s)})(k_S + p_S(\eta U(S) + \frac{\overline{\eta} k_S}{1-p_S})) \tag{7.14}$$
$$+ \frac{\int_{\underline{w(S,s)}}(wG(S,s) + B(S,s) + \eta p_S \overline{p}_s U(S) + \frac{\overline{\eta} p_S k_S}{1-p_S})dF(w)}{1-\eta p_S p_s}$$

Let $\widehat{q(S')}$ be an eligible man's subjective probability that he will meet an eligible woman of type S'. An eligible man will not meet any woman with subjective probability $(1-\sum_{S'} \widehat{q(S')})$. Then the expected utility of an eligible man of type s is:

$$u(s) = (1 - \sum_{S'} \widehat{q(S')})(k_s + p_s u(s)) + \sum_{S'} \widehat{q(S')} x(S', s) \tag{7.15}$$

Let $\widehat{Q(s')}$ be an eligible woman's subjective probability that she will meet an eligible man of type s'. Since women are scarce, we assume that every eligible woman will meet a man in each period. Then the expected utility of an eligible woman of type S is:

$$U(S) = \sum_{s'} \widehat{Q(s')} X(S, s') \tag{7.16}$$

We can substitute the values of $b(S,s)$, $B(S,s)$, $g(S,s)$ and $G(S,s)$ from the appendix into (7.9) and (7.11) to get the binding reservation match value:

$$\omega(S,s) = \Omega(S,s) = \underline{w(S,s)} \tag{7.17}$$

$$= \frac{k_s\left(1 - \frac{p_s p_S \overline{\eta p_S p_s}}{\overline{p_s p_S}}\right) + k_S\left(\overline{\eta p_S p_s}\frac{\overline{\eta p_S}}{\overline{p_S}} - \frac{\overline{\eta} p_S}{\overline{p_S}}\right) + \left(u(s)\frac{\overline{\eta p_S p_s}\ \overline{p_s}}{\overline{p_s p_S}} + U(S)\overline{\eta p}_S \eta\right) p_S p_s}{2\gamma(S,s)}$$

In the discussion thus far, individuals take the matching probabilities, $q(S)$ and $Q(s)$, as given. These matching probabilities in turn are determined by the aggregation of the behavior of these individuals. The next section describes how we compute the marriage market equilibrium in such a model.

7.3.2 Marriage market equilibrium

Let individuals' subjective steady state matching probabilities be $\widehat{q(S)}$ and $\widehat{Q(s)}$. Given these subjective matching probabilities, $\underline{w(S,s)}$, the binding reservation match values, are determined as in the previous section. Substitute $F(\underline{w(S,s)})$ for $\lambda(S,s)$ in equations (7.1) to (7.5). Given $F(\underline{w(S,s)})$, a_h, p_h, p_l and η, equations (7.1) to (7.5) may be solved to provide realized steady state values for $q(S)$ and $Q(s)$. If the realized steady state values for $q(S)$ and $Q(s)$ are equal to the individuals' expectations of those values, $\widehat{q(S)}$ and $\widehat{Q(s)}$, we have found a rational expectations marriage market equilibrium for the behavioral model.

We do not show that a marriage market equilibrium exists for all admissible parameter values. Rather, we will study the existence of market equilibria that are consistent with the reduced form estimates of the model.

7.3.3 Empirical methodology

Our approach is to employ the reduced form parameter estimates to calculate relevant parameters of the behavioral model. We estimate the systematic gains from marriage, $\gamma(H,h)$, $\gamma(H,l)$, $\gamma(L,h)$, $\gamma(L,l)$, and the per period returns to being single, k_h and k_l. We assume that $F(w)$ is the standard uniform distribution, hence $\lambda(S,s) = F(\underline{w(S,s)}) = \underline{w(S,s)}$. Thus no parameter of the distribution has to be estimated. Since there are only four reservation match values $\underline{w(S,s)}$, we can estimate at most four behavioral parameters. Thus these parameters must be restricted such that there will be at most four unknown parameters. We set $k_l = 1$ and $\gamma(H,l) = \gamma(L,h)$. Using equation (7.17) as the reservation match value, the remaining unknown behavioral parameters, $\gamma(H,h)$, $\gamma(H,l) = \gamma(L,h)$, $\gamma(L,l)$, and k_h are linear in the reduced form parameters (see equations 7.9, 7.11, and 7.13 to 7.16). Thus the estimates will be unique.

