

## The CS Model

- Becker introduced transferable utility model of marriage market 30 years ago.
- It is the standard model of the marriage market but has never been estimated.
- Two problems deter estimation:
  1. Equilibrium transfers unobserved.
  2. No natural ordering for spouses (E.g. Individuals differ by age, education, religion, ethnicity). But no structure means loss of identification.

# 1 Identification problem

- $I$  types of men and  $J$  types of women
- For each type of woman,  $I$  preference parameters to characterize her utility from each type of spouse and remaining single. For men and women, there are  $2 \times I \times J$  preference parameters.
- Researcher observes the quantity of each type of women (men) in the marriage market,  $f_j$  ( $m_i$ ) for type  $j$  ( $i$ ) women (men) ( $I + J$  observations), and the quantity of type  $i$  men married to type  $j$  women,  $\mu_{ij}$  ( $I \times J$  observations). Total number of observables are  $I + J + I \times J$ .
- For  $I, J > 2$ , number of observables are less than the number of unknown preference parameters.

## 2 Marriage matching function

- Reduced form approach.
- $M$  vector with element  $m_i$ .  $F$  vector with element  $f_j$ .  $\Pi$  vector of parameters.
- A marriage matching function is an  $I \times J$  matrix  $\mu(M, F; \Pi)$  whose  $i, j$  element is  $\mu_{ij}$ :

$$\sum_l \mu_{lj} + \sum_{i=1}^I \mu_{ij} = f_j \quad \forall j \quad (1)$$

$$\sum_k \mu_{ik} + \sum_{j=1}^J \mu_{ij} = m_i \quad \forall i \quad (2)$$

$$\sum_l \mu_{lj}, \sum_k \mu_{ik}, \mu_{ij} \geq 0 \quad \forall i, j \quad (3)$$

- Zero spillover matching rule:

$$\mu_{ij}(M, F) = \mu_{ij}(m_i, f_j)$$

- Eg. Schoen's harmonic mean matching rule:

$$\mu_{ij}(M, F) = \frac{\alpha_{ij} m_i f_j}{m_i + f_j}$$

- This paper proposes and estimates a transferable utility model of the marriage market.
  
- There are three conceptual benefits for considering transferable utility models of the marriage market.
  1. Marriage market equilibrium must satisfy all the accounting constraints.
  2. Reduced form for equilibrium quantities of a market clearing model do not include equilibrium transfers.
  3. Transferable utility models can solve the identification problem.

### 3 Identification strategy

- Marital output of an  $i, j$  pair depends on  $i$  and  $j$ .
- $I \times J$  marital outputs plus  $I + J$  outputs of types being single.
- Transferable utility models maximize the sum of marital output in the society.

- Our marriage matching function:

$$\frac{\mu_{ij}}{(m_i - \sum_k \mu_{ik})(f_j - \sum_l \mu_{lj})} = \Pi_{ij}$$

- Non-parametric marriage matching function with spillover effects which will fit any observed marriage distribution.

## 4 The model

- Let the utility of male  $g$  of type  $i$  who marries a female of type  $j$  be:

$$V_{ijg} = \tilde{\alpha}_{ij} - \tau_{ij} + \varepsilon_{ijg}, \quad \text{where} \quad (4)$$

$\tilde{\alpha}_{ij}$ : Systematic gross return to male of type  $i$  married to female of type  $j$ .

$\tau_{ij}$ : Equilibrium transfer made by male of type  $i$  to spouse of type  $j$ .

$\varepsilon_{ijg}$ : i.i.d. random variable with type I extreme value distribution.

- The payoff to  $g$  from remaining unmarried, denoted by  $j = 0$ , is:

$$V_{i0g} = \tilde{\alpha}_{i0} + \varepsilon_{i0g} \quad (5)$$

where  $\varepsilon_{i0g}$  is also an i.i.d. random variable with type I extreme value distribution.

Individual  $g$  will choose according to:

$$V_{ig} = \max_j [V_{i0g}, \dots, V_{ijg}, \dots, V_{iJg}] \quad (6)$$

Obtain quasi-demand function for  $j$  type spouse:

$$\begin{aligned} \ln \mu_{ij}^d &= \ln \mu_{i0}^d + \tilde{\alpha}_{ij} - \tilde{\alpha}_{i0} - \tau_{ij} \\ &= \ln \mu_{i0}^d + \alpha_{ij} - \tau_{ij} \end{aligned}$$

Expected gain to entering marriage market and marriage rate:

$$V_i = EV_{ig} = c + \tilde{\alpha}_{i0} + \ln\left(1 + \sum_j \exp(\alpha_{ij} - \tau_{ij})\right)$$

$$n_i = \ln\left(1 + \sum_j \exp(\alpha_{ij} - \tau_{ij})\right) = \ln\left(\frac{m_i}{\mu_{i0}^d}\right) \approx \frac{\sum_j \mu_{ij}^d}{m_i} = r_i$$

Quasi supply function of  $j$  type spouse:

$$\ln \mu_{ij}^s = \ln \mu_{0j}^s + \gamma_{ij} + \tau_{ij}$$

Market clearing

$$\mu_{ij} = \mu_{ij}^d = \mu_{ij}^s$$

Marriage matching function

$$\ln \mu_{ij} - \frac{\ln \mu_{i0} + \ln \mu_{0j}}{2} = \frac{\alpha_{ij} + \gamma_{ij}}{2} = \pi_{ij}$$

$$\frac{\mu_{ij}}{\mu_{i0}\mu_{0j}} = \Pi_{ij}$$

## 5 Empirical evidence

- The data from 1970 and 1980 *US Census* and *Vital Statistics*. We seek to explain the bivariate age distributions of marriages in 1970 and 1980.
- Type space consists of ages of individuals between 16-75.

	1970	1980	$\Delta$
• $M^t$	15.14 <i>mil</i>	22.07 <i>mil</i>	46%
$F^t$	18.61 <i>mil</i>	25.78 <i>mil</i>	38%
$\mu^t$	1.62 <i>mil</i>	1.69 <i>mil</i>	4.3%

Mar. rates, gains to marriage, fell between decade.

- Expand the type space to include education.
  1. Fit improved.
  2. Still unable to account for drop in mar. rates, gains to marriage, among young adults.
  3. Fractions of college graduates in 70 and 80 about the same for young adults.