Children as Public Goods

1 A single parent

- An endowment of e.

- Child's consumption be q

- Then her consumption is:

$$c = e - q \tag{1}$$

(1) is the budget constraint of the mother.

- Let mother's utility function be:

$$U(c, q) \tag{2}$$

- Substituting $e-q$ for c in the utility function, she will want to choose q to solve:

$$\max_q H(q) = U(e-q, q) \tag{3}$$

Let q^* be her optimal choice of child's consumption.

$$H_q(q^*) = -U_c(e-q^*, q^*) + U_q(e-q^*, q^*) = 0 \tag{4}$$

$$U_c(e-q^*, q^*) = U_q(e-q^*, q^*)$$

- If she increases q from q^* to $q^* + \Delta$ where Δ is a small amount, her increase in utility from the increase in child's consumption is approximately $U_q(e-q^*, q^*)\Delta$.

- When she increases q by Δ, she is substracting Δ from her own consumption. Her decrease in utility from the decrease in her own consumption is $U_c(e-q^*, q^*)\Delta$.

- (4) says that when $q = q^*$, the gain in utility from increasing her child's consumption by Δ is exactly equal to the loss in utility from decreasing her own consumption by Δ.

- Put another way, she cannot increase her utility by marginally deviating from q^*.

- Her optimal amount of own consumption is $c^* = e - q^*$ and the level of utility that she can achieve is $U^* = U(c^*, q^*)$.

2 Married parents

- Let the father's endowment be h.

- Let his utility function be:

$$V(g, q)$$

where $V(.,.)$ is concave and increasing in g, his consumption, and q, his child's consumption.

- The public good aspect of children within marriage is captured by q.

- Let the father offer the mother a reservation utility of U^r for marrying him. $U^r > U^*$

- Then the father's objective is to solve:

$$\max_{g,q} V(g, q)$$

subject to:

$$U(e+h-g-q,q) \geq U^r$$

- Write the father's constrained optimization problem as a Lagrangian problem:

$$L(g,q,\lambda) = V(g,q) + \lambda(U(e+h-g-q,q) - U^r)$$

where λ is the lagrange mutiplier for the problem.

- Let $\widehat{x}$ denote the optimal choice of x and for a function $Z(x_1,..,x_n)$, $\widehat{Z}$ denotes the value of $Z(\widehat{x}_1,..,\widehat{x}_n)$.

- The first order conditions are:

$$L_g = \widehat{V}_g - \lambda\widehat{U}_c = 0 \tag{5}$$
$$L_q = \widehat{V}_q - \lambda\widehat{U}_c + \lambda\widehat{U}_q = 0 \tag{6}$$
$$L_\lambda = \widehat{U} - U^r = 0 \tag{7}$$

(7) says that the mother will get her reservation utility.

(5) and (6) may be combined to get:

$$\widehat{V}_q + \frac{\widehat{U}_q}{\widehat{U}_c}\widehat{V}_g = \widehat{V}_g \tag{8}$$

where V_i is the father's marginal utility of consuming good i, $i = g, q$.

- Consider increasing $\widehat{q}$ by a small amount Δ. The increase in utility that the father gets from the increase in child consumption is $\widehat{V}_q\Delta$.

- The mother also gets an increase in utility of $\widehat{U}_q\Delta$.

- Since he is not interested in increasing her utility, he may lower her consumption by $\frac{\widehat{U}_q}{\widehat{U}_c}\Delta$ and leave her as well off as before.

- His utility will from the increased consumption will increase by $\left(\frac{\widehat{U}_q}{\widehat{U}_c}\Delta\right)\widehat{V}_g$. $\left(\frac{\widehat{U}_q}{\widehat{U}_c}\Delta\right)\widehat{V}_g$ is the gain from marriage.

- Since his wife is better off when he increases child's consumption, she is willing to compensate him for doing so. This compensation is missing if he is a single parent.

- So under marriage, his total change in utility from increasing q by Δ is the left hand side of (8) mutiplied by Δ.

