3 Dynamic Considerations

In the previous chapter, agents can see all the attributes of potential spouses. However in many circumstances, the knowledge about other agents' attributes is imperfect. So an agent only learns about the other agent's attribute after at least one interaction with the other agent. Sometimes, you may need more than one interaction if the signal about attribute is noisy.

What kind of results do you get with this experience matching? Consider the case of purely idiosyncratic matching. That is, different agents do not have the same ranking of potential partners. The fact that your friend turns down a partner does not imply whether that partner is suitable for you or not.

With idiosyncratic matching and imperfect information, an agent learns about potential partners by interacting with them. If an interaction is unfavorable, the match breaks up and the agents choose again. Agents stop searching when they find an acceptable match.

In this chapter, we will use non-transferable utility models because transfers to clear the marriage market is not of importance for the issues considered here.

3.1 Search and Marriage

Consider a society with many males and females. Each adult lives two periods. Each individual has has a reservation utility per period of remaining single, s.

Assume that the ages of the participants are observed by everyone. For now, we will assume that adults are willing only to marry others of the same age.

When a young male and a young female meet in the first period, they draw an indiosyncratic match value θ from the cumulative distribution $F(\theta)$. If they marry, each individual gets the match value θ for every period of marriage. There is no divorce. If individuals who draw a match value in the first period and decide not to marry, they can return to the marriage market in the second period to draw another match value from the same match value distribution as in the first period. There is no correlation between the match value drawn in the first period and the second period match value that a single person may draw.

Since an individual is willing to marry only if the return to marriage is better than being single, a necessary condition for marriage is:

$$\theta \geq s \tag{3.1.1}$$

But this condition is not sufficient for first period matches to be formed. The reason is that the alternative to marriage in the first period is not being single forever but rather being single in the first period and returning to the marriage market in the second period.

If two single individuals are searching for matches in the second period, and they will never marry if they cannot agree to a match in the second period, then (3.1.1) is sufficient for match formation. In this case, if $\theta \geq s$, they will marry and each of them will enjoy θ in the second period. If $s > \theta$, they will both agree not to marry and remain single. We call s the reservation match value for a second period single person. Before drawing θ, a single individual at the beginning of the second period will enjoy an expected utility of

$$E\max\{s, \theta\} = sF(s) + (1 - F(s))E(\theta|\theta \geq s)$$

E is the expectations operator. The interpretation of the above equation is as follows. With probability $F(s)$, the couple will draw a value of θ which is smaller than s. In this case, the couple will agree not to marry and enjoy s, the value of remaining single. With probability $(1 - F(s))$, the couple will draw a value of θ which is greater than s. In this case, they will choose to marry and enjoy their realized value of θ. The expected value of θ given that the realization of θ is greater than s is denoted by $E(\theta|\theta \geq s)$. So before drawing θ, their expected utility is the right hand side of the above equation. Note that $E\max\{s, \theta\} > s$.

Now consider a young female's decision problem. Let her draw θ_1 in the first period. Her present value from marriage in the first period without discounting is:

$$V_m(\theta_1) = \theta_1 + \theta_1 \tag{3.1.2}$$

Her expected present value from remaining single is:

$$V_s = s + E\max\{s, \theta\} \tag{3.1.3}$$

Since $E\max\{s,\theta\} > s$, $V_s > 2s$.
Her decision whether to marry or remain single depends on:

$$\begin{aligned} V(\theta_1) &= \max\{V_m(\theta_1), V_s\} \\ &= \max\{2\theta_1, V_s\} \end{aligned}$$

$\frac{V_s}{2}$ is her reservation match value. That is, if $\theta_1 < \frac{V_s}{2}$, she will not marry in the first period. Since $\frac{V_s}{2} > s$, she will reject matches, θ_1, in which $\frac{V_s}{2} > \theta_1 \geq s$. What is the intuition? If she accepts a match which is just marginally better than remaining single in the first period, she deprives herself a chance of searching in the second period. There is no guarantee that she will successfully marry in the second period either if she searches. She may also fail to marry in the second period. However the cost of failure is not large if θ_1 is just slightly higher than s. The value of delay in the hope of getting something better in the future is known as the option value of waiting.

