

1 Positive assortative matching

A society has a high ability man, h , a high ability woman, H ,

a low ability man, l , and a low ability woman, L .

The Hh match produce $Y_{Hh} = 10$ units of marital output.

The Ll match produces $Y_{Ll} = 5$ units of marital output.

The Hl Lh matches produce $Y_{Hl} = Y_{Lh} = 7$ units each.

What is the equilibrium configuration of matches with transferable utilities?

A configuration matches is stable if no two individuals want to separate from their current matches to form new matches.

$$Y_{Hh} + Y_{Ll} = 10 + 5 = 15$$

$$Y_{Hl} + Y_{Lh} = 7 + 7 = 14$$

Is positive assortative matching where both spouses divide output equally stable?

In this case, a high ability spouse gets 5 and a low ability spouse gets 2.5.

Consider H breaking away from her match and marrying the l male. What must he get to be willing to leave his current wife?

He must get at least 2.5. How much is the H wife left?
She can get at most $7 - 2.5 = 4.5$

So she cannot benefit from leaving her h husband.

Can a l female benefit from leaving her L husband and marrying a h husband? Also no.

So positive assortative matching is stable.

Is negative assortative matching stable?

Assume that negative assortative matching is stable. Then

$$S_{Hl} + s_{Hl} = 7$$

$$S_{Lh} + s_{Lh} = 7$$

Let H woman consider offering the h man s_{Lh} to break away. So the most she can get for herself is

$$S_{Hh} = 10 - s_{Lh} \quad (1)$$

She will not break away if

$$S_{Hl} > S_{Hh} = 10 - s_{Lh}$$

$$7 - s_{Hl} > 10 - s_{Lh} \quad (2)$$

$$s_{Lh} - s_{Hl} > 3 \quad (3)$$

Now let the L woman consider offering the l man s_{Hl} to break away. She can get for herself

$$S_{Ll} = 5 - s_{Hl} \quad (4)$$

She will not break away if

$$S_{Lh} > S_{Ll} = 5 - s_{Hl} \quad (5)$$

$$7 - s_{Lh} > 5 - s_{Hl} \quad (6)$$

$$2 > s_{Lh} - s_{Hl} \quad (7)$$

(3) and (7) are contradictory. So at least one pair will break away which means negative assortative matching is not stable.

2 General result with transferable utilities

$$Y_{Hh} + Y_{Ll} \geq Y_{Hl} + Y_{Lh}$$

Then positive assortative matching is stable with transferable utilities.

$$Y_{Hh} + Y_{Ll} \leq Y_{Hl} + Y_{Lh}$$

Then negative assortative matching is stable with transferable utilities.

3 General result with non-transferable utilities

Assume each spouse gets half of marital output. Then if output is increasing in spousal ability, will get positive assortative matching.

E.g.

The Hh match produce $Y_{Hh} = 10$ units of marital output.

The Ll match produces $Y_{Ll} = 5$ units of marital output.

The Hl Lh matches produce $Y_{Hl} = Y_{Lh} = 8$ units each.

With transferable utilities, we know negative assortative matching is stable.

What about non-transferable utilities? Negative assortative matching is not stable because the high ability spouse gets 4. They can break their current matches and match with each other and get 5 each.