- On the other hand, when he increases q by Δ, he decreases his own consumption by Δ. His fall in utility from the drop in own consumption is $\widehat{V}_g\Delta$.

- At the optimum, represented by (8), he is indifferent between an additional transfer of Δ to his child or not.

3 Free Riding within a Household

- The mother will choose c and leave $q^m = e - c$ for the child.

- The father will choose g and leave $q^f = h - g$ for the child.

- Total child's consumption will be $q = q^m + q^f = e - c + h - g$.

- If both parents act purely in their own self interest, what are the equilibrium allocations?

- We will use the concept of Nash Equilibrium from game theory to find the equilibrium allocation.

- Let q^{mn} be the equilibrium allocation of the mother to the child and q^{fn} be equilibrium allocation of the father to the same child.

- If the pair of equilibrium allocations $\{q^{mn}, q^{fn}\}$ constitutes a Nash Equilibrium, then $\{q^{mn}, q^{fn}\}$ must simultaneously satisfy

(1) $U(e - q^{mn}, q^{mn} + q^{fn}) \geq U(e - q^{m}, q^{m} + q^{fn})$ for all feasible q^{m}.

(2) $V(h - q^{fn}, q^{mn} + q^{fn}) \geq V(h - q^{f}, q^{mn} + q^{f})$ for all feasible q^{f}.

- Condition (1) says that if q^{fn} is the equilibrium allocation of the father to the child, then the mother's best response is to choose q^{mn} as her allocation.

- Condition (2) says that if q^{mn} is the equilibrium allocation of the mother to the child, then the father's best response is to choose q^{fn} as his allocation.

- When (1) and (2) are satisfied simultaneously, $\{q^{mn}, q^{fn}\}$ is a pair of equilibrium allocations of this parental allocation game.

- A justification for focussing on Nash Equilibrium for the parental allocation game is as follows. If the mother chooses q^{mn}, then the father can do no better than choosing q^{fn} and vice versa. So if the parents choose $\{q^{mn}, q^{fn}\}$, there is no incentive for either of them to deviate from their equilibrium allocation. Thus we should not be surprized to observe parents choosing $\{q^{mn}, q^{fn}\}$.

- So from a predictive point of view, we expect to see parents choosing $\{q^{mn}, q^{fn}\}$ rather than other allocations.

- Let q^{mn} be the mother's allocation. Then the father will solve

$$\max_{q^f} V(h - q^f, q^{mn} + q^f) \tag{9}$$

- If his optimal allocation to the child (best response), q^{fn} is strictly larger than zero, it will satisfy the first order condition:

$$V_g(h - q^{fn}, q^{fn} + q^{mn}) = V_q(h - q^{fn}, q^{fn} + q^{mn}) \tag{10}$$

- When the father contributes q^{fn} to the child, the mother's optimal choice (best response), q^{mn}, if strictly larger than zero will satisfy:

$$U_c(e - q^{mn}, q^{mn} + q^{fn}) = U_q(e - q^{mn}, q^{mn} + q^{fn}) \tag{11}$$

- If q^{mn} and q^{fn} are strictly larger than zero, then the equilibrium allocation is obtained by solving (10) and (11) for q^{mn} and q^{fn}.

- Consider a transferable utility example where $e = h$, the per period utility of the mother is $U = c + \ln(2e - c - g)$ and the per period utility of the father is $V = g + \ln(2e - c - g)$.

- Solving (10) and (11), there are multiple per period Nash equilibria all of which satisfy:

$$q^{fn} + q^{mn} = 1$$

- In general, the Nash Equilibrium is inefficient.

4 Efficient Allocations Again

- In example above, consider the static Nash equilibrium with symmetric contributions, $q^{fn} = q^{mn} = \frac{1}{2}$.

- If each parent treats each period as separate and chooses the static Nash Equilibrium contribution in each period, the per period equilibrium level of utilities are:

$$U^n = e - \frac{1}{2} \tag{12}$$

$$V^n = e - \frac{1}{2} \tag{13}$$

- The efficient level of child consumption, from (??), is $\widehat{q} = 2$.