Since $\frac{V_s}{2}$ is her reservation match value in the first period, the probability that she will marry in the first period is $1 - F(\frac{V_s}{2})$. If she is single at the beginning of the second period, the probability that she will marry is $1 - F(s) > 1 - F(\frac{V_s}{2})$ because $s < \frac{V_s}{2}$. Single second period individuals are more likely to marry than first period individuals because there is no futher option value from from delaying.

3.2 Divorce

Now let us allow young married adults to divorce when they are old. The rest of the model remains unchanged. Let a young married couple experience θ_1 in the first period. In this case, in the second period, they will want to divorce if

$$\theta_1 < E\max\{s,\theta\}$$

A young married woman with match value θ_1 will receive in the second period:

$$V_2(\theta_1) = \max[\theta_1, E\max\{s,\theta\}]$$

No matter what the value of θ_1 is, $V_2(\theta_1) \geq E\max\{s,\theta_2\}$. That is, the value of being married at the beginning of the second period is always at least as large as being not married at the beginning of the second period. Put another way, from the perspective of the beginning of the second period, there is no cost to being married in the first period.

Thus if $\theta_1 > s$, the individual should marry in the first period. So the reservation match value for first period marriage falls when divorce is feasible. The explanation is as follows. With no divorce cost, it is optimal to marry as long as the return to marriage in the first period exceeds the value of being single. In the second period, the married person can divorce and re-enter the marriage as if he or she never married. Thus there is no cost of being married in the first period in terms of what is feasible in the second period.

We will also observe positive assortative matching by age in marriage. No young single individual will choose to match with an older single person because the young person will not have the option of remaining married in the second period. With divorce, there is no cost to having this option. Without divorce, a young individual has to consider the benefit of matching with an older individual because the young individual can return to the marriage market in the second period. But with divorce, young adults will strictly prefer to match with each other.

Divorce is strictly welfare improving in this society. More individuals will marry in a sociey with divorce than without.

In this environment, there are some prefectly predictable divorces. Marriages whose first period match value θ_1 satisfy:

$$s < \theta_1 < E \max\{s, \theta\}$$

will end in divorce in the second period. These divorces occur even though there is no new information about the marriage which occured in the first period. An interpretation of marriage with perfectly anticipated divorce is that it describes young couples who cohabitate with little expectation of a long term relationship.

3.2.1 Arrival of new information in a marriage

We now let the quality of a first period match change in the second period. Let θ_1 be the match value in the first period. Let θ_2 be the match value in the second period where

$$\begin{aligned} \theta_2 &= \theta_1 + \lambda + \omega \qquad (3.2.1) \\ E\theta_2 &= \theta_1 + \lambda \end{aligned}$$

ω is drawn from a density $g(\omega)$ and distribution function $G(\omega)$ and has a mean of 0. λ is interpreted at the average increase in the value of the match (match specific capital). In general we expect $\lambda > 0$. Even if a person marries in the first period, she may divorce in the second if the value of divorce, V_d, is larger than θ_2:

$$\begin{aligned} \theta_2 &< E\max\{s,\theta\} = V_d \\ \theta_1 + \lambda + \omega &< V_d \\ \omega &< V_d - \lambda - \theta_1 \end{aligned} \tag{3.2.2}$$

Let $\Omega(\theta_1, V_d, \lambda) = V_d - \lambda - \theta_1$. $\Omega(\theta_1, V_d, \lambda)$ is the reservation match value for remaining married in the second period. If $\omega < \Omega(\theta_1, V_d, \lambda)$, then the pair gets divorced. $\Omega(\theta_1, V_d, \lambda)$ is decreasing in λ. Good initial matches, i.e. high values of θ_1, leads to low values of $\Omega(\theta_1, V_d, \lambda)$, and a lower divorce probability. Note the importance of new information, ω, which leads to a divorce. The divorce may result in a remarriage or not. Given that she marries in the first period with a match value, θ_1, her expected utility in the second period is:

$$\begin{aligned} V_2(\theta_1, \lambda) &= E(\max\{\theta_2, V_d\}|\theta_1) \\ &= \overline{G(\Omega)}(\theta_1 + \lambda + \Gamma(\omega, \Omega)) + G(\Omega)V_d \\ &= \overline{G(\Omega)}(\theta_1 + \lambda + \Gamma(\omega, \Omega)) + G(\Omega)(\theta_1 + \lambda + \Omega) \\ &= \theta_1 + \lambda + \overline{G(\Omega)}\Gamma(\omega, \Omega) + G(\Omega)\Omega \\ &> \theta_1 + \lambda \end{aligned} \tag{3.2.3}$$

where $\Omega = \Omega(\theta_1, V_d, \lambda)$, $\overline{G(\Omega)} = 1 - G(\Omega)$ and $\Gamma(\omega, \Omega) = E(\omega|\omega > \Omega)$. (3.2.3) says that the possibility of divorce increases expected utility in a marriage. Note that remarriage is not necessary for this result. If divorce allows a couple to get out of a bad marriage, it is enough to show that divorce is welfare improving. Even if $\lambda = 0$, $V_2(\theta_1, 0) > \theta_1$. Also from the definition of $V_2(\theta_1, \lambda)$, $V_2(\theta_1, \lambda) > V_d$.

So her total utility from marriage in the first period is:

$$V_m(\theta_1, \lambda) = \theta_1 + V_2(\theta_1, \lambda) \tag{3.2.4}$$

Compared with (3.1.2), her expected utility is higher than when she could not get divorced. If she does not marry in the first period, she will get:

$$V_s = s + E\max(s, \theta) = s + V_d \tag{3.2.5}$$

So in the first period she will choose:

$$\begin{aligned} V(\theta_1) &= \max\{V_s, V_m(\theta_1, \lambda)\} \\ &= \max\{s + V_d, \theta_1 + V_2(\theta_1, \lambda)\} \end{aligned} \tag{3.2.6}$$

Since $V_2(\theta_1, \lambda) > V_d$, she may choose to marry even when $\theta_1 < s$. There are two reasons for such marriages. First, due to the accumulation of marriage specific capital, she expects the match value in the second period to be higher than from meeting someone new. Second, even if $\lambda = 0$, when there is no accumulation of marriage specific capital, marrying allows her to see how the match will evolve. If the match evolves poorly, she can divorce and try to match with someone else. Thus if she marries in the first period, she gets two draws for a good match in the second period. If she chooses not to marry in the first period, she will only get one draw for a good match in the second period. She values an additional draw in the second period and therefore is willing to marry someone in the first period for some $\theta_1 < s$.

Thus in general, more individuals are likely to marry in the first period when divorce is feasible. In the above setup, the possibility of divorce is again welfare enhancing.

3.3 Under investment in marriage specific capital

Assume that a man, h, and a woman, w, with match value θ_1 decides to marry. Let $\lambda = e_h + e_w$, where e_i is the investment of individual i in the first period of marriage. Let the cost of investment to individual i be e_i^2.

Each individual will choose effort independently. So we want to find the Nash equilibrium (NE) in effort contribution. As discussed in chapter **??**, the NE effort levels within marriage are inefficient (too low) relative to the efficient solution. We show here that this under investment is exacerbated by the possibility of divorce.

Let $\Omega(\theta_1, e_h, e_w) = V_d - \theta_1 - e_h - e_w$. Using (3.2.3), the efficient solution, which maximizes total marital output minus costs, solves:

$$\begin{aligned} &\max_{e_h,e_w}\{2V_2(\theta_1, e_h + e_w) - e_h^2 - e_w^2\} \qquad (3.3.1)\\ = &\max_{e_h,e_w}\{2[\theta_1 + e_h + e_w + \overline{G(\Omega(\theta_1, e_h, e_w))}E(\omega|\omega \geq \Omega(\theta_1, e_h, e_w)) \\ &+G(\Omega(\theta_1, e_h, e_w))\Omega(\theta_1, e_h, e_w)] - e_h^2 - e_w^2\} \end{aligned}$$

Let e_i^* be the efficient choice of effort for individual i. For an interior maximum, the first order condition for e_i is:[1]

$$2\frac{\partial V_2(\theta_1, e_i^* + e_{\neq i}^*)}{\partial e_i^*} = 2e_i^* \tag{3.3.2}$$

$$1 - G(V_d - \theta_1 - e_i^* - e_{\neq i}^*) = e_i^*$$

The left hand hand side of (3.3.2) is the marginal benefit of effort. The right hand side is the marginal cost of effort. At the optimum, the marginal benefit of effort is equated to the marginal cost of effort.

