1 Slope of an indifference curve

- Let a utility function of two goods, c and q, be:

$$U(c, q)$$

- An indifference curve is:

$$U(c, q) = U_0$$

  where U_0 is a specific level of utility.

- Along an indifference curve, consider increasing c by dc marginal units and q by dq marginal units such that the level of utility remains the same. Then we must have:

$$U_c dc + U_q dq = 0 \tag{1}$$

where U_c is the marginal utility of c and U_q is the marginal utility of q.

- Rearrange (1) to get the slope of the indifference curve:

$$\frac{dc}{dq} = -\frac{U_q}{U_c} \tag{2}$$

which is also known as the marginal rate of substitution between c and q.

- In the single parent case, the slope of the budget constraint is

$$\frac{dc}{dq} = -1 \tag{3}$$

- At the optimum, equate the slope of the indifference curve to the slope of the budget constraint to get:

$$\begin{aligned} -\frac{U_q(c^*, q^*)}{U_c(c^*, q^*)} &= -1 \\ U_c(c^*, q^*) &= U_q(c^*, q^*) \end{aligned}$$

2 Children as Public good

- Family endowment is

$$h + e - g - c - q = 0$$

- Mother's utility is

$$U(c, q) = U(h + e - g - q, q)$$

- Let the mother's reservation utility be U_r. Consider increasing g by dg marginal units and q by dq marginal units such that her level of utility remains the same. Then we must have:

$$U_c(-dg - dq) + U_q dq = 0 \qquad (4)$$

- (4) may be rewritten to get:

$$\begin{aligned} \frac{dg}{dq} &= \frac{U_q - U_c}{U_c} \\ &= \frac{U_q}{U_c} - 1 \\ &> -1 \end{aligned}$$

- The father's utility function is:

$$V(g, q)$$

- His indifference curve is:

$$\frac{dg}{dq} = -\frac{V_q}{V_g}$$

- If he chooses allocations to maximize his utility subject to giving the mother her reservation utility, U_0, his optimal choices, $\widehat{g}$ and $\widehat{q}$ will equate his marginal rate of substitition with her marginal rate of substitution:

$$\begin{aligned} \frac{\widehat{U}_q}{\widehat{U}_c} - 1 &= -\frac{\widehat{V}_q}{\widehat{V}_g} \\ \widehat{V}_q + \frac{\widehat{U}_q}{\widehat{U}_c}\widehat{V}_g &= \widehat{V}_g \end{aligned}$$