Solve mother's problem given father's consumption of g^n:

$$\max_c U(c, g^n) = c^{\alpha}(2e - c - g^n)^{(1-\alpha)}$$

$$\frac{\partial U(c,g)}{\partial c} = c^{-\beta}\alpha\,(2e - c - g^n)^{\beta} - c^{\alpha}\,(2e - c - g^n)^{-\alpha}\,\beta$$

$$\beta = 1 - \alpha$$

$$\frac{\partial U(c,g)}{\partial c} = 0 \Longrightarrow \alpha\,(2e - c^n - g^n) = c^n(1-\alpha)$$

Best response functions:

$$\alpha\,(2e - c^n - g^n) = g^n(1-\alpha)$$
$$\alpha\,(2e - c^n - g^n) = c^n(1-\alpha)$$

NE

$$c^n = g^n = 2\alpha\frac{e}{1+\alpha}$$
$$U(c^n, g^n) = \frac{2e\alpha^{\alpha}(1-\alpha)^{(1-\alpha)}}{1+\alpha}$$

Efficient solution:

$$\max_g E(g) = 2[g^{\alpha}(2e - 2g^{n})^{(1-\alpha)}]$$
$$\frac{\partial E(g)}{\partial g} = 0 \Longrightarrow g^{*} = c^{*} = e\alpha$$

Equilibrium investment in children is less than efficient investment:

$$c^{*} - c^{n} = \frac{-e\alpha(1-\alpha)}{1+\alpha}$$