If there is complementarity in marital production, as is implied by assortative matching, then:

$$\gamma(H,h) > \gamma(H,l) = \gamma(L,h) > \gamma(L,l) \tag{7.18}$$

and

$$\gamma(H,h) + \gamma(L,l) > 2\gamma(H,l) \tag{7.19}$$

As outlined in section 7.3.1, $g(S,s)$, $G(S,s)$, $b(S,s)$ and $B(S,s)$ are determined by the Nash Bargaining solution. This solution is fully characterized by the previously discussed parameters and does not add additional unknown parameters. Thus the behavioral model is in principle identified.

7.3.4 Structural estimates

Table 7.3, column (1) presents parameter estimates for the model for the NB sample. Because $k_l = 1$, all parameter estimates are interpreted relative to the per period return of a single low status individual. The estimate for k_h is 12.95 (with a standard error of 4.20). This means that single high status individuals had a much higher per period payoff than single low status individuals. The estimate for $\gamma(H, h)$ is 20.03, compared to 8.06 for $\gamma(H, l)$. Thus there was a relatively large systematic loss for a high status individual who married a low status individual rather than another high status individual. This large loss explains the reluctance of high status individuals to marrying down and also explains their relatively low marriage rates. On the other hand, the estimate for $\gamma(L, l)$ is 1.81 (with a standard error of 0.06), so low status individuals gained significantly from marrying up. Our estimates of the systematic returns to marriage satisfies complementarity in marital production, (7.18) and (7.19).

Because high status individuals lost and low status individuals gained in a mixed-status marriage, there were transfers from the low status individuals to the high status individuals (rows 6 and 7). On the other hand, the transfers involved in own-status matches were much smaller. In both Hh and Ll matches, men paid small transfers to women in marriage, which implies that women had higher outside options than men.

In terms of expected lifetime discounted consumption, rows 9 to 12 show that high status individuals were distinctly better off than low status individuals, with utility levels over nine times higher than that of their low status neighbors. Holding status constant, men entering the marriage market enjoyed slightly lower utilities than women. Thus even though men were always fecund relative to women, the relative scarcity of women benefited women.[40]

Table 3, column (2) provides structural estimates for the NBQ sample. The point estimate for k_h is 2.62. Not only is it much lower than in the NB sample, but with a standard error of 2.19, the per period return of a single low status individual is not necessarily lower than that of a high status individual. The estimated lifetime utilities of low status individuals, however, are roughly half that of high status individuals. The NBQ results satisfy marital complementarity, (7.18) and (7.19), but some of the estimates are imprecise. The estimate of the systematic return to a low-low marriage, $\gamma(L, l)$, is 2.11 (with a standard error of 1.49), while the estimate of $\gamma(H, l)$ is 2.37 (with a standard error of 0.38). Hence the gain to low status individuals from marrying up is positive but small. The loss to high status individuals from marrying down is clearer, given that the estimate of $\gamma(H, h)$ is 5.02, with a standard error of 1.53.[41] The smaller variation in the gains from

[40] This welfare comparison is conditional on the assumed market structure.

[41] The structural results are sensitive to the entry age. If people enter at age 15, marital

marriage in the NBQ sample produces smaller estimated transfers from the low to the high status individual in mixed status pairings. Men continue to make small transfers to women in own-status marriages. Their lifetime utility was also marginally lower than women's utility, consistent with the NB results.

Thus across both samples, high status individuals had low marriage rates because they had little to gain from marriage, and the gains to Hh matches far outweighed mixed-status matches. There were systematic transfers between spouses in these marriages. Controlling for status, fecund women fared marginally better than men. Men fared better than menopausal women.

