2 Gender Neutral Marriages I: Static Considerations

When a couple marry, the married household produces different kinds of marital output. Examples of marital output include the incomes (market goods) that are earned by the husband and wife, children that they have, housework and so on. Some of this output can be divided between the two spouses and consumed separately. These divisible output are private goods. For example, food eaten by one spouse cannot be eaten by the other spouse. So holding the total divisible output constant, more divisible output for one spouse imply less divisible output for the other spouse.

On the other hand, there are other forms of marital output that are not divisible. For example, as discussed in the previous chapter, children are local public goods in marriage.[1]

Since a marriage produces both private goods and local public goods, an individual may attract a spouse by promising to allocate more of the divisible marital output to the potential spouse. If the promise is credible, this promised allocation may result in a marriage that may otherwise not occur. Moreover, changing marriage market conditions may result in changes in the initial allocation of divisible marital output between spouses.

In most of this book, we will study models of the marriage market in which we assume that the promised division of marital output is feasible and credible. Occasionally, we will also study marriage markets in which the promised division of marital output is not feasible or credible.

[1] Sometimes, an individual may exclude his or her spouse from consuming the local public good. For example, a couple may not want to watch a rental video together even though it is feasible for both of them to watch the same rental video together without additional cost. Since the main local public good in marriage are children, we will assume that the other spouse cannot be excluded from consuming the marital public goods in marriage.

2.1 Monagamy without matching

The objective of this model is to describe a marriage market which behaves as closely as possible to a textbook model of a competitive industry. For each gender, most marriage markets have many participants who compete with each other for spouses. Thus a primary assumption of a competitive industry, many participants on both sides of a market, is satisfied.

Consider a marriage market with M available men and F available women. Each woman can marry at most one man and vice versa. There is no divorce and no remarriage even if a spouse dies. While different women have different attitudes toward marriage, they are indifferent to whom they may marry. Similarly, different men have different attitudes toward marriage but they are also indifferent to whom they may marry.

Since all individuals of the opposite gender are perfect substitutes as spouses, there is no matching in this marriage market. That is, we do not have to keep track of who marries whom. Matching in marriage is an important consideration and we will discuss matching considerations later.

Although individuals do not care to whom they are married to, each of them have to choose whether to marry or not. The number of males who want to marry may not be equal to the number of females who want to marry. How does the marriage market clear? That is, if the number of men and women who want to marry are not the same, who gets to marry and who will remain single?

In this model, we will assume that the marriage market clears with a transfer τ, paid by a husband to his wife. In other words, if a man wants to marry, he must transfer some resources, τ, to her. τ may be a positive or negative number. If τ is negative, then the wife is transferring resources to her husband. So when we say that a man pays a transfer in marriage, this is purely a convention, with no normative (welfare) or positive (predictive) consequence.[2] Each individual in the marriage market takes τ as given. That is, they do not believe that their particular decision to marry or not will affect τ. We expect that as τ increases, the number of men who will want to marry will fall whereas the number of women who will want to marry will increase. Thus there may be values of τ in which the numbers of men who want to marry will be equal to the numbers of women who want to marry. Our objective is to solve for

[2] In this book, we will follow the convention that men are demanders in the marriage market and women are suppliers. This convention will have no behavioral or welfare consequence.

these market clearing values of τ and the numbers of marriages when the market clears.

Since different individuals have different attitudes toward marriage, let different individuals derive different utilities from being single. For male m, let his utility from being unmarried be s_m. Let his gross utility from being married be α_m. He also has to transfer τ units of resources to a woman if he wants to marry her. We will assume that one unit of resource transferred is one unit of utility less for the man. After substracting the utility cost of the transfer, his net utility from marriage is:

$$\alpha_m - \tau \tag{2.1.1}$$

His net gain, in utility terms, from marriage relative to remaining single, n_m, is:

$$n_m = \alpha_m - s_m - \tau \tag{2.1.2}$$

$$= y_m - \tau \tag{2.1.3}$$

y_m is the gross gain from marriage which is independent of τ. The net gain from marriage, n_m, is a function of τ. As τ increases, n_m will fall. For τ sufficiently large, n_m, the net gain to marriage will be negative.

We index the males such that the gross gain from marriage, y_m, is decreasing as the index m increases. m ranges from 0 to M. Assume that the distribution of y_m is continuous in m. Then $y_{m'} < y_m$ for $m' > m$.

Similarly, for female f, let her utility from remaining unmarried be S_f. Let her gross utility of being married be γ_f. She receives τ units of resources from a man if she is willing to marry him. Let her gain in utility terms from the transfer τ be τ. Adding the utility benefit of the transfer, her net utility from being married is:

$$\gamma_f + \tau \tag{2.1.4}$$

Her net gain, in utility terms, from marriage relative to remaining single, N_f, is:

$$N_f = \gamma_f - S_f + \tau \tag{2.1.5}$$

$$= Y_f + \tau$$

Y_f, the gross gain from marriage is independent of τ. The net gain, N_f, is a function of τ. As τ increases, N_f will increase. We index the females such that the gross gain from marriage, Y_f, is increasing as the index f increases. f ranges from 0 to F. Let the distribution of Y_f be continuous in f. So $Y_{f'} > Y_f$ for $f' > f$.

How is τ determined? Who will marry and who will remain single?

2.1.1 Marriage market clearing

We begin the analysis by constructing the supply of women to the marriage market as follows. We assume that every woman will choose to marry or not in order to maximize her utility from her action.