Since the two spouses have symmetric preferences, we may look for a symmetric equilibrium in which $e_i^* = e_{\neq i}^* = e^*$. In this case, (3.3.2) becomes:

$$2\frac{\partial V_2(\theta_1, 2e_i^*)}{\partial e_i^*} = 2e_i^* \tag{3.3.3}$$

$$1 - G(V_d - \theta_1 - 2e_i^*) = e_i^* \tag{3.3.4}$$

(3.3.4) is an equation in one unknown, e^*, and therefore we can solve for e^*, the efficient level of effort. The left hand side of (3.3.4) is bounded above by 1. The right hand side is increasing in e_i^* and unbounded. Thus we can find a solution to (3.3.4).

Instead of investing efficiently, both individuals may choose invest non-cooperatively. In this case, we will solve for the Nash equilibrium level of invesments. In the Nash equilibrium, each agent takes the other's effort level as given, $e_{\neq i}^n$, and solve

$$\max_{e_i}\{V_2(\theta_1, e_i + e_{\neq i}^n) - e_i^2\} \tag{3.3.5}$$

The first order condition for effort in this case is:

$$\frac{\partial V_2(\theta_1, e_i^n + e_{\neq i}^n)}{\partial e_i^n} = 2e_i^n \tag{3.3.6}$$

Again since both individuals have symmetric preferences, we may look for a symmetric equilibrium in which $e_i^n = e_{\neq i}^n = e^n$. In this case, (3.3.6)

[1] Given a random variable ω with cumulative distribution $G(\omega)$, if

$$Y(x) = G(h(x))h(x) + (1 - G(h(x)))E(\omega|\omega \geq h(x))$$

then

$$\frac{\partial Y}{\partial x} = G(h(x))h'(x)$$

becomes:

$$\frac{\partial V_2(\theta_1, 2e^n)}{\partial e^n} = 2e^n \tag{3.3.7}$$

$$\frac{1 - G(V_d - \theta_1 - 2e_i^n)}{2} = e_i^n \tag{3.3.8}$$

(3.3.8) is an equation in one variable, e^n, and so we can solve for e^n, the non-cooperative equilibrium level of effort.

Comparing (3.3.3) with (3.3.7), the marginal costs of effort are the same in the NE and in the efficient case. But the marginal benefits of effort are twice as large in the efficient case than in the NE. Thus individuals will invest less in the NE than in the efficient case. The inefficiency is due to the fact that in the non-cooperative equilibrium, each individual ignores his or her own's contribution on the welfare gain of the other person.

Can we change divorce cost to get to a more efficient solution? If you increase the cost of getting a divorce, V_d, the returns to divorce will fall. Individuals will be less likely to get a divorce. But if a couple is more likely to spend time together, they will also find it more profitable to invest more in the marriage. The change in effort choice may be obtained by differentiating (3.3.8) with respect to V_d:

$$\frac{\partial e_i^n}{\partial V_d} = \frac{-g(V_d - \theta_1 - 2e_i^n)}{1 - g(V_d - \theta_1 - 2e_i^n)}$$

The sign of $\frac{\partial e_i^n}{\partial V_d}$ depends on the sign of $1 - g(V_d - \theta_1 - 2e_i^n)$. When there is an interior solution to (3.3.5), in addition to satisfying the first order condition (3.3.6), we must also satisfy the second order condition which reduces to $1 - g(V_d - \theta_1 - 2e_i^n) > 0$. In other words, $\frac{\partial e_i^n}{\partial V_d} < 0$. That is, when the value of divorce increases, non-cooperative investment in the marriage will fall. But here there is a policy tradeoff. Increasing the cost of divorce will increase marriage specific investments which moves towards the efficient amount of investment. But after the investment, it is inefficient to hold matches together if it is efficient for them to split up. Thus there is a policy tradeoff in how high should divorce costs be.