Our approach provides a new interpretation of women's lower average age of first marriage. Impending menopause will lower women's outside options (ceteris paribus), but it also makes women relatively scarce in the marriage market. This improves their chances of meeting a mate (thus raising their outside options). Thus, whether differential fecundity lowers women's average age of first marriage relative to men is theoretically ambiguous. Our structural estimates show that on balance, women had higher outside options than men in the marriage market.

7.4 Bibilographic notes

This chapter is complementary to other recent empirical equilibrium random matching models of marriage (Aiyagari, Greenwood and Güner (2000), Seitz (1999) and Wong (2000)).[43] The common theme in these papers is the use of equilibrium random matching models to rationalize different regularities in the marriage market. Wong estimates the determinants of inter-racial marriages. Aiyagari et al. calibrate a model of marriage, divorce, work and parental investments.[44] Their focus is on the determinants of single parenthood, the distribution of income and policy simulations. Empirically,

complimentarity is violated with the per period return to a mixed status match marginally lower than in either low-low or high-high matches. Part of the problem with the NBQ sample stems from sample size. While there are over 2,000 observations in the sample, the actual number of high-high and mixed status marriages is quite small (under 25 in each case).

[43] Other empirical analyses include Becker, Landes and Michael (1977), Chiappori, Fortin and Lacroix (1998), Grossbard-Shechtman (1993), Brien, Lillard and Stern (forthcoming).

[44] Calibration models of the family are appearing rapidly. Also see Greenwood et al. (1999); Regalia and Rios-Rull (1999).

Seitz's paper is the most ambitious. Using micro data, Seitz estimates a non-stationary model of marriage, divorce and work for young adults.

7.5 Problems

1. Consider a simplification of the statistical model presented above. Assume that there is no high type individual in the society. That is, there is no class distinction. Let the number of new individuals of each gender enter the marriage market each period be 1. Let $\lambda = 0$. I.e. all matches are accepted. So this model has two unknown parameters, p and η.

(a) What are the analogs to equations (7.1) to (7.5)? Solve for n, N, π, Π as functions of p and η.

(b) Let n, the number of unmarried males in the marriage market, be 7.

Let π, the number of marriages, be 13. What is your estimate of p and η?

2. Following question 1, further, assume that θ fraction of males and θ fraction of females in every birth cohort never enter the marriage market at all. These individuals die at the same rates as individuals who enter the marriage market. So from our perspective, we cannot distinguish between an individual who never entered the marriage market from another who entered the marriage market but died before marrying.

(a) Now n_o, the number of observed unmarried males, consists of both males who are in the marriage market, n, and males who are not in the marriage market, n_θ. What are the analogs to equations (7.1) to (7.5)? What are n_θ and n_o in terms of p, η and θ?

(b) Let $\theta = 0.1$. Let $n_o = 7$ and $\pi = 13$. What is your new estimate of p and η?

(c) Since question 1 essentially sets θ to zero, in which direction are the estimates of the parameters of p and η biased if you do not know what θ is?

Explain the direction of the bias.

(d) What data from Table 1 will be useful for you to estimate θ as well?

7.6 The Nash Bargaining Solution

Let a man of status s meets a woman of status S and they draw a match value of w. Let $\beta 2w\gamma(S,s)$ be the per period marital output that the man receives in a fecund marriage. The woman will receive $(1-\beta)2w\gamma(S,s)$. If they marry, the value of marriage to the man is:

$$v(S,s,w,\beta) = \frac{\beta 2w\gamma(S,s) + p_s\overline{p}_S(1+\frac{\overline{\eta}p_s p_S}{\overline{p_s p_S}})u(s) + \frac{\overline{\eta}p_s p_S k_s}{\overline{p_s p_S}}}{1-\eta p_s p_S}$$

The value of marriage to the woman is:

$$V(S,s,w,\beta) = \frac{(1-\beta)2w\gamma(S,s) + \eta p_S\overline{p}_s U(S) + \frac{\overline{\eta}p_S k_S}{1-p_S}}{1-\eta p_S p_s}$$