For a given value of τ, the woman f will marry only if

$$N_f = Y_f + \tau \geq 0$$

Given her optimal behavior, her utility, U_f, will be:

$$\begin{aligned} U_f &= \max[\gamma_f + \tau, S_f] \\ &= \max[N_f, 0] + S_f \end{aligned}$$

The above equation tells us that the woman f will receive a minimum utility of S_f . In general, her utility, if she marries, will be higher. The marginal woman is one who is indifferent between marriage and remaining unmarried. Let $\underline{f}$ index the marginal woman. Then for the marginal woman,

$$N_{\underline{f}} = 0 \Rightarrow Y_{\underline{f}} = -\tau$$

And

$$N_{\underline{f}+h} > 0 > N_{\underline{f}-k} \; ; \; h, k > 0$$

Given τ, $F - \underline{f}$ is the number of women who are willing to marry. Since N_f is increasing in τ, as τ increases, $\underline{f}$ decreases and more women will want to marry. Figure (2.1) shows the supply curve of wives (SS) as a function of τ. τ is measured on the vertical axis. $F - \underline{f}$, the number of married women is measured on the horizontal axis.

As drawn in the figure, the gross gains from marriage, $Y_f < 0$ for all f. Since the gross gains from marriage are negative, women require a positive τ to enter the marriage market.[3] As Y_f is increasing in f,

[3] Some women will be willing to enter the marriage market for a negative τ if the SS curve intersects the vertical axis below $\tau = 0$.

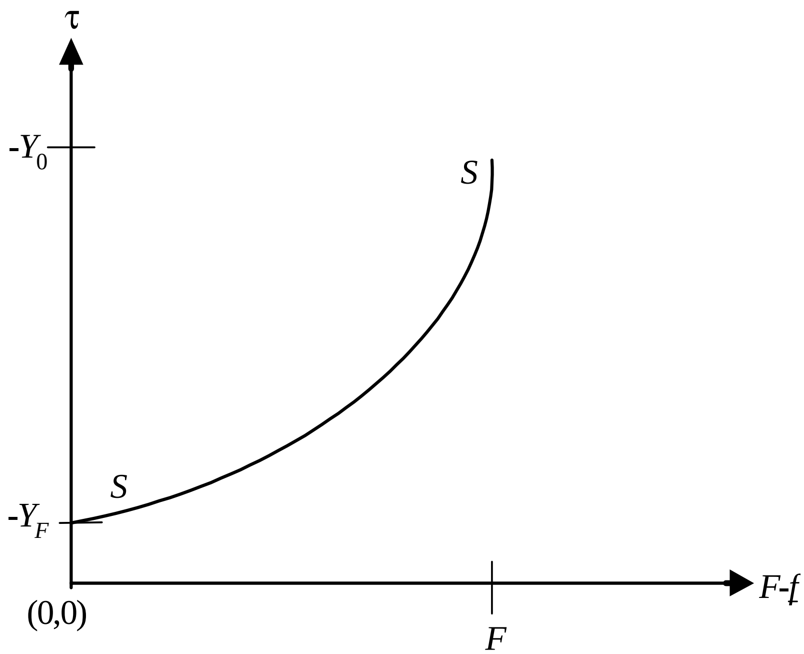

Figure 2.1.: Supply of women to marriage market.

the first woman will enter when $\tau = -Y_F$. As τ increases, $\underline{f}$ decreases. The supply of women to the marriage market ends when $\tau = -Y_0$ and $\underline{f} = 0$. Other than upward sloping, the particular shape of the supply curve depends on the distribution of the gross gains to marriage. Since we have not specified a particular distribution for Y_f, the exact shape of the supply curve, SS, in figure (2.1) has no significance.

Similarly, for a given value of τ, the man m will marry only if

$$n_m = y_m - \tau \geq 0$$

Given his optimal behavior, his utility, u_m, will be:

$$\begin{aligned} u_m &= \max[\alpha_m - \tau, s_m] \\ &= \max[n_m, 0] + s_m \end{aligned}$$

The marginal man is one who is indifferent between marriage and remaining unmarried. We will assume that the marginal man will marry. Let $\underline{m}$ index the marginal man. Then

$$n_{\underline{m}} = 0 \Rightarrow y_{\underline{m}} = \tau \qquad (2.1.6)$$

$$n_{\underline{m}-k} > 0 > n_{\underline{m}+h} \ ; \ h, k > 0 \qquad (2.1.7)$$

Given τ, $\underline{m}$ is also the number of men who want to marry. As τ decreases, $\underline{m}$ increases and more men will want to marry. Figure (2.2) shows the demand curve for wives (DD) as a function of τ. Since y_m is decreasing in m, the first man will enter when $\tau = y_0$. The demand curve ends at $\tau = y_M$ and $\underline{m} = M$. Since we did not specify a particular distribution for y_m, other than downward sloping, there is no other restriction on DD.[4]

In order to find the equilibrium value of τ, we find the intersection of the demand and supply curves as in a standard textbook discussion of a competitive market. Figure (2.3) shows the intersection of the demand and supply curves in the marriage market. As shown in the figure, the equilibrium or market clearing value of τ is τ^*. The number of married males which also equals the number of married females is $\mu^* = \underline{m}^* = F - \underline{f}^*$. The number of unmarried males is $M - \mu^* = M - \underline{m}^*$ and the number of unmarried females is $F - \mu^* = \underline{f}^*$.

The model does not predict whether τ^* is positive or negative. In figure (2.3), τ^* is positive but this is simply illustrative of a possible equilibrium.

So far, the model has minimal empirical content. It predicts that there will be some married individuals and some unmarried individuals. In order to generate some empirical content, we will do some comparative static exercises with the model.

