332F: Mid Term

NAME:_____

STUDENT NO:_____

Answer all questions.

1. Consider the investment in public good discussed in the text. Let the mother's utility be:

$$U(c, q) = q + \ln c$$

Let the father's utility be:

$$V(g, q) = q + \ln g$$

Let mother's endowment and father's endowment be:

$$e = h = 2$$

Note that mother's marginal utility of own consumption is:

$$U_c = \frac{1}{c}$$

Father's marginal utility of own consumption is

$$V_g = \frac{1}{g}$$

Mother's marginal utility of child's consumption is equal to the father's marginal utility of child's consumption:

$$U_q = V_q = 1$$

(a) Find the equilibrium level of child's consumption if both parents act non-cooperatively.

(b) Find the equilibrium level of child's consumption if both parents act cooperatively to maximize the sum of parental utilities.

(a) Given the father's contribution, $2 - g_h$, the mother will choose

$$\max_c (2 - g_h + 2 - c) + \ln c$$

Her choice of c, c^*, will satisfy:

$$\begin{aligned} -U_q + U_c &= 0 \\ -1 + \frac{1}{c^*} &= 0 \\ c^* &= 1 \end{aligned}$$

So she will give 1 unit of output to the child. Since the father has the same utility function and endowment as the mother, he will also in equilibrium give 1 unit to the child. So total child consumption is 2.

(b) Parents solve:

$$\max_{c,g} 2(2 - c - g) + \ln c + \ln g$$

The optimal choices for c and g, $\widehat{c}$ and $\widehat{g}$ satisfy:

$$\begin{aligned} -2U_q + U_c &= 0 \\ -2V_q + V_g &= 0 \end{aligned}$$

which implies:

$$\begin{aligned} -2 + \frac{1}{\widehat{c}} &= 0 \\ -2 + \frac{1}{\widehat{g}} &= 0 \end{aligned}$$

So

$$\widehat{c} = \widehat{g} = \frac{1}{2}$$

and $\widehat{q} = 3$.

2. (a) Explain the difference between transferable versus non-transferable models of marriage.

(b) In Canada, there is more positive assortative matching in marriage by years of education today than 25 years ago. Can you explain this trend? Make a reference to (a) in your explanation.

(a) Transferable models of marriages assume that the marital output is flexibly divided between the potential husband and the wife. Moreover the potential husband and the wife knows that the agree upon division of marital output will be carried out in the marriage.

Non-transferable models of marriage assume that it is not possible to flexibly divide marital output between a potential husband and wife. That is, there is no credible way for husbands and wives to commit to a particular division of marital output after the output is produced.

(b) An increase in positive assortative matching by education can be generated by (1) an increase in complementarity in the marital output production function between husband's and wife's education, (2) if marital output becomes more non-transferable (i.e. a public good). To the extent that it becomes more difficult to enfore division of marital output in marriage, then marital output becomes a public good. In this case, individuals will want to sort more by education if more education generates more marital output. There is also an argument that complementarity in spouse's education has risen as more wives enter the labor market.

3. Consider a society with M men where M is large. All men have the same utility from being single, 0. There are two types of men with respect to marriage, h and l types. $\frac{1}{2}$ of the men are h types and the other half are l types. There are $\frac{3}{4}M$ number of women in this society. All women have the same utility from being single, 0. There are also two types of women, H and L types, equally divided among the female population.

The total marital output generated by an l, L marriage is 1. The total marital output generated by an $\{l, H\}$, $\{h, L\}$ or $\{h, H\}$ marriage is 2.

For a marriage between type m male and type f female, the male will transfer τ_{mf} to the female as her share of the marital output. He will keep $Z_{mf} - \tau_{mf}$ for himself where Z_{mf} is the total marital output produced by that pair.

The marriage market clears when given τ_{mf} for every type of marriage, every individual can find a spouse of his or her choice if he or she wants to.

(a) Is there complementarity in marital output production in this society? How can you motivate this kind of marital output production function?

(b) Find the equilibrium types of marriages, number of marriages of each type and transfers which clear the marriage market. (Hint: is there an advantage to a high type male or female to marrying another high type?)

(a) There is no complimentarity in marital production.

$$G(h, H) + G(l, L) < G(h, L) + G(l, H)$$

which violates complementarity. This production function implies that there is one job where ability matters and the other job ability does not matter.

Moreover there is specialization such that one person does the job where ability matters and the other person does the other job. In this case, the high ability person does the job where ability matters.

(b) There are more men than women in the society. Note that high ability women do not care about who they marry, high or low ability men. Assume for the time being that they marry low ability men. In this case, $\frac{3}{8}$ high low ability men are married to high ability women. Assume for the time being that the rest of the low ability men is unmarried and therefore receive a utility of zero. In this case, the low ability men who are married will be willing to give all the surplus from marriage to the women. So high ability women receive 2 units from marriage.

Now there are also $\frac{3}{8}$ low ability women to match with $\frac{3}{8}$ high ability men. $\frac{1}{8}$ high ability men are unmarried. Since unmarried men get zero utility, the high ability men will also be willing to give 2 units to the women to remain married. So in equilibrium men give away all the output from marriage. Can high ability men do better? No. An unmarried high ability man cannot propose to a high ability woman since he cannot offer to give her more than 2 units which is what she is already getting from her low ability husband.

In this example, we initially assume that $\frac{1}{8}$ low ability men and $\frac{1}{8}$ high ability men is unmarried. This is unnecessary. All that matters is that all married men get no marital output. And $\frac{3}{8}$ high ability men marry low ability women. High ability women dont care who they marry.

4. Consider the paper by Gruber on the impact of unilateral divorce. He provide results on the impact of unilateral divorce on the divorce rate, the impact on adults who are impacted by the law change as adults, and the impact on adults who were impacted by the law change when they were children. Which models discussed in the course so far are supported by his results? Do his results contridict any of the models?

For adults living with unilateral divorce, (a) they are more likely to divorce, (b) but not necessarily more likely to marry.

For children who grew up with unilateral divorce, (c) they are more likely to marry and divorce as adults, (d) they are more likely to have adverse socioeconomic outcomes as adults.

Most of the models discussed show that the reservation match value for marriage falls when divorce is allowed. Thus the marriage rate should increase with unilateral divorce which makes divorce easier. There should also be a higher divorce rate. Thus (a) and (c) supports the models. (b) does not support the models.

With marriage specific investments, a lower divorce cost will decrease marriage specific investments. Also, divorced parents will allocate less resources to children. (d) supports the hypothesis that children is a public good in marriage.