Applying the Nash bargaining solution with equal bargaining power,[46] $\beta(S,s,w)$ is determined by:

$$\beta(S,s,w) = \arg\max_{\beta}[v(S,s,w,\beta)-(k_s+p_s u(s))][V(S,s,w,\beta)-(k_S\frac{\overline{\eta p_S}}{\overline{p_S}}+\eta p_S U(S))]$$

$$= \frac{1}{2} + \frac{k_s(1-\frac{p_s p_S\overline{\eta p_S p_s}}{\overline{p_s p_S}}) - k_S(\overline{\eta p_S p_s}\frac{\overline{\eta p_S}}{\overline{p_S}} - \frac{\overline{\eta}p_S}{\overline{p_S}}) + (u(s)\frac{\overline{\eta p_S p_s}\ \overline{p_s}}{\overline{p_s p_S}} - U(S)\overline{\eta p}_S\eta)p_S p_s}{4w\gamma(S,s)}$$

Thus:

$$wg(S,s)+b(S,s) = \beta(S,s,w)2w\gamma(S,s)$$

$$= w\gamma(S,s) + \frac{k_s(1-\frac{p_s p_S\overline{\eta p_S p_s}}{\overline{p_s p_S}}) - k_S(\overline{\eta p_S p_s}\frac{\overline{\eta p_S}}{\overline{p_S}} - \frac{\overline{\eta}p_S}{\overline{p_S}}) + (u(s)\frac{\overline{\eta p_S p_s}\ \overline{p_s}}{\overline{p_s p_S}} - U(S)\overline{\eta p}_S\eta)p_S p_s}{2}$$

and:

$$wG(S,s)+B(S,s) = (1-\beta(S,s,w))2w\gamma(S,s)$$

$$= w\gamma(S,s) - \frac{k_s(1-\frac{p_s p_S\overline{\eta p_S p_s}}{\overline{p_s p_S}}) - k_S(\overline{\eta p_S p_s}\frac{\overline{\eta p_S}}{\overline{p_S}} - \frac{\overline{\eta}P_S}{\overline{p_S}}) + (u(s)\frac{\overline{\eta p_S p_s}\ \overline{p_s}}{\overline{p_s p_S}} - U(S)\overline{\eta p}_S\eta)p_S p_s}{2}$$

[46]E.g. Chapter 6, Osborne and Rubinstein (1990).