First, consider an exogenous increase in the number of men to the marriage market from M to M'. This will increase the sex ratio, which is the ratio of available men to available women from $\frac{M}{F}$ to $\frac{M'}{F}$. Let the increase in men be distributed across all the different types of men. That is, for every y_m type, there are more available men of that type than before. So the demand curve for marriage shifts to the right from DD to $D'D'$ in Figure (2.4). The equilibrium value of τ increases from τ^* to τ'. Married women now derive a large share of the marital output. As a result, more women are induced to marry. The number of marriages increases from μ^* to μ'. The marriage rate for women, $\frac{\mu}{F}$, increases

[4] Some men will require a negative τ to enter the marriage market if the curve DD intersects the horizontal axis before M.

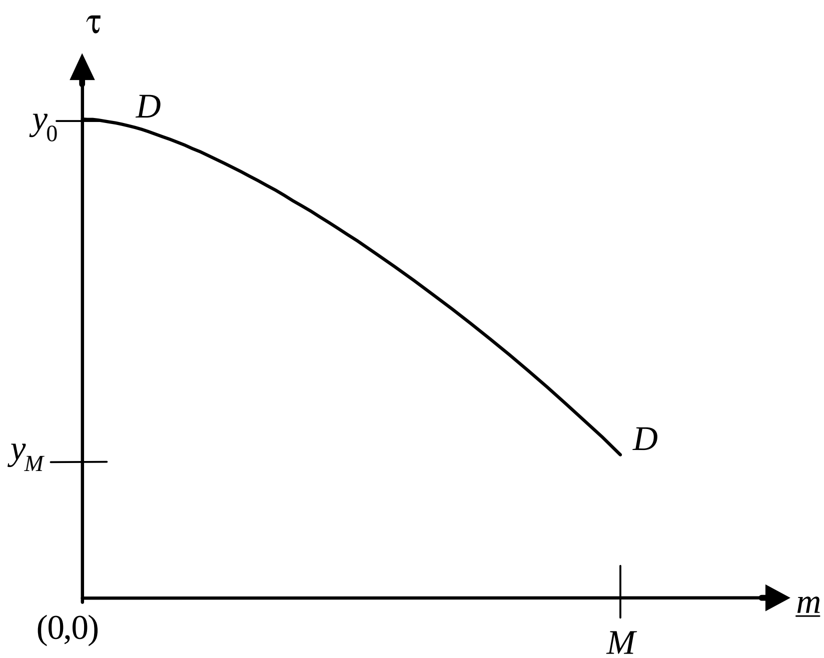

Figure 2.2.: Demand of men in marriage market

from $\frac{\mu^*}{F}$ to $\frac{\mu'}{F}$. However the effect on the marriage rate for men, $\frac{\mu}{M}$, is ambiguous. μ increases from μ^* to μ' but M also increases to M'.

Now consider an exogenous decrease in the number of available men to the marriage market. This exogenous decrease will reduce the sex ratio. Similar to the argument above, the exogenous decrease in the number of available men will decrease the equilibrium value of τ and also decrease the number of marriages. The marriage rate for women will decrease. Again the change in the marriage rate for men is ambiguous.

Similarly, an increase in the supply of available women to the marriage market will decrease the sex ratio, decrease the equilibrium value of τ, increase the number of marriages, increase the marriage rate for men and have an ambiguous effect on the marriage rate for women. Finally, a decrease in the supply of available women to the marriage market will increase the sex ratio, increase the equilibrium value of τ, decrease the number of marriages, decrease the marriage rate for men and have an ambiguous effect on the marriage rate for men.

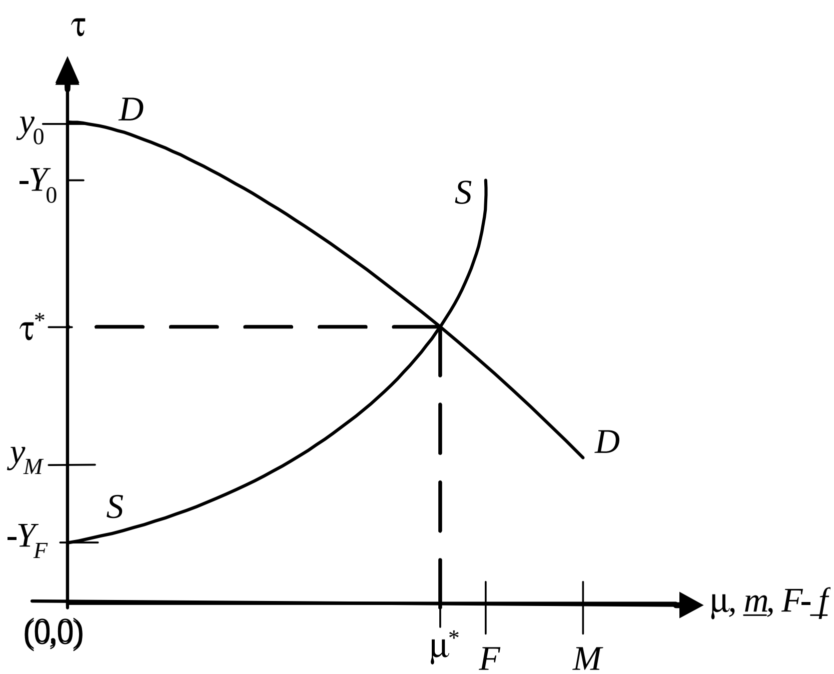

Figure 2.3.: Equilibrium in marriage market.

Comparing the four different comparative static results, an increase in the sex ratio increases the equilibrium value of τ, the transfer that husbands make to their wives. A decrease in the sex ratio decreases the equilibrium value of τ. The effects of changes in the sex ratio on the number of marriages and marriage rates are ambiguous.