3.4 Divorce and Marriage Market Externalities

Is a married couple more likely to divorce if there are more divorced individuals in the population? If the answer is yes, will there be mutiple equilibrium divorce rates in this society? These issues are known as search externalities in the marriage market.

Consider the above society with K number of men and K number of women. Let us assume that divorced individuals only marry each other. This assumption simplifies the analysis because we do not have to worry about search externalities in the singles market. Let there be k^s divorced men and women, let the probability that a divorced man will find a divorced woman to match with be $p(k^s)$. Each individual contemplating divorce takes $p(k^s)$ as given. The value of divorce now is:

$$V_d(k^s) = (1 - p(k^s))s + p(k^s)E\max(s, \theta) \tag{3.4.1}$$

Note that V_d is increasing in k^s.

$$\begin{aligned} V_d' &= (E\max(s,\theta) - s)p_{k^s} > 0 \\ V\text{"}_d &= (E\max(s,\theta) - s)p_{k^s k^s} \end{aligned} \tag{3.4.2}$$

Abstract from the investment in match specific capital, i.e. let λ be exogenous. Also let all couples begin with the same match value θ_1 in the first period of marriage. As before, the minimum value of ω for initiating a divorce is:

$$\Omega(\theta_1, k^s) = V_d(k^s) - \lambda - \theta_1 \tag{3.4.3}$$

As k^s increases, $\Omega(\theta_1, k^s)$ increases which means that married couples need higher minimum match values to stay together. That is, $\Omega(\theta_1, k^s)$ is an increasing function of k^s. See Figure (**??**) below.

Let the number of individuals who want to get divorced be k^d. As $\Omega(\theta_1, k^s)$ increases, more married couples want to get divorced. This means that k^d increases. i.e. k^d is an increasing function of Ω. Let $k^d = h(\Omega)$. See **??** below.

In equilibrium, the number of individuals who want to divorce must equal the supply of divorcees, that is $k^d = k^s$. As can be seen in **??**, there are potentially multiple equilibrium number of divorced individuals. The multiple solutions are self-fullfilling prophesies. If individuals anticipate a low rate of divorce, that is a low expected return to divorce, they will not get divorce. But if individuals do not get divorce, the low rate of divorce is self-fullfilling.

There is an externality here. The equilibrium divorce rate is too low to be efficient. Why? The reason is that each couple contemplating divorce do not take into account their actions on the decisions of others. That is, they do not care that if they get divorce, they will induce other individuals to be also more likely to get divorce.

The search externality says that the equilibrium divorce rate is too low to be efficient. But the non-cooperative investment in marriage specific

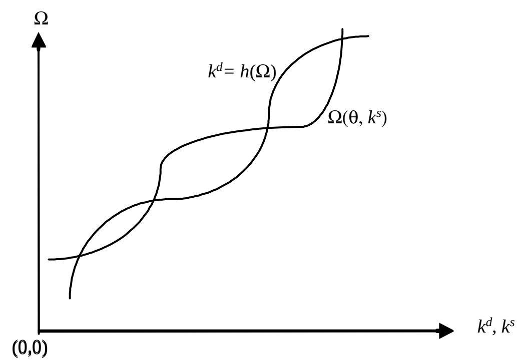

Figure 3.1. Multiple divorce equilibria

capital model says that the divorce rate is too high to be efficient. At this stage, we do not have enough information to know whether the equilibrium divorce rate in a particular society is efficient or not.

So far we have been looking at divorce from the perpective of adults. What about the welfare of children? We can think of at least part of the investment in match specific capital as investment in children. In this case, increasing the cost of divorce may improve the welfare of children in the sense that it promotes increased investment in children. It does bind some marriages together where the parents would prefer to separate.

3.5 Amicable versus unamicable divorce

All the models that have been discussed to date involve amicable divorces. That is, if one spouse wants to divorce, the other spouse also wants to do so. In this case, at the time of the decision to separate or not, both spouses will choose the same decision. The reason for this coincidence of preferences is that we have assumed throughout this chapter that both parties get the same expected payoff from marriage or divorce in every period. Thus there is no reason for either party to disagree on whether to separate or not.