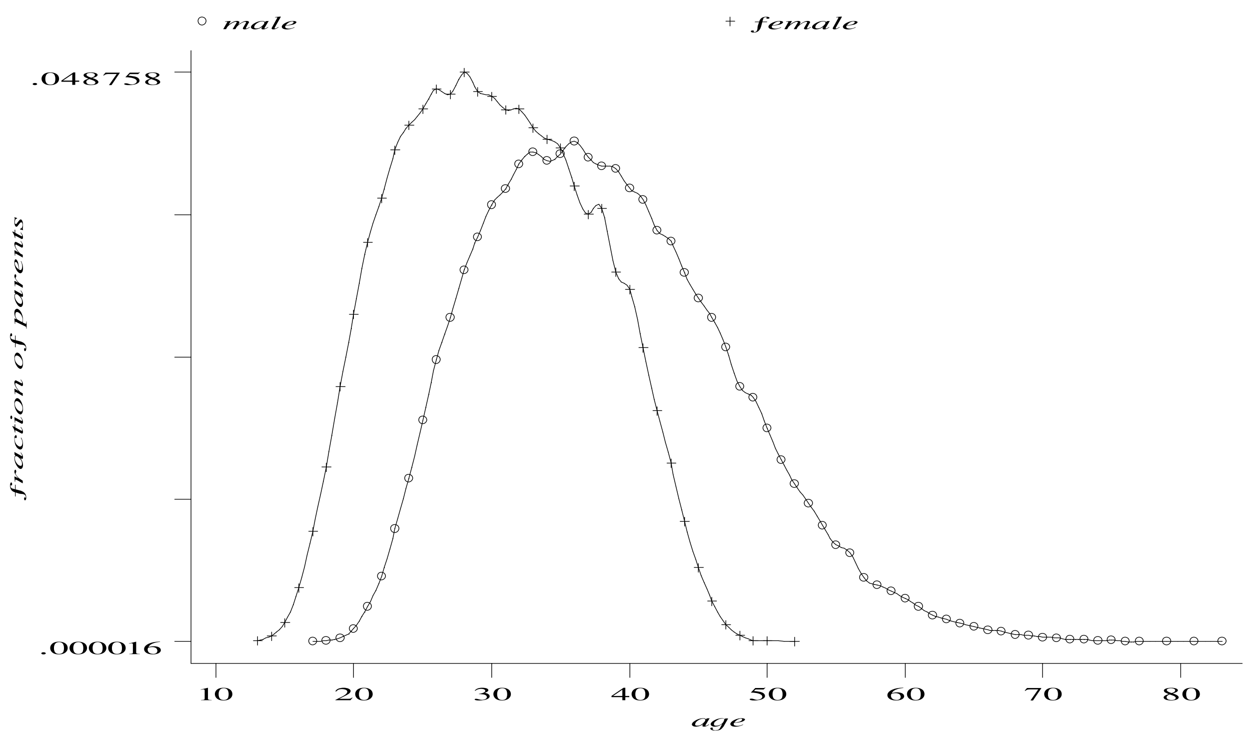

Figure 7.1: Densities of Parent's Age at Birth of Child

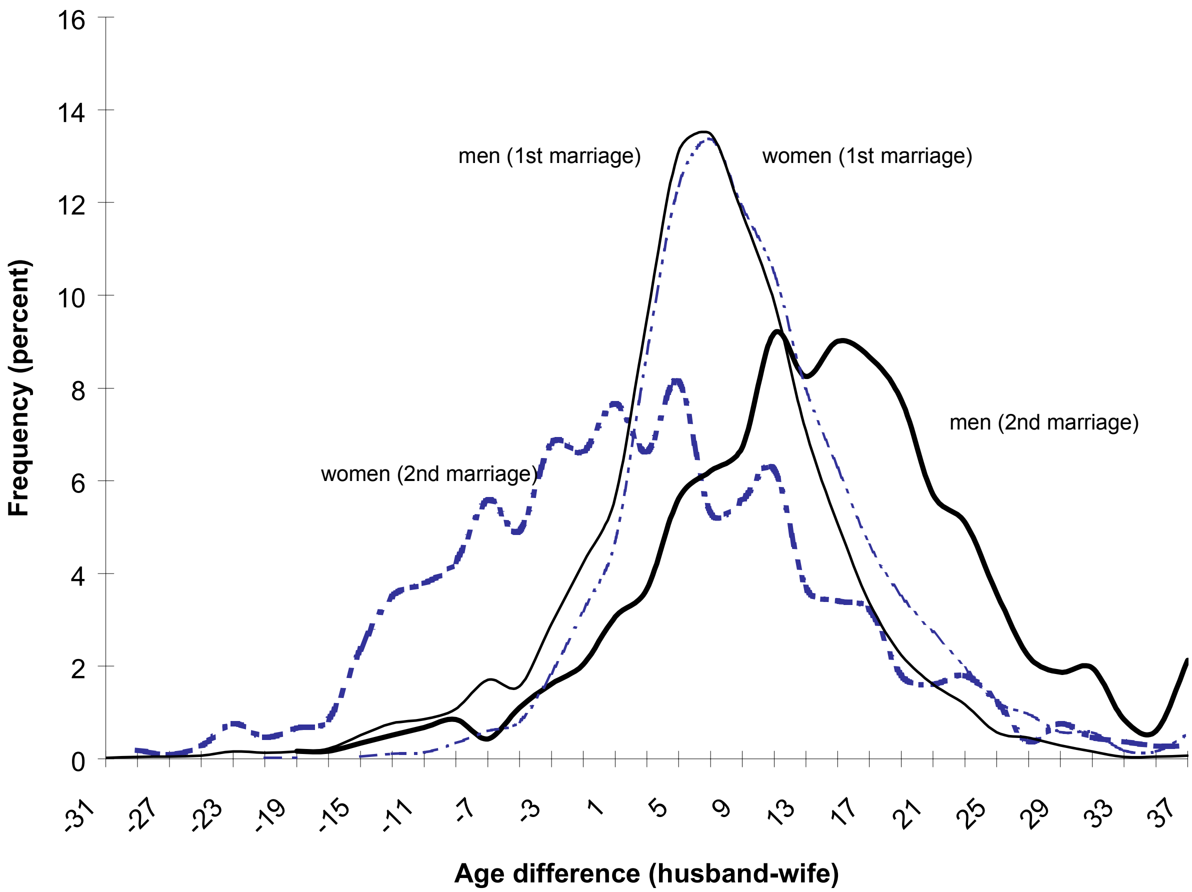

Figure 2: Spousal Age Differences: by Gender and Marriage Rank

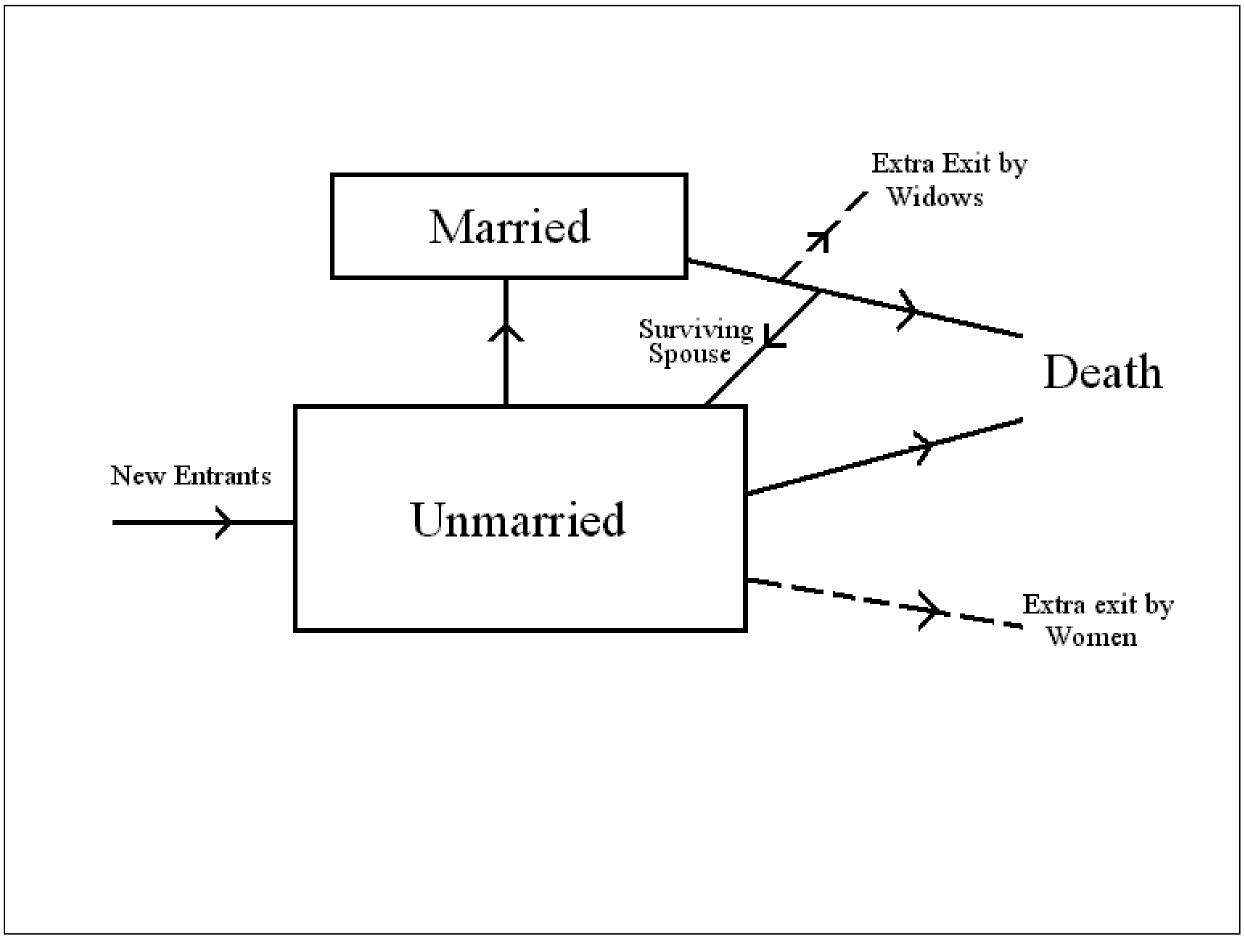

Figure 3: Flow chart of marriage market

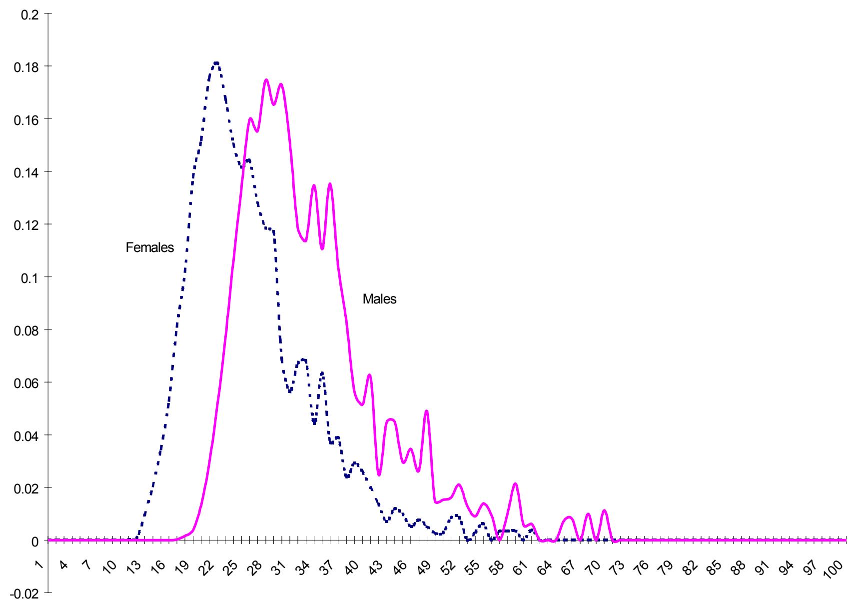

Figure 4: Marriage Hazard Rates by Gender

		Observed means			Estimated means			
		(1)	(2)		(3)		(4)	
		NB	NBQ		NB		NBQ	
(1)	% h^*	5.89	10.79	η	0.988	(0.0008)	0.974	(0.401)
(2)	Life span h^*	52.37	49.57	p_h	0.968	(0.002)	0.968	(0.035)