2.1.2 Social welfare

Other than the marginal man and woman, all the other married men and women strictly prefer to be married. Their utilities from marriage are strictly larger than remaining unmarried. In economic terminology, we say that they derive rents from marriage. The measure of rent for a married woman, f, is N_f, the net gain from marriage. Similarly, the measure of rent for a married man, m, is n_m.

All the unmarried men and women also strictly prefer to remain unmarried. These men and women derive rent from remaining unmarried.

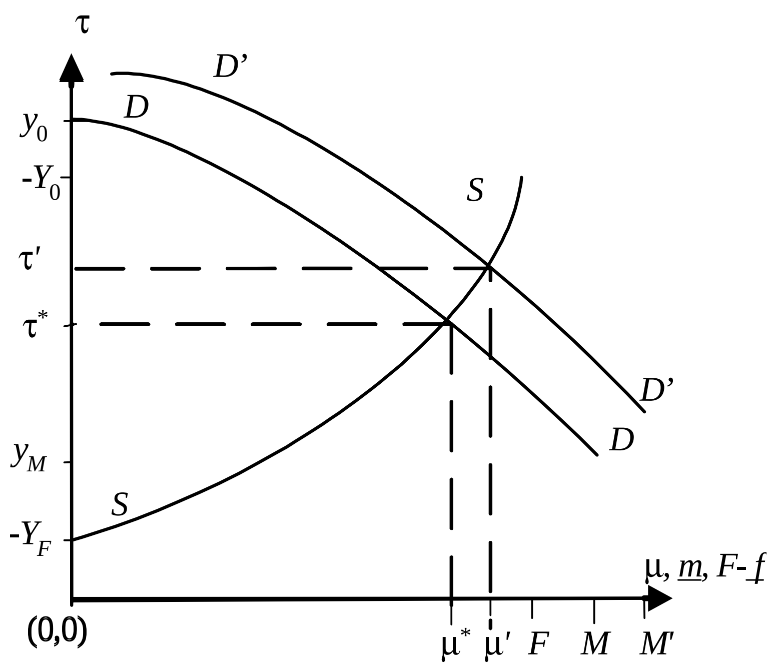

Figure 2.4.: An increase in the number of available men.

The measures of their rent are $-N_{f'}$ for unmarried female f' and $-n_{m'}$ for unmarried male m'.

If woman f marries, we know that she prefers marrying to not marrying. But we do not know the level of utility that is achieved by that individual. Similarly for a woman f' who chooses not to marry, we know that her utility is higher from remaining single than otherwise. But we do not know the level of utility that she obtains. This is also true for men.

While the marginal man or woman earns no rent in marriage, there is no reason to assume that the aggregate amount of rent from marriage is evenly distributed between gender. Even if we can observe τ^*, it would not be informative about the gender difference in the distribution of the aggregate amount of rents from marriage.

What then can we know about aggregate welfare in this society? Consider a utilitarian social welfare function, W, which is the sum of the utilities of all individuals in the society:

$$W = \sum_{0}^{M}(\iota_m \alpha_m + (1-\iota_m)s_m) + \sum_{0}^{F}(\iota_f \gamma_f + (1-\iota_f)S_f) \qquad (2.1.8)$$

where ι_m is an indicator function which equals one if male m is married and zero otherwise. And ι_f is an indicator function which equals one if female f is married and zero otherwise. Depending on who is assigned to be married and who is assigned to be single, W shows the aggregate welfare achieved by that society. W ignores transfers between individuals which are purely redistributive and do not affect aggregate welfare.

We will show that our marriage market equilibrium maximizes the social welfare function, W. Applying (2.1.8) to our equilibrium marital matches, the aggregate welfare that is achieved, W^*, is:

$$W^* = \sum_{0}^{\mu^*}\alpha_m + \sum_{\mu^*}^{M} s_m + \sum_{0}^{F-\mu^*} S_f + \sum_{F-\mu^*}^{F} \gamma_f$$

Equilibrium transfers, τ^*, do not appear in the above expression because the transfers are purely redistributive. For every married couple, the husband will prefer a lower τ^* and his wife will prefer the reverse. But the fall in utility of the husband is offset exactly by the increase in the utility of his wife. So whatever the value of τ^*, W^* is unaffected. With marriage market clearing, the number of married men is equal to the number of married women, , $\underline{m}^* = F - \underline{f}^* = \mu^*$.

We will now show that we cannot rearrange the marrieds and the unmarrieds to increase W over W^*. Any other configuration of matches may involve the following changes: (1) Switching an umarried male m' with a married male $m^{\#}$, (2) Switching an umarried female with a married female. (3) Marrying two previously unmarried individuals. (4) Dissolving a marriage of two previously married couple.

First, holding the number of marrieds constant, consider switching an unmarried male, m', with a married male, $m^{\#}$. The change in social welfare will be

$$[\alpha_{m'} - s_{m'}] - [\alpha_{m^{\#}} - s_{m^{\#}}] = y_{m'} - y_{m^{\#}} < 0$$

The change in social welfare is negative because $y_{m^{\#}} > y_{m'}$ from (2.1.7). Similarly, we cannot increase social welfare by switching an unmarried female with a married female. The generalization to making multiple simultaneous switches is straightforward.

What about increasing the number of marrieds? Consider marrying an unmarried male, m', and an unmarried female, f' . The change in social welfare is:

$$[\alpha_{m'} - s_{m'}] + [\gamma_{f'} - S_f] = [(\alpha_{m'} - \tau^*) - s_{m'}] + [(\gamma_{f'} + \tau^*) - S_{f'}]$$
$$= n_{m'} + N_{f'} < 0$$

The change in social welfare is negative because $n_{m'}$ and $N_{f'}$ are negative for all individuals who choose to remain unmarried. Similarly, we cannot increase social welfare by switching a married man and woman to being unmarried. Since no *other* man or woman is affected by these kinds of switches, making multiple simultaneous switches will also not increase social welfare.