If we relax the assumption that both spouses get the same expected payoff from marriage or divorce, there may be a disagreement between them as to whether they want to separate or not. For example, consider the model of divorce discussed in Section 3.2.1. Let the husband in the second period draw an idiosyncratic match value ω_h from $g(\omega_h)$. Let the wife draw an independent second period match value ω_f from $g(\omega_f)$. In this case, ω_h will in general not be equal to ω_f. Given their own match values in the second period, one spouse may prefer to divorce and the other spouse may prefer to remain married. In this case, whether they remain married or not will depend on the laws governing divorce and whether spouses can transfer resouces to each other to effect a divorce or not. These extensions are beyond the scope of this chapter.

3.6 Empirical evidence

There is large variation on the age of first marriage within each gender. This variation may be taken as evidence that search frictions are an important phenomenon in the marriage market. However, the variation may also be due to other factors. For example, some individuals may choose to delay entering the marriage market because they wish to focus on accumulating more human capital (such as schooling). Thus quantification of the importance of search frictions in the marriage market is thin. An example in this direction is chapter **??**.

There is evidence on the importance of new information arriving in a marriage which leads to divorce. Weiss and Willis (1996) analyzed data from a cohort of young adults. They showed that an unexpected increase in husband's income lowered the divorce hazard and an unexpected increase in wife's income increased the divorce hazard.[2] The level of expected earnings, formed at the time of marriage, did not affect the divorce hazard. Thus these authors interpret their findings on providing support for the hypothesis that the arrival of new information in a marriage affects the stability of that marriage.

There is also evidence on how social policies affect marital behavior. Prior to the 1970's, most US states required a finding of "fault" on one of the spouses or the consent of both spouses wthout a "fault" finding before a divorce would be granted. Such regimes were known as "consent" divorces. Many states changed their divorce laws in the seventies to "unilateral" regimes where a divorce would be granted if one spouse wanted it. There have been many studies on the impact of this change on marital behavior including marriage rates and divorce rates. For example, Gruber (2000) showed that both marriage rates and divorce rates increased in states after they switched from "consent" regimes to "unilateral" regimes. His analysis also showed that children that grew up in "unilateral" regimes had worse socioeconomic outcome than children that grew up in "consent" regimes. His results are suggestive of the

[2] The hazard rate of an event at period t is the probability that the event will occur in period t conditional on it not having occurred by period $t-1$.

hypothesis that parents invest more in marriage specific capital if divorce is harder to come by.[3]

3.7 Problems

1. Let $f(\theta)$ be the uniform distribution where θ is distributed between 0 and 1. Then

$$f(\theta) = 1$$
$$F(X) = \int_0^X 1 d\theta = X$$
$$E(\theta|\theta > X) = \frac{1}{1-X}\int_X^1 \theta d\theta = \frac{1+X}{2}$$

Consider the model in Section 3.1. If $s = \frac{1}{2}$, find the reservation match value for marriage in period one with and without divorce.

2. Let $g(\omega)$ be the uniform distribution where ω is distributed between $-\frac{1}{2}$ and $\frac{1}{2}$. Then

$$g(\omega) = 1$$
$$G(\Omega) = \int_{-\frac{1}{2}}^{\Omega} 1 d\omega = \Omega + \frac{1}{2}$$
$$E(\omega|\omega > \Omega) = \frac{1}{\frac{1}{2}-\Omega}\int_{\Omega}^{\frac{1}{2}} \omega d\omega = \frac{\Omega + \frac{1}{2}}{2}$$

Consider the model in Section 3.2.1. Let θ be uniformly distributed between 0 and 1. Let $s = \frac{1}{2}$ and $\lambda = 0$. Find the reservation match value for remaining married as a function of θ_1. Find the reservation match value of θ_1 for getting married.

3. Consider the model in Section 3.3. Let the cost of effort e_i for individual i be $2e_i^2$. Using the same distributions and parameters as in the previous questions, find the non-cooperative levels of investment in marriage specific capital, e_h and e_w, as a function of θ_1. What happens to the investment levels as θ_1 increases? What is the economics of this comparative static?

[3] On the other hand, one may argue that parents exogenously lowered their investment in marriage specific capital. As a result, more marriages failed and legislatures responded to this increased failure rate by switching from consent to unilateral regimes.