(3)	Life span l^*	56.74	54.75	p_l	0.971	(0.002)	0.972	(0.031)
(4)				δ	1.178	(0.070)	1.084	(1.190
(5)	% unmarried $h\ m^*$	43.31	53.91	$q(H)$	0.055		0.062	
(6)	% unmarried $h\ f^*$	37.56	37.42	$q(L)$	0.478		0.292	
(7)	% unmarried $l\ m^*$	19.84	24.45	$Q(h)$	0.086		0.150	
(8)	% unmarried $l\ f^*$	14.02	15.24	$Q(l)$	0.914		0.850	
(9)	% widowers remarry	50.38	50.31	$\lambda(H,h)$	0.695	(0.039)	0.615	(0.737)
(10)	% widows remarry	32.54	34.45	$\lambda(H,l)$	0.954	(0.004)	0.965	(0.036)
(11)				$\lambda(L,h)$	0.950	(0.005)	0.989	(0.016)
(12)	$\frac{\pi_{Hl}}{\pi_{Ll}}^*$	0.023	0.026	$\lambda(L,l)$	0.787	(0.011)	0.735	(0.227)
(13)	$\frac{\pi_{Lh}}{\pi_{Ll}}$	0.022	0.022	$\frac{\pi_{Lh}}{\pi_{Ll}}$	0.021		0.007	
(14)	$\frac{\pi_{Hh}}{\pi_{Ll}}$	0.017	0.034	$\frac{\pi_{Hh}}{\pi_{Ll}}$	0.014		0.047	
(15)	MAFM $h\ m$	30.75	30.15	MAFM $h\ m$	32.42		34.61	
(16)	MAFM $h\ f$	21.46	20.83	MAFM $h\ f$	26.75		23.77	
(17)	MAFM $l\ m$	27.35	26.96	MAFM $l\ m$	25.04		26.29	
(18)	MAFM $l\ f^*$	21.09	20.04					
(19)	N (births)	11865	2502					
(20)	N ($\pi' s$)	3316	677					
(21)	% m	46.62	44.48					