We have shown a special case of a general result that a transferable utility model of the marriage market maximizes the sum of individual utilities, the utilitarian social welfare function, in the society (E.g. Roth and Sotomayor 1992; Chapter 8).

2.1.3 A sex ratio of 1

In this model, there is nothing special about a sex ratio of 1, due to the same number of available men and women. Even with a sex ratio of 1, τ^*, the equilibrium value of τ is uniquely determined and there will be married and unmarried individuals. Moreover the equilibrium value of τ changes continuously for small changes in the sex ratio away from 1. That is, τ^* does not change discretely for small deviations in the sex ratio away from 1.

2.1.4 Empirical evidence

There are many scholarly articles which studied the empirical correlation between female labor supply and the sex ratio across different marriage markets. Assuming that leisure is a normal good, an increase in resources available to an individual, holding all other factors constant, will lead the individual to consume more leisure and work less. The above model predicts that an increase in the sex ratio will increase the share of marital output given to wives in that marriage market. Those wives should increase their consumption of leisure and reduce their hours of work in response to the increase in their share of marital output. Single women in that marriage market may even reduce their hours of work in anticipation of a larger share of the marital output when they marry. Thus the theory predicts that observed proxies for female labor supply should be negatively correlated with the sex ratio across marriage markets.

Most studies of this correlation construct one sex ratio for each marriage market.[5] For example, if the marriage market is a city at a point in time, the empirical sex ratio is the ratio of men aged 20-30 to women aged 18-28 in that city at that time. The average age for women is lower than that for men in constructing the sex ratio in recognition of the fact that most men marry younger women.[6] Since there is only one sex ratio for each marriage market, the researcher is assuming that all individuals of the same gender in that age range are close substitutes in marriage.

Some studies measure female labor supply with the employment rate and or average hours of work for women in the age range for which the sex ratio was constructed. Other studies use individual level data and study the labor supply of individual women.

Grossbard-Shectman 1993, South and Lloyd 1992, South and Trent 1988, Chiappori, et. al. 2001, Angrist 2002 have all studied the correlation between female labor supply and the sex ratio.[7] The studies differ primarily in their definitions of a marriage market (within country across time, across states at a point in time, across cities at a point in time, across countries, across ethnic groups), and whether they study individual level or market level outcomes. Angrist also worried about the endogeneity of the sex ratio. I.e., individuals may change marriage markets in response to differences in equilibrium $\tau's$ across different marriage markets. In general, they find that female labor supply is negatively correlated with the sex ratio in a marriage market. Given the different empirical techniques and marriage markets studied, this finding provides strong support for the above model of marriage.

The implication of the above empirical finding should not be underestimated. It says that the net gains to marriage for husbands and wives are systematically related to marriage market conditions. In particular, a large sex ratio favors women and a small sex ratio favors men.

Grossbard-Shectman 1993, South and Lloyd 1992, South and Trent 1988, and Angrist 2002 also report the effect of variations in the sex ratio on the marriage rate of women. More often than not, they find that the sex ratio is positively correlated with the marriage rate of women across

[5] The set of individuals used to construct the sex ratio is chosen primarily by pragmatic considerations.

[6] It should be clear that the model does not have anything to say about this gender difference in spousal ages. We will address this gender difference later.

[7] Few studies focus on male labor supply because most men work full time and so there is little variation in male labor supply across marriage markets.

marriage markets. The weak evidence in favor of a positive correlation is not surprising because the model showed that an increase in the sex ratio does not always imply an increase in the marriage rate of women.

2.2 Caring About Spousal Type & Assortative Matching

In the previous model, all individuals within a gender provide the same marital output to their potential spouses. Thus individuals are indifferent to whom they are married to. But in general, individuals within a gender value different types of spouses differently. When agents are heterogenous in terms of tastes and endowments, they will not be indifferent to whom they are matched with.

When individuals are heterogenous, a common observation is that agents that are alike in the relevant dimensions tend to match with each other. People who have a lot of a relevant attribute match with other people who also have a lot of that attribute. People who have little of an attribute tend to match with other people who also have little of that attribute. This kind of matching is known as positive assortative matching.

What assumptions do you need to make to get positive assortative matching in marriage? The first assumption that we will consider is complementarity in marital ouput production in a model with transferable utilities.

Consider a society with two types of men, m_1 and m_2 where the ability of m_2 men is higher than that of m_1 men. Let there also be two types of women, f_1 and f_2 where the ability of f_2 women is higher than that of f_1 women. The number of m_i men is equal to the number of f_j women for $i = j$.

Let the total monetary equivalent of the match between m_i men and f_j women be $\pi(i, j)$ which we also denote as π_{ij}.

Assumption 1:

$$G_{22} + G_{11} > G_{12} + G_{21}$$

Assumption 1 says that for any two pairs of matches, the total monetary output from positive assortative matching is larger than negative assortative matching. If π is differentiable, Assumption 1 says that $\frac{\partial^2 \pi}{\partial i \partial j} > 0$.

Consider now the highest ability types, m_2 and f_2. Who would they like to match with? Assume that they are not matched with each other. That is, let m_2 be matched with f_1 and f_2 matched with m_1. For a match between m_i and f_j, let the share of gains from the match for m_i be v_{ij} and the share of f_j is by residual $U_{ij} = \pi_{ij} - v_{ij}$.