- Assuming symmetric contributions, per period utility under the efficient contribution level is:

$$\widehat{V} = \widehat{U} = e - 1 + \ln 2 > e - \frac{1}{2} \qquad (14)$$

- Let both parents have infinite horizons and let the discount factor be δ, $0 < \delta < 1$.

- The present value of utility for each parent from the efficient action is:

$$\sum_{i=0}^{\infty} \delta^i \widehat{U} = \frac{\widehat{U}}{1-\delta} = \frac{e - 1 + \ln 2}{1-\delta}$$

- If the father deviates from the efficient action in a period, let the mother punish the father by choosing q^{mn} thereafter and vice versa.

- If the mother chooses q^{mn} every period, the best response for the father is also to choose q^{fn} and vice versa.

- In otherwords, if one parent deviates from the efficient action in a period, both parents will revert to the per period Nash Equilibrium behavior thereafter.

- If the mother chooses the efficient level and the father chooses to optimally deviate for a period, his return in that period is e which is larger than $e - 1 + \ln 2$, his return from also acting efficiently. But both parties will revert to static Nash Equilibrium behavior thereafter.

- The present value of utility for the husband from deviating in a single period is:

$$V^d = e + \delta \sum_{i=0}^{\infty} \delta^i (e - \frac{1}{2}) \quad (15)$$
$$= e + \frac{\delta(e - \frac{1}{2})}{1 - \delta}$$

- The father will not choose to cheat if:

$$\frac{e - 1 + \ln 2}{1 - \delta} > e + \frac{\delta(e - \frac{1}{2})}{1 - \delta}$$

That is, if $\delta > 2(1 - \ln 2)$, neither spouse will find it profitable to deviate from the efficient investment level in each period.

5 Empirical evidence

- Waite (1995) summarizes many studies. These studies overwhelmingly conclude that children who grow up with two parents fare better than those with one parent.

- While consistent with the hypothesis that children with two parents do better, there are other differences between the environments of the two groups of children other than the number of parents present. These other differences also affect the socioeconomic outcomes of children. Put another way, individuals who are ill equipped to be parents may also be less able to marry or remain married when they have children. In that case, children who grow up with married parents are more likely to have parents who are also better equipped to bring up children well. Then if we compare children growing up with one parent or two parents, we are comparing both the difference between household structures (married or otherwise) and parenting abilities. We cannot disentangle the effects of parenting abilities from the effects of household structures.

- The ideal empirical experiment would be to compare children growing up in different household structures, holding parenting abilities constant. Corak (2001) uses the death of a parent to approximate this experiment. Consider two groups of otherwise statistically identical children: (i) Children growing up with two parents, and (ii) children growing up families where one parent has died. Let the death of a parent be unrelated to parental ability. Then the socioeconomic difference between children in group (i) and group (ii) captures the change in socioeconomic outcomes due to a change in household structure from two parents to one parent, holding parental ability constant. Corak showed that the death of a parent when a child is young lowered the average annual earnings of that adult child by about 5%. These adult children were also 30% more likely to receive income assistance from the government. Thus Corak's evidence imply, all other things equal, that there is a significant benefit to children growing up in two parents households.

- Parents may disagree about how much resources to allocate to their children. Many studies show that resources allocated to children change when the distribution of labor earnings between husbands and wives change, holding total labor earnings of the family constant. In general, children gain resources when the mother earns relatively more (Strauss and Thomas 1995).

- One criticism of these studies is that when we compared resources of children across different families with different distribution of labor earnings, the different distribution of labor earnings may also be correlated with different parental preferences.

- Lundberg, Pollak and Wales (1997) deal with this criticism by comparing outcomes to British children when the British government changed a government monetary transfer from the father to the mother, holding total monetary transfer to the family fixed. They showed that children obtained more resources after mothers obtain the transfers rather than fathers.

- What about efficient versus non-efficient allocations within the family? The empirical evidence on this question is mixed.