* moments used in estimation. All age related data are measured in years. m = males; f = females; h = high; π_{Ss}=number of Ss marriages; MAFM = mean age at first marriage; η = per period probability that women remain in the marriage market; p_s= per period survival probability for type s; $\lambda(S,s)$ = rejection probability for an Ss match; δ= length of a period (in years); $q(S)$= probability of a male meeting a female of type S. *Source:* Samples include individuals with known life span who lived until at least age 10. NBQ sample is restricted to those born in Quebec City. π_{Ss} sample restricted to cases in which the individual is male, in his first marriage, and both he and his spouse were born in the province.

Table 1: Reduced form estimates

	All	Female	Male
	(1)	(2)	(3)
Male widowed from first marriage	**0.314**		
	(0.018)		
No children in first marriage (NC)	**-0.159**	**-0.148**	-0.141
	(0.058)	(0.029)	(0.169)
Number of children in first marriage	**-0.006**	**-0.012**	0.009
	(0.002)	(0.003)	(0.004)
Age at first spouse's death (AGE)	**-0.024**	**-0.018**	**-0.029**
	(0.001)	(0.001)	(0.001)
First married in a city	**-0.038**	-0.020	**-0.072**
	(0.017)	(0.017)	(0.035)
Year of first marriage	**-0.004**	**-0.004**	**-0.004**
	(0.0004)	(0.0004)	(0.001)
NC $\times$ AGE	**0.004**	**0.005**	0.004
	(0.002)	(0.002)	(0.004)
Signed first marriage record	0.031	-0.015	**0.112**
	(0.020)	(0.020)	(0.036)
Noble parents	-0.090	-0.099	0.001
	(0.050)	(0.037)	(0.118)
Bourgeois parents	**-0.089**	**-0.073**	-0.076
	(0.035)	(0.030)	(0.089)
F-test on high status variables	**0.040**	**0.039**	**0.021**
N	4609	2923	1686
Pseudo R-squared	0.409	0.425	0.361

The dependent variable equals 1 if the individual remarried, 0 otherwise. Values reported are maximum likelihood probit estimates of the change in probability of a one-unit change in the independent variable, evaluated at the means of the independent variables. Bold type indicates significance at the 5% level. White corrected standard errors are in parentheses. The F-test tests that the high-status coefficients (noble parents, bourgeois parents, and signing) are jointly zero. *Source:* The sample consists of individuals known to have been widowed from their first marriage, subsequent remarriages are ignored.

Table 2: Incidence of Remarriage: probit estimates

		Sample			
		(1) NB		(2) NBQ	
(1)	k_h	12.95	(4.20)	2.62	(2.19)
(2)	$\gamma(H,h)$	20.03	(5.92)	5.02	(1.53)
(3)	$\gamma(H,l)$	8.06	(2.32)	2.37	(0.38)
(4)	$\gamma(L,l)$	1.81	(0.06)	2.11	(1.49)
(5)	$b(H,h)$	-0.13		-0.13	
(6)	$b(H,l)$	-6.36		-0.93	
(7)	$b(L,h)$	6.14		0.62	
(8)	$b(L,l)$	-0.10		-0.18	
(9)	$U(h)$	431.92		93.18	
(10)	$u(h)$	428.35		90.02	
(11)	$U(l)$	48.29		50.51	
(12)	$u(l)$	44.61		45.42	

Table 3: Unequal sharing estimates