We want to ask the following question? Will any man and woman prefer to leave their current marriages to form new marriages. In order for there to be no rematching (i.e. current matches to be stable), we must have the following two conditions:

$$v_{21} + U_{12} \geq \pi_{22} \quad (2.2.1)$$

$$v_{12} + U_{21} = \pi_{12} - U_{12} + \pi_{21} - v_{21} \geq \pi_{11} \quad (2.2.2)$$

If (2.2.1) does not hold, then f_2 and m_2 can leave their existing unions and form a new union together which can give each of them higher utilities than what they get currently. If (2.2.2) does not hold, then m_1 and f_1 can also leave their existing unions and form a new union together which can give each of them higher utilities than what they currently get. Adding the two equations, we get:

$$\pi_{12} + \pi_{21} \geq \pi_{22} + \pi_{11}$$

which violates Assumption (1). That is, negative assortative matching is not stable.

We will now show that there are v_{11} and v_{22} such that positive assortative matching is stable. Let m_i be matched with f_i, $i = 1, 2$. In particular, choose v_{11} and v_{22} such that:

$$\pi_{22} - \pi_{12} > v_{22} - v_{11} > \pi_{21} - \pi_{11} \quad (2.2.3)$$

By Assumption 1, (2.2.3) is feasible.

For m_2 and f_1 to leave their current matches to form a new match, we need:

$$v_{22} < v_{21}$$

$$\pi_{11} - v_{11} < \pi_{21} - v_{21}$$

Add the two restrictions to imply:

$$v_{22} - v_{11} < \pi_{21} - \pi_{11} \quad (2.2.4)$$

By assumption, (2.2.3) contridicts (2.2.4) and so m_2 and f_1 will not leave their current matches to form a new match.

For m_1 and f_2 to leave their current matches to form a new match, we need:

$$\pi_{22} - v_{22} < \pi_{12} - v_{12}$$
$$v_{11} < v_{12}$$

Add the two restrictions to imply:

$$\pi_{22} - \pi_{12} < v_{22} - v_{11} \tag{2.2.5}$$

By assumption, (2.2.3) contridicts (2.2.5) and so m_1 and f_2 will not leave their current matches to form a new match.

So when Assumption 1 applies, $\pi_{22} - \pi_{12} > \pi_{21} - \pi_{11}$ and we can find v_{22} and v_{11} which will satisfy (2.2.3). In this case, positive assortative matching in marriage is stable.

2.2.1 Examples

Example 1: Positive externalities in the household

Let the output produced by a household of m_i and f_j be $m_i f_j$. The production function satisfies Assumption 1. So we know in this case there will be positive assortative matching. To the extent that there are complimentarities in abilities in household production, we will tend to get positive assortative matching.

Example 2: Specialization in the household

Assume that only one spouse is in the labor force and the other spouse engage in household activity. All individuals are equally skilled in household production but the output of a household of m_i and f_j is:

$$\max[m_i, f_j]$$

Note that Assumption 1 fails in this case. So we should not get positive assortative matching. Assume that there is positive assortative matching. Let the share of output of f_2 be v_2. Her spouse will get $f_2 - v_2$.Let the share of output of f_1 be v_1. Her spouse will get $f_1 - v_1$.Will these matches break up? Will m_1 be tempted to leave his spouse for f_2. He will do so if:

$$f_2 - v_2 > f_1 - v_1 \tag{2.2.6}$$

So positive assortative matching can survive only if (2.2.6) is negated, i.e.

$$f_2 - v_2 < f_1 - v_1 \tag{2.2.7}$$

But if (2.2.7) holds, m_2 will leave his mate for f_1. So assortative matching is not stable. Negative assortative matching is stable. What does this mean? There should be negative assortative matching on wages. Empirically, there is assortative matching on wages and education. Thus if gains from specialization in the household are important, as in households where only one spouse works in the labor market, there has to be other factors which mitigate the gain from negative assortative matching.

Example 3: In the above section, we assumed that the number of m_i men is equal to the number of f_j women for $i = j$. What happens when this assumption is not valid? Problem 3 at the end of this chapter provides an example in which this assumption is relaxed.

2.3 Non-transferable utilities

The models in the previous sections assume that marital output is perfectly divisible between a married couple. We will now look a marriage market model without matching but marital output is not divisible between a marital couple.

Let there be M men and F women. The utilities of being single are as in the earlier model without matching. Let the marital output of a marriage between any man and any woman be Z. Here, we assume that each spouse will consume Z in marriage. So the marital output is a local public good in marriage.

In this case, all men with values of being unmarried, s_m, less than Z will want to marry. Let the number of men who want to marry be μ_m. All women with values of being unmarried, S_f, less than Z will also want to marry. Let the number of women who want to marry be μ_f. If μ_m is not equal to μ_f, not everybody who wants to marry will be able to marry. The maximum number of marriages, which we will assume is equal to the actual number of marriages will be

$$\min[\mu_f, \mu_m]$$

Without loss of generality, assume that $\mu_f < \mu_m$. Then all the women who want to marry will marry. $\mu_m - \mu_f$ men will want to marry but be unable to. They will have to remain unmarried and receive their utilities from being single which will be less than Z. Each married individual will receive Z.

Who are the men who want to marry but have to remain unmarried? This question cannot be addressed in our simple setup. But when there are individuals rationed out of marriage, they will try to attract spouses by giving them additional resources in marriage. To the extent that such diversion of resources in a marriage is feasible, we return to marriage models with divisible output. If transfers are infeasible, individuals will try in invest in themselves to make themselves more attractive in the marriage market.

2.3.1 Non-transferable utilities and assortative matching

When individuals within a gender are heterogenous, another rationale for positive assortative matching in marriage is when marital output is a public good. Let $\pi(m, f)$ be the output of marriage with a man of skill m and a woman of skill f. $\frac{\partial \pi}{\partial m} > 0$ and $\frac{\partial \pi}{\partial m} > 0$. Marital output is a public good for the two spouses. That is, the gain from marriage for the husband is $\pi(m, f)$ and and the gain to marriage for the wife is also $\pi(m, f)$. There is no redistribution of the output from marriage between the couple. So we are in a non-transferable utility framework.

Given such a marital production function, each man will want to marry the highest ability woman who is willing to marry him and vice versa. Only positive assortative matches are stable. A man of lower ability will not be able to persuade a woman who is matched with a higher ability man to leave her current match to form a new match with him. Similarly, a woman of lower ability will not be able to persuade a man who is matched with a higher ability woman to leave his current match to form a new match with her.

We can make related arguments to show that negative assortative matches are not stable.

Note that we do not need complementarity in the production of marital output, $\frac{\partial^2 \pi}{\partial m \partial f} > 0$, to get positive assortative matching if marital output is a public good.

2.4 Pre-Marital Investments

The difference between transferable utility models of the marriage market and their non-transferable counterparts can be exaggerated.

When individuals care about the attributes of their spouses, there is an incentive for individuals to invest in themselves to make themselves more attractive in the marriage market. In a non-transferable utility model of the marriage market, these pre-marital investments play a similar role as transfers in a transferable utility model. This section will sketch such a model.

Consider a large heterogenous population of men and women who want to marry. Let y be the income of a man and y is distributed in the male population according to the continuous density $f(y)$. Let Y be the income of a woman and Y is distributed in the female population according to the continuous density $g(Y)$. Without loss of generality, assume that there are more men than women. So in equilibrium some men will not marry.

Income may be consumed or invested in pre-marital capital. Pre-marital capital is a public good in marriage. Let a man with income y invests k units of pre-marital capital and consumes $y-k$. A woman of income Y invests K units of pre-marital capital and consumes $Y-K$. If they marry each other, each of them will consume $\pi(K,k)$ marital output where $\pi(.,.)$ is increasing in both arguments. We will assume that there is no transfer between spouses. There is no other gain to marriage. Then let $u(y-k)+\pi(K,k)$ be his utility from consumption and marriage where $u(.)$ is an increasing concave function. If he does not marry, he will enjoy a utility of $u(y)$. For simplicity, we are assuming here that pre-marital capital has no value to a single individual. Similarly, let $U(Y-K)+\pi(K,k)$ be her utility from consumption and marriage where $U(.)$ is an increasing concave function. If she does not marry, she will enjoy $U(Y)$. Given these preferences, a man wants to marry a woman with as much K as possible. Similarly, a woman wants to marry a man with as much k as possible.

Without marriage market competition, the investment levels of a potential husband and wife will be similar to the case of post marital investments. As discussed in the previous chapter, since $\pi(K,k)$ is a local public good, the equilibrium investment levels will be inefficient.

With marriage market competition, the situation is different. We first have to discuss how men and women will match in this marriage market. Consider a marriage matching function $K=\beta(k)$ whereby if a man has k amount of pre-marital capital, he expects to attract a spouse with $\beta(k)$ units of pre-marital capital. If a woman has K units of pre-marital capital, she expects to attract a spouse with $\beta^{-1}(K)$ units of pre-marital capital. Since individuals prefer their partner to have as much pre-marital investments as possible, $\beta(k)$ should be increasing in k.

Given $\beta(k)$, a man with income y will solve

$$\max\{u(y), \max_k u(y-k)+\pi(\beta(k),k))\} \tag{2.4.1}$$

(2.4.1) shows that the man chooses between being single and being married, assuming an optimal choice of investment and spouse. Let the man's optimal choice of k be $k^*(y)$.

Similarly, a woman with income Y will solve

$$\max\{U(Y), \max_K U(Y-K) + \pi(K, \beta^{-1}(K)\} \qquad (2.4.2)$$

Let her optimal choice of K be $K^*(Y)$.

Then $\beta(k)$ clears the marriage market if the number of men who invest $\widehat{k}$ or more is equal to the number of women who invest $\beta(\widehat{k})$ for any $\widehat{k} > 0$. See Peters and Siow for details on how to construct a market clearing matching function.

There does not need to be the same number types of males and females. For example, assume that market clearing requires different females with the same income Y to be matched with two different types of males with different levels of k, k_1 and k_2. Then the women who want to match with k_i men will have to invest $\beta(k_i)$. The market clearing matching function will make women with the same income Y indifferent between the different types of men. So in equilibrium, these women will be willing to match with the two different types of men.

In this non-transferable utility model, pre-marital investments play a similar role as transfers in a transferable utility model.

We will now study the welfare properties of this marriage market.

Consider a man with income y. We can construct combinations of K and k which will keep him at a constant level of utility u_1. These combinations of points (K, k) form his indifference curve u_1 in Figure (2.5) below. Given any k, he will obtain higher utility if he has more K. But it is not true that for any K, he prefers less k. That is, the indifference is not always upward sloping. Since he values k, for sufficiently small values of k, he has to get more K in order to reduce k further. Thus u_1 is downward sloping for low levels of k. The locus of points (K, k) which form his indifference curve u_2 gives him a higher level of utility. For every k, he gets more K with locus u_2 than with locus u_1. Given $\beta(k)$, the matching function, he will optimally choose to invest k^* and match with a bride with capital $K^* = \beta(k^*)$. He will obtain a utility level of u^* as shown in the figure. k^* solves (2.4.1). Note that his indifference curve u^* is tangent to $\beta(k)$ at point (K^*, k^*).

Consider a woman with income Y. Her indifference curve between k and K are as shown in the curve U_1 in the figure. Given $\beta(k)$, the matching function, she will optimally choose to invest K^* and match with a groom with capital $k^* = \beta^{-1}(K^*)$. She will obtain a utility level of U^* as shown in the figure. K^* solves (2.4.2). Note that her indifference curve U^* is tangent to $\beta(k)$ at point (K^*, k^*). The man with income y who marries the woman with income Y have indifference curves whose slopes are tangent to each other. Thus there is no way for either him or

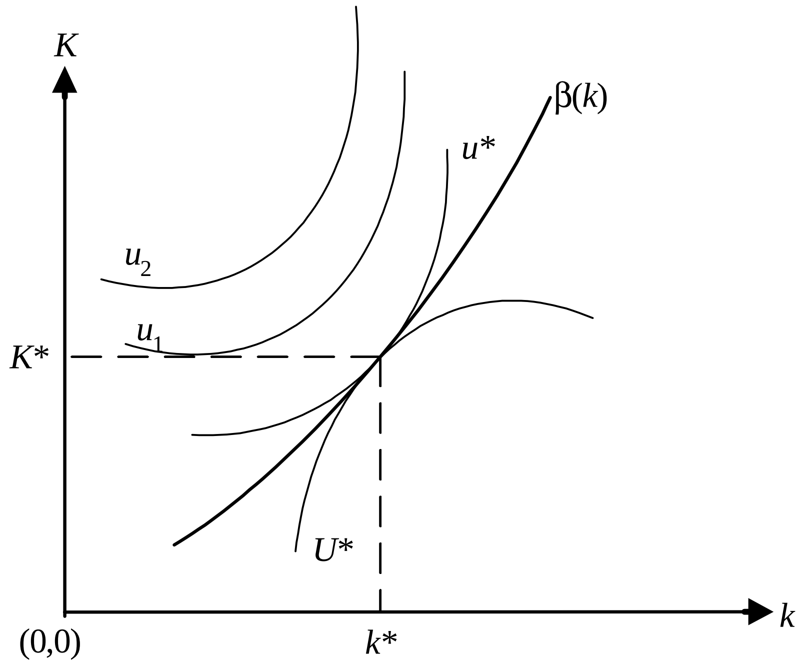

Figure 2.5.: Equilibrium investments in K and k.

her to improve on their own welfare without making the spouse worse off. In fact, for every married couple, the wife's indifference curve is tangent to that of her husband. So no wife can improve her welfare without making her husband worse off. Moreover, since $\beta(k)$ is market clearing, no spouse prefers to invest another level of capital to marry someone else. Thus pre-marital investments are efficient.

2.5 Bibilographic Notes

Many of the issues discussed in this chapter was first raised by Becker 1973 and 1974, and summarized in his 1991 book. Our discussion of assortative matching with transferable utilities follows Weiss 1997. Lam 1988 discussed the argument for positive assortative matching with non-tranferable utilities. The argument for efficient premarital investments with non-transferable utilities follows Peters and Siow 2002.

2.6 Problems

1. Consider a society in which all men have the same gross utility from being married, z. All women also have the same gross utility from being married, Z. Men differ in their utilities from being single. Let τ be the equilibrium transfer that men pay to women in marriage. Then a man of index m gets s_m if single and $z - \tau$ in marriage. Assume that s_m is increasing in m. Women also differ in their utilities from being single. A woman of index f gets S_f if single and $Z + \tau$ from being married. S_f is increasing in f. Men and women are indifferent to which spouse they are married to.

(a) Show in a diagram, the equilibrium level of τ.

(b) The government wants to encourage marriage and therefore provides a subsidy of π per married couple. Show the new equilibrium level of τ. What will happen to the number of marriages?

(c) Consider an increase in the supply of men to the society. What will happen to the equilibrim level of τ and the marriage rates of men and women?

(d) Compare this society to the one discussed in Section 2.1. Can you tell which society you are living in by comparing the comparative statics between the two societies?

2. Consider the model of the marriage market discussed in Section 2.1. Assume that $y_m = y$ for all m and $Y_f = Y$ for all f.

(a) Assume that $M > F$. What is the equilibrium transfer τ^*?

(b) Assume that the sex ratio is one, i.e. $M = F$. What is the equilibrium transfer τ^*? Is it unique?

(c) Can social norms play a role in the above society in determining τ^*? Is the assumption of $y_m = y$ for all m and $Y_f = Y$ for all f reasonable?

3. Consider a society with M men where M is large. All men have the same utility from being single, s. There are two types of men with respect to marriage, h and l types. $\frac{1}{2}$ of the men are h types and the other half are l types. There are $\frac{3}{4}M$ number of women in this society. All women have the same utility from being single, S. There are also two types of women, H and L types, equally divided among the female population.

The total marital output generated by an l, L marriage is $Z > s + S$. The total marital output generated by an l, H or h, L marriage is $2Z$. The total marital output generated by an h, H marriage is $4Z$.

For a marriage between type m male and type f female, the male will transfer τ_{mf} to the female as her share of the marital output. He will keep $Z_{mf} - \tau_{mf}$ for himself where Z_{mf} is the total marital output produced by that pair.

The marriage market clears when given τ_{mf} for every type of marriage, every individual can find a spouse of his or her choice if he or she wants to.

(a) Since women are scarce in this society, some men will not be married. Which type of men is likely to be these unmarried men? If some type m men are married and others are not in equilibrium, what must the equilibrium transfers in marriage be to make these men indifferent between being single versus being married?

(b) Since there are more h type men than h type women, some L type women will marry h type men. Other L type women will marry l type men. What must be the relationship between τ_{hL} and τ_{lL} such that L type women are indifferent between the two types of marriages?

(c) Find the equilibrium types of marriages, number of marriages of each type and transfers which clear